INSTRUCTIONS

1. This is a closed book and closed notes examination. You are provided with an equation sheet and all the needed property tables.
2. Do not hesitate to ask the instructor if you do not understand a problem statement.
3. Start each problem on the same page as the problem statement. Write on only one side of the page. Materials on the back side of the page will not be graded. There are blank pages following problems 2 and 3 for your work.
4. Put only one problem on a page. Another problem on the same page will not be graded.
5. Identify system boundary, list relevant assumptions, and provide solution with appropriate basic equations for problems 2 and 3. Do not specify “Given” or “Find” on these problems.
6. If you give multiple solutions, you will receive only a partial credit although one of the solutions might be correct. Delete the solution you do not want graded.
7. For your own benefit, please write clearly and legibly. Maximum credit for each problem is indicated below.
8. After you have completed the exam, at your seat put your papers in order. This may mean that you have to remove the staple and re-staple. Do not turn in loose pages.
9. Once time is called you will have three minutes to turn in your exam. Points will be subtracted for exams turned in after these three minutes.

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Problem 1 (25 points) Answer the following questions. For Problem 1 only, assumptions need not be stated. No credit will be given without correct justification even if the answer is correct.

(a) Consider isothermal heating processes for two different fluids.

What happens to internal energy during the heating process for the following types of fluids? (6 points)

<table>
<thead>
<tr>
<th>Type</th>
<th>Increases</th>
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<tr>
<td>Ideal Gas</td>
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<td></td>
<td>+1</td>
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<tr>
<td>Saturated Vapor</td>
<td>+1</td>
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</table>

For ideal gas, internal energy depends only on temperature or \( \Delta u = \int C_p \, dT = 0 \) +2

Saturated vapor becomes superheated vapor upon isothermal heating; pressure must change either in non-flow or flow system +2

(b) In a certain process, the temperature of an ideal gas increases by 800 K. The change in specific internal energy and specific enthalpy during this process is measured to be 1600 kJ/kg and 1200 kJ/kg, respectively.

Are the given measurements possible? Justify your answer with equation(s). (6 points)

Yes +1  
No +1  
Insufficient Information +1

\[ \Delta u = 1600 \text{ kJ/kg} \]

\[ \Delta h = 1200 \text{ kJ/kg} \]

\[ h = u + PV \] For an ideal gas: \( PV = RT \) \( \Rightarrow h = u + RT \) \( \Rightarrow \Delta h = \Delta u + R \Delta T \) \( \Rightarrow \Delta h > \Delta u \) +1 +1 +2  

Impossible +1
Problem 1 (continued)

(c) An ideal gas flows steadily through a rigid, constant area duct. Its temperature increases from inlet to exit \((T_2 > T_1)\) and its pressure decreases from inlet to exit \((P_2 < P_1)\) due to friction.

What happens to its density from inlet to exit? Justify your answer with equation(s). (4 points)

- Increases
- Decreases \[+1\]
- Remains Same

For an ideal gas: \(Pv = RT\) \(\Rightarrow\) \(\rho = \frac{1}{v} = \frac{P}{RT}\) \(\Rightarrow\) density decreases since pressure decreases and temperature increases \[+3\]

What is the heat transfer for the duct? Justify your answer with equation(s). (4 points)

- Into the system \[+1\]
- Out of the system
- No heat transfer

\[
\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_i - h_f) + \left( \frac{v_i^2 - v_f^2}{2} \right) + g (z_i - z_f) \right]
\]

Steady \(\Rightarrow\) \(\frac{dE_{cv}}{dt} = 0\); Rigid \(\Rightarrow\) \(\dot{W}_{cv} = 0\); Constant area \(\Rightarrow\) \(\forall_1 = \forall_2\)

For ideal gas, enthalpy depends only on temperature \(\Rightarrow h_f > h_i\) when \(T_2 > T_1\)

\(\dot{Q}_{cv} = \dot{m} (h_f - h_i) > 0\) \[+3\]

(d) Which of the following most closely approaches an ideal gas state? (5 points)

- \(P << P_{critical}, T << T_{critical}\)
- \(P >> P_{critical}, T << T_{critical}\)
- \(P << P_{critical}, T > T_{critical}\)

Ideal gas is low density state i.e. applicable at low pressure and high temperature \[+5\]
**Problem 2 (25 points)** Air flowing steadily at the rate of 10 kg/s is compressed from 18.85 bar and 133 K (State 1) to 32.05 bar and 152.95 K (State 2) in a cryogenic application. Air exits the compressor with a velocity of 100 m/s.

Critical pressure of air = 37.7 bar
Critical temperature of air = 133 K
Molecular weight of air = 28.97 kg/kmol

Calculate the flow area (cm²) at the exit of the compressor.

Identify system boundary, list assumptions, and provide solution with basic equations.

**Assumptions**
- Steady state
- One-dimensional flow

**Basic Equation(s)**
\[
\frac{dm}{dt} = \sum_{i} \dot{m}_i - \sum_{e} \dot{m}_e
\]

**Solution**
Reduced pressure of air at the compressor exit: 
\[
P_R = \frac{P}{P_{\text{critical}}} = \frac{32.05 \text{ bar}}{37.7 \text{ bar}} = 0.85
\]

Reduced temperature of air at the compressor exit: 
\[
T_R = \frac{T}{T_{\text{critical}}} = \frac{152.95 \text{ K}}{133 \text{ bar}} = 1.15
\]

Figure A-1 for compressibility factor: 
\[
Z_{\text{air}} = 0.8 \Rightarrow \text{air is not ideal gas} \Rightarrow P_R v_2 = Z_{\text{air}} R_{\text{air}} T_2
\]
Problem 2 (continued)

Specific volume of air at the compressor exit: 
\[ v_2 = \frac{Z_2 R_{\text{air}} T_2}{P_2} \]

\[ R_{\text{air}} = \frac{8.314 \text{ kJ/kmol-K}}{28.97 \text{ kg/kmol}} = 0.287 \text{ kJ/kg-K} \]

\[ v_2 = \frac{0.8 \times 0.287 \text{ kJ/kg-K} \times 152.95 \text{ K}}{(32.05 \times 100) \text{ kPa}} = 0.011 \text{ m}^3/\text{kg} \]

Considering mass balance for the air compressor: 
\[ \dot{m}_{\text{air}} = \dot{m}_1 = \dot{m}_2 = \frac{A_2 v_2}{v_2} \]

Flow area at the compressor exit: 
\[ A_2 = \frac{\dot{m}_{\text{air}} v_2}{\rho} = \frac{10 \text{ kg/s} \times 0.011 \text{ m}^3/\text{kg}}{100 \text{ m/s}} = 1.1 \times 10^{-3} \text{ m}^2 = 11 \times 10^{-4} \text{ m}^2 \]

\[ A_2 = 11 \text{ cm}^2 \]
Problem 3 (50 points) A power cycle using steam/water as the working substance is shown below. Steam/water flows steadily through the system at the rate of 25 kg/s. Heat is added in the boiler and rejected from the condenser. The turbine produces power while the pump consumes power. Steam enters the turbine at 100 bar and 400°C (State 1) and expands to 0.1 bar and 90% quality (State 2). Saturated liquid at 0.1 bar (State 3) leaves the condenser and it is pumped to the boiler pressure (State 4) in an isothermal process.

(a) Calculate the power developed (kW) by the turbine.
(b) What is the rate of heat transfer (kW) for the boiler?
(c) Apply an overall energy balance for the power cycle as the system and verify that the overall energy balance is satisfied with appropriate calculations.
(d) Calculate thermal efficiency (%) of the power cycle.

Identify system boundary, list assumptions, and provide solution with basic equations.

Assumptions
- Steady state
- One-dimensional flow
- Ignore KE and PE changes
- Boiler and condenser: No work \( W_{cv} = 0 \)
- Turbine and pump: No heat transfer \( Q_{cv} = 0 \)
- Incompressible liquid water in pump
Problem 3 (continued)

Basic Equation(s)
\[ \frac{dm}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e \Rightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_4 = \dot{m} \]
\[ \frac{dE}{dt} = \dot{Q}_{CV} - \dot{W}_{CV} + \sum_i \dot{m}_i \left( h_i + \frac{V_i}{2} + gZ_i \right) - \sum_e \dot{m}_e \left( h_i + \frac{V_i}{2} + gZ_i \right) \]

Solution

(a) Considering energy balance for the turbine (CV I), power developed by the turbine:
\[ \dot{W}_{turbine} = \dot{m} (h_1 - h_2) \]

State 1: \( P_1 = 100 \) bar, \( T_1 = 400^\circ C \)
Table A-3 for water: \( T_{sat}(P_1) = 311.1^\circ C \) \( \Rightarrow T_1 > T_{sat}(P_1) \) \( \Rightarrow \) superheated vapor (SHV)
Table A-4 for superheated water vapor: \( h_1 = 3096.5 \frac{kJ}{kg} \)

State 2: \( P_2 = 0.1 \) bar, \( x_2 = 0.9 \) \( \Rightarrow \) saturated liquid-vapor mixture (SLVM)
Table A-3 for water: \( h_f (P_2) = 191.83 \frac{kJ}{kg} \) and \( h_{fg} (P_2) = 2392.8 \frac{kJ}{kg} \)
\[ h_2 = h_f (P_2) + x_2 h_{fg} (P_2) = 191.83 \frac{kJ}{kg} + 0.9 \times 2392.8 \frac{kJ}{kg} = 2345.35 \frac{kJ}{kg} \]

\[ \dot{W}_{turbine} = 25 \frac{kJ}{s} \times (3096.5 - 2345.35) \frac{kJ}{kg} \Rightarrow \dot{W}_{turbine} = 18,778.8 \text{ kW} \]

(b) Considering energy balance for the boiler (CV II), rate of heat transfer for the boiler:
\[ \dot{Q}_{CV} = \dot{Q}_{boiler} = \dot{m} (h_3 - h_4) \]

State 4: \( P_4 = 100 \) bar, \( T_4 = T_3 = T_{sat}(P_3) = 45.81^\circ C \)
Table A-3 for water: \( T_{sat}(P_4) = 311.1^\circ C \) \( \Rightarrow T_4 < T_{sat}(P_4) \) \( \Rightarrow \) sub-cooled liquid

For incompressible liquid water in the pump: \( \Delta h = \Delta u + v\Delta P = C_{water} x + v\Delta P \)
Problem 3 (continued)

State 3: \( P_3 = 0.1 \text{ bar}, \text{saturated liquid} \)

Table A-3 for water: \( v_3 = v_f \left( P_3 \right) = 1.0102 \times 10^{-3} \frac{\text{m}^3}{\text{kg}} \) and \( h_3 = h_f \left( P_3 \right) = 191.83 \frac{\text{kJ}}{\text{kg}} \)

\[
h_4 = 191.83 \frac{\text{kJ}}{\text{kg}} + 1.0102 \times 10^{-3} \frac{\text{m}^3}{\text{kg}} \left( 100 - 0.1 \right) \times 100 \text{ kPa} = 201.92 \frac{\text{kJ}}{\text{kg}}
\]

\[
\dot{Q}_{\text{boiler}} = 25 \frac{\text{kg}}{\text{s}} \times (3096.5 - 201.92) \frac{\text{kJ}}{\text{kg}} \Rightarrow \dot{Q}_{\text{boiler}} = 72,364.5 \text{ kW}
\]

(c) Considering energy balance for the pump (CV III), power consumed by the pump:

\[
\dot{W}_{\text{CV}} = \dot{W}_{\text{pump}} = \dot{m} \left( h_3 - h_4 \right) = 25 \frac{\text{kg}}{\text{s}} \times (191.83 - 201.92) \frac{\text{kJ}}{\text{kg}} = -252.3 \text{ kW}
\]

Considering energy balance for the condenser (CV IV), rate of heat transfer for the condenser:

\[
\dot{Q}_{\text{CV}} = \dot{Q}_{\text{condenser}} = \dot{m} \left( h_3 - h_2 \right) = 25 \frac{\text{kg}}{\text{s}} \times (191.83 - 2345.35) \frac{\text{kJ}}{\text{kg}} = -53,838 \text{ kW}
\]

Net rate of heat transfer for the power cycle:

\[
\dot{Q}_{\text{cycle}} = \dot{Q}_{\text{boiler}} + \dot{Q}_{\text{condenser}} = 72,364.5 \text{ kW} - 53,838 \text{ kW} = 18,526.5 \text{ kW}
\]

Net power output for the power cycle:

\[
\dot{W}_{\text{cycle}} = \dot{W}_{\text{turbine}} + \dot{W}_{\text{pump}} = 18,778.8 \text{ kW} - 252.3 \text{ kW} = 18,526.5 \text{ kW}
\]

Considering energy balance for the entire power cycle (CV V): \( \dot{Q}_{\text{cycle}} = \dot{W}_{\text{cycle}} \) is valid

(d) Thermal efficiency of the power cycle:

\[
\eta_{\text{thermal}} = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_m} = \frac{\dot{W}_{\text{turbine}} - |\dot{W}_{\text{pump}}|}{\dot{Q}_{\text{boiler}}} = \frac{18,526.5 \text{ kW}}{72,364.5 \text{ kW}} \Rightarrow \eta_{\text{thermal}} = 25.6\%
\]