Exam 3

This exam is open book/open notes.

Use the standard solution format for Problems 2-4, except that Given and Find are already listed. A system sketch is also provided for each problem. Show your work clearly and list all needed assumptions and basic equations. Show clearly your control volume for each problem.

Put your name and Thermo Number on top of every page turned in and make sure the pages are in the correct order.

Work only on the front side of each page. If you need extra space, work on the extra yellow paper that is provided and clearly indicate which problem is being solved on each extra sheet of paper.

Place your final answers in the boxes provided for each part.

Erase or mark through work that you do not wish to have graded. You will receive only partial credit if multiple calculations are presented for the same problem even if one is completely correct.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Score</th>
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<tbody>
<tr>
<td>1</td>
<td>/10</td>
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<tr>
<td>2</td>
<td>/35</td>
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<td>3</td>
<td>/20</td>
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<td>4</td>
<td>/35</td>
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<td>Total</td>
<td>/100</td>
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(a) A reversible heat engine operates between a high-temperature reservoir at 1000°C and a low-temperature reservoir at 0°C. The rate of heat input from the high-temperature reservoir is 100 kW. What is the rate of heat rejection to the low-temperature reservoir? (5 pts)

\[ \dot{Q}_C = 21.4 \text{ kW} \]

\[ \eta_{rev} = 1 - \frac{\dot{Q}_C}{\dot{Q}_H} \]

\[ = 1 - \frac{T_C}{T_H} = 1 - \frac{273}{1273} = 0.786 \]

\[ Q_H = W + Q_C \]

\[ W = \eta_{rev} Q_H = 78.6 \text{ kW} \]

\[ Q_C = Q_H - W = 21.4 \text{ kW} \]

(b) An irreversible refrigeration cycle operates between a high-temperature reservoir at 40°C and a low-temperature reservoir at -20°C. The manufacturer claims that the COP for the refrigeration cycle is 3.8. Is this COP possible or impossible to achieve? Justify your answer with a calculation of the maximum theoretical COP for the cycle. (5 pts)

COP is possible to achieve.

\[ \text{COP}_{rev, ref} = \frac{Q_C}{Q_H - Q_C} = \frac{T_C}{T_H - T_C} = \frac{253}{313 - 253} \]

\[ \text{COP}_{rev, ref} = 4.21 \]

Since COP_{actual} < COP_{rev}, the manufacturer's claim is possible.
Problem 2: Given: Air is contained in a well-insulated piston-cylinder device. At state 1, the initial volume of the air is 0.8 m³, the initial temperature is 310 K, and the initial pressure is 150 kPa. The air is compressed in a reversible, adiabatic process until the final volume at state 2 is 0.08 m³. The piston is then locked in place and paddle wheel work is performed on the air until the air pressure inside the cylinder doubles (\(P_3 = 2P_2\)).

Find:
a. Complete the table of properties below for each state of the system. Use the ideal gas air table to analyze the process. Do not interpolate, pick the closest value.
b. Calculate the compression work \(W_{12}\) in kJ.
c. Calculate the paddle-wheel work \(W_{23}\) in kJ.
d. Calculate the overall entropy change \(S_2 - S_1\) in kJ/K.

<table>
<thead>
<tr>
<th>State</th>
<th>(T) (K)</th>
<th>(P) (kPa)</th>
<th>(v) (m³/kg)</th>
<th>(U) (kJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>310</td>
<td>150</td>
<td>0.593</td>
<td>2486.5</td>
</tr>
<tr>
<td>2</td>
<td>750</td>
<td>3630</td>
<td>0.0593</td>
<td>744.6</td>
</tr>
<tr>
<td>3</td>
<td>1500</td>
<td>7260</td>
<td>0.0593</td>
<td>1626</td>
</tr>
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\[(b) W_{12} = -4466.1 \text{ kJ} \quad (c) W_{23} = -861 \text{ kJ} \]

\[(d) S_2 - S_1 = +0.808 \text{ kJ/K} \]

Assumptions:
1. \(Q = 0\)
2. Air is ideal gas
3. \(\Delta KE = 0\)
4. \(\Delta PE = 0\)
5. Closed system
6. No friction

Basic Equations:
Conservation of Mass: \(m = m_1 = m_2 = m_3\)
Conservation of Energy: \(Q - W = \Delta U\)
Entropy Change, Ideal Gas: \(S_f - S_i = S_f^0 - S_i^0 - R \ln \left( \frac{P_f}{P_i} \right)\)
Ideal Gas EOS: \( P \Delta V = mRT \)
\( R = 0.287 \text{ kJ/kg} \cdot \text{K} \)

Isentropic Process: \( \frac{V_{r2}}{V_{r1}} = \frac{V_2}{V_1} \quad \frac{P_{r2}}{P_{r1}} = \frac{P_2}{P_1} \)

Solution:
State 1: \( T_1 = 310 \text{ K} \quad P_1 = 150 \text{ kPa} \)
\( V_1 = 0.8 \text{ m}^3 \quad m = \frac{P_1 V_1}{RT_1} = \frac{(150 \text{ kJ/m}^3/\text{kg})(0.8 \text{ m}^3)}{(0.287 \text{ kJ/kg} \cdot \text{K})(310 \text{ K})} = 1.349 \text{ kg} \)
\( V_1 = V_1/m = (0.8 \text{ m}^3/1.349 \text{ kg}) = 0.593 \text{ m}^3/\text{kg} \)
\( U_1 = mU_1 = (1.349 \text{ kg})(721.25 \text{ kJ/kg}) = 298.5 \text{ kJ} \)

State 2: \( S_2 = S_1 \quad \frac{V_2}{V_1} = 0.1 = \frac{V_{r2}}{V_{r1}} \)
\( V_{r2} = 0.1V_{r1} = (0.1)(577.3) = 57.73 \text{ m}^3 \)
\( \Rightarrow T_2 = 750 \text{ K} \quad U_2 = mU_2 = (1.349)(551.99) = 744.6 \text{ kJ} \)
\( S_2 = 2.6474 \text{ kJ/kg} \cdot \text{K} \)

State 3: \( V_2 = V_3 \quad P_3 = 2P_2 \quad T_3 = 2T_2 = 1500 \text{ K} \)
\( U_3 = mU_3 = (1.349)(1205.4) = 1626 \text{ kJ} \)
\( S_3 = 3.4452 \text{ kJ/kg} \cdot \text{K} \)

\( W = -(U_2 - U_1) = U_1 - U_2 = 298.5 - 744.6 = -446.1 \text{ kJ} \)
\( W_3 = U_2 - U_3 = 744.6 - 1626 = -881 \text{ kJ} \)
\( S_3 - S_1 = S_3 - S_2 = S_3^0 - S_2^0 = R \ln \left( \frac{P_3}{P_2} \right) \)
\( = 3.4452 - 2.6474 \approx 0.8599 \text{ kJ/kg} \cdot \text{K} \)
\( S_3 - S_1 = m(S_3 - S_1) = (1.349 \text{ kg})(0.8599 \text{ kJ/kg} \cdot \text{K}) = 1.108 \text{ kJ/kg} \cdot \text{K} \)
Problem 3: Given: A copper block with a mass of 10 kg and an initial temperature of 900 K is brought into contact with an aluminum block with a mass of 50 kg and an initial temperature of 200 K in a well-insulated enclosure. Let the copper block be labeled block A and the aluminum block be labeled block B. The specific heat for the copper is given by

\[ C_A(T) = \alpha + \beta T = 0.100 + 5.0 \times 10^{-4} T \quad \text{[}C_A\text{]} = \text{kJ/} (\text{kg} \cdot \text{K}) \quad T \text{ is abs temp in K} \]

The specific heat for the aluminum block is \( C_B(T) = C_A = 0.895 \text{ kJ/}(\text{kg} \cdot \text{K}) \), and can be assumed constant as a function of temperature. The final temperature of the blocks is determined to be 256 K from a first law analysis.

Find:

a. Calculate the total entropy change for the process. For this calculation, do not use an average specific heat for block A, include the temperature dependence in the calculation.

b. \[ S_2 - S_1 = +6.57 \frac{kJ}{K} \]

Assumptions:
1. Closed system
2. Incompressible solid
3. \( Q = 0 \)
4. \( W = 0 \)

Basic Equations: Incompressible solid

\[ S_2 - S_1 = \int_{T_1}^{T_2} \frac{C(T)}{T}dT \]
Solution:

\[ S'_2 - S'_{1} = m_A \int_{T_{1A}}^{T_2} \left( \frac{c_A(T)}{T} \right) dT + m_B c_B \ln \left( \frac{T_2}{T_{1B}} \right) \]

\[ = m_A \int_{T_{1A}}^{T_2} \left( \frac{\alpha}{T} + \beta \right) dT + m_B c_B \ln \left( \frac{T_2}{T_{1B}} \right) \]

\[ = m_A \alpha \ln \left( \frac{T_2}{T_{1A}} \right) + m_A \beta \left( T_2 - T_{1A} \right) + m_B c_B \ln \left( \frac{T_2}{T_{1B}} \right) \]

\[ = (10 \text{ kg})(0.1 \text{ kJ/kg-K}) \ln \left( \frac{256}{700} \right) + (10 \text{ kg})(5 \times 10^{-4} \text{ kJ/kg-K}) \]

\[ (256 - 900 \text{ K}) + (50 \text{ kg})(0.895 \text{ kJ/kg-K}) \ln \left( \frac{256}{700} \right) \]

\[ = -1.257 - 3.220 + 11.047 \frac{\text{kJ}}{\text{K}} \]

\[ = \boxed{+6.57 \frac{\text{kJ}}{\text{K}}} \]

Note:

\[ \alpha = \frac{\text{kJ}}{\text{K} \cdot \text{kg}} \quad \beta = \frac{\text{kJ}}{\text{K}^2} \]
Problem 4: Given: An adiabatic steam turbine operates at steady state with water as the working fluid. At the turbine inlet, section 1, the pressure is 20 MPa and the temperature is 700°C. At the turbine exit, section 2, the pressure is 30 kPa. The isentropic efficiency of the turbine is 0.80. The power output from the turbine is 20 MW. Neglect kinetic and potential energy changes.

Find:
(a) What is the state of the fluid at the turbine exit?  
(b) Determine the exit temperature (°C). 
(c) Draw the process on a T-s diagram, indicating states 1, 2, and 2s. 
(d) Calculate mass flow rate in kg/s.

Basic Equations:

\[ \dot{m} \text{in} = \dot{m} \text{out} \]

Conservation of Energy:

\[ \frac{dE}{dt} = 0 = a h' \dot{m} + \dot{W} + \dot{m} h_1 - \dot{m} h_2 \]

\[ \dot{W} = \dot{m} (h_1 - h_2) \]

\[ M_{turb} = \frac{h_1 - h_2}{h_1 - h_{2s}} \]

Solution: Steady 1: \( P_1 = 20 \text{ MPa} \) \( T_1 = 700°C \)

\( h_1 = 3809 \text{ kJ/kg} \) \( s_1 = 6.7993 \text{ kJ/kg-K} \)
State 25:
\[ P_{25} = 30 \text{ kPa} \quad T_{25} = T_1 \]
\[ S_f(30 \text{ kPa}) = 0.9439 \quad \frac{kJ}{K} \quad S_g(30 \text{ kPa}) = 7.7686 \quad \frac{kJ}{K} \]
\[ x_{25} = \frac{6.7493 - 0.9439}{7.7686 - 0.9439} = 0.858 \]
\[ h_{25} = (1-x_{25})h_f + x_{25}h_g = 0.142(289.72) + 0.858(7625.3) = 2294 \quad \frac{kJ}{kg} \]

State 2:
\[ n_{turb} = 0.80 = \frac{h_1 - h_2}{h_1 - h_{25}} \]
\[ h_2 = h_1 - 0.80(h_1 - h_{25}) = 3809 - (0.8)(3809 - 2294) \]
\[ h_2 = 2597 \quad \frac{kJ}{kg} \quad h_2 > h_f(30 \text{ kPa}) \]
\[ h_2 < h_g(30 \text{ kPa}) \Rightarrow \text{State 2 is SLV} \]
\[ x_2 = \frac{2597 - 289.72}{7625.3 - 289.72} = 0.988 \]

\[ T_2 = 69.10^\circ C \]
\[ W_t = 20 \text{ MW} = 20 \times 10^3 \frac{kJ}{s} = m(h_1 - h_2) \]
\[ m = \frac{W_t}{h_1 - h_2} = \frac{20 \times 10^3}{(3809 - 2597) \frac{kJ}{kg}} = 165 \frac{kg}{s} \]