Problem 1: (60/200 points)

The standard problem solving procedure is not required for problem 1, but to receive any credit for Problem 1 your answer must be supported by providing appropriate analysis, equations, tables, and/or charts.

(a) (15/200 points) An ideal gas has a molar mass of 31 kg/kmol and a specific heat at constant pressure of 28.5 kJ/(kmol·K). Find (i) the gas constant, in kJ/(kg·K) and (ii) the specific heat at constant volume, in kJ/(kg·K).

\[ R = \frac{R}{M} = \frac{8.314 \text{ kJ/} \text{kmol} \cdot \text{K}}{31 \text{ kg/} \text{kmol}} = 0.268 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \]

\[ C_v = C_p - R = \frac{28.5 \text{ kJ/} \text{kmol} \cdot \text{K}}{31 \text{ kg/} \text{kmol}} - 0.268 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 0.651 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \]

\[(i) \quad 0.268 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \quad (ii) \quad 0.651 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \]

(b) (20/200 points) Carbon dioxide gas, initially at a pressure of 15 psia and a temperature of 1500 R undergoes a polytropic compression with \( pV^n = \) constant, where \( n = 1.2 \). The amount of work done on the gas is 45 Btu/lbm of CO\(_2\). Determine the final temperature, in R. [\( W_b = \frac{(P_2V_2 - P_1V_1)}{(1 - n)} \) for a polytropic process.]

From Table A-1E, \( T_{\text{crit}} = 547.5 \text{ R} \quad T/T_{\text{crit}} > 2 \)

\[ \Rightarrow \text{ideal gas} \quad pV = mRT \]

\[ W = \frac{P_2V_2 - P_1V_1}{1-n} = mR \frac{T_2 - T_1}{1-n} \]

\[ T_2 = \frac{W(1-n)}{mR} + T_1 \]

\[ = \left(-45 \frac{\text{Btu}}{\text{lpm}}\right) \left(\frac{1-1.2}{0.04513 \frac{\text{Btu}}{\text{lpm} \cdot \text{R}}}\right) + 1500 \text{ R} \]

\[ = 1699.4 \text{ R} \]

\[ 1699.4 \text{ R} \]
(c). (25/200 points) A engineering student at Purdue is measuring the performance of a new heat engine working between a hot reservoir of 700 K and a cold reservoir of 300 K. The student measures the work out of the engine to be 100 kJ and the heat leaving the hot reservoir to be 167 kJ. Find: (i) What is the maximum thermal efficiency that any engine can have operating between these to temperatures. (ii) Using the student numbers, determine the thermal efficiency of this engine. (iii) Can the student's numbers be correct? Why?

\[
\text{(i)} \quad \eta_{\text{max}} = 1 - \frac{T_c}{T_H} = 1 - \frac{300 \text{K}}{700 \text{K}} = 0.571 \quad (57.1\%) \\
\text{(ii)} \quad \eta_{\text{student}} = \frac{W}{Q_{\text{in}}} = \frac{100 \text{kJ}}{167 \text{kJ}} = 0.6 \quad (60\%) \\
\text{(iii)} \quad \text{The numbers are incorrect, because} \\
\eta_{\text{student}} > \eta_{\text{max}} \text{ that is not possible!}
\]
The standard problem solving procedure is required for this problem.

Problem 2: (40/200 points)

Given: An isolated system consists of a closed aluminum vessel of mass 0.2 lbm containing 2 lbm of liquid water, each initially at 40°F, immersed in a 40-lbm bath of liquid water, initially at 70°F. The system is allowed to come to equilibrium. The specific heat of aluminum and water can be considered constant at 0.212 Btu/(lbm °F) and 1.0 Btu/(lbm °F) respectively.

Find: (a). The final temperature, in °F.
(b). The entropy changes, each in Btu/R, for the aluminum vessel and each of the liquid water masses.
(c). The amount of entropy produced, in Btu/R

System Sketch:

![System Sketch Diagram]

Assumptions:
- The aluminum and each of the liquid is incompressible
- Constant specific heat
- \( \Delta P.E = 0, \Delta K.E = 0 \)

Basic Equations:
\[
Q - W = \Delta U + \Delta K.E + \Delta P.E
\]
\[
\Delta U = \int C_v dT \quad \Delta S = C \ln \frac{T_2}{T_1}
\]
\[
\frac{dS}{dt} = \frac{Q_i}{U_j} + S
\]

Solution:
(a) \( M_1 C_w (T_f - T_{c1}) + M_a C_a (T_f - T_{c2}) + M_2 C_w (T_f - T_{c2}) = 0 \)
(b) \( T_f = \frac{M_1 C_w T_{c1} + M_a C_a T_{c2} + M_2 C_w T_{c2}}{M_1 C_w + M_a C_a + M_2 C_w} \)
(c) \( T_f = 68.54 °F \)
(Continued on Problem 2)

\( \Delta S_1 = m_1 C_w \ln \frac{T_f}{T_i} = 2 \, \text{lb} \cdot \left( \frac{1}{\text{lb}^\circ F} \right) \ln \frac{68.54 + 460}{40 + 460} \approx 0.1110 \, \frac{\text{Btu}}{\text{R}} \)

\( \Delta S_a = m_a C_a \ln \frac{T_f}{T_i} = 0.2 \, \text{lb} \cdot \left( \frac{1}{\text{lb}^\circ F} \right) \ln \frac{68.54 + 460}{40 + 460} \approx 0.0024 \, \frac{\text{Btu}}{\text{R}} \)

\( \Delta S_2 = m_2 C_w \ln \frac{T_f}{T_i} = 40 \, \text{lb} \cdot \left( \frac{1}{\text{lb}^\circ F} \right) \ln \frac{68.54 + 460}{70 + 460} \approx -0.1103 \, \frac{\text{Btu}}{\text{R}} \)

\[ \Delta S_1 = 0.1110 \, \frac{\text{Btu}}{\text{R}}, \quad \Delta S_a = 0.0024 \, \frac{\text{Btu}}{\text{R}}, \quad \Delta S_2 = -0.1103 \, \frac{\text{Btu}}{\text{R}} \]

\( \Delta S = \sigma \)

\[ \sigma = \Delta S_1 + \Delta S_a + \Delta S_2 = (0.1101 + 0.0024 - 0.1103) \, \frac{\text{Btu}}{\text{R}} \]

\[ \sigma = 0.0031 \, \frac{\text{Btu}}{\text{R}} \]
The standard problem solving procedure is required for this problem.

Problem 3: (50/200 points)

Given: In an open gas turbine running under Brayton cycle at steady state, air enters the compressor at 0.95 bar and 295K and exits at 5.69 bar. The air then passes through a heat exchanger before entering the turbine at 1080K, 5.69 bar. Air exits the turbine at 0.95 bar. The compressor and turbine operate adiabatically and kinetic and potential energy effects can be ignored. The specific heat of air cannot be assumed as constant. Do not use interpolation but the closest table values.

Find: (a) The net power developed by the plant, in kJ/kg, if the compressor and turbine operate without internal irreversibilities.
(b) If the heat source temperature in the heat exchanger is 1100 K and a control volume consisting of the heat exchanger and part of the heat source is considered, is the heat transfer process for the heat exchanger possible? Explain why.
(c) The net power developed by the plant, in kJ/kg, if the compressor and turbine isentropic efficiencies are 82 and 85%, respectively.
(d) Draw the cycle with and without internal irreversibilities in the p-V and T-s chart. Label the states on the charts accordingly.

System sketch:

Assumptions:
steady state, steady flow
\( \dot{Q} = 0 \) for turbine and compressor
\( \Delta KE = 0, \Delta PE = 0 \)
Air is ideal gas

Basic Equations:
\[
\left( \frac{p_2}{p_1} \right)_s = \frac{p_{r2}}{p_{r1}}, \quad n_{turbine} = \frac{W_{act}}{W_{isen}} \quad n_{comp} = \frac{W_{isen}}{W_{act}}
\]
\[
dm_{CV} = \sum_{in} m_{in} - \sum_{out} m_{out} \quad \Delta s = s_2 - s_1 - R \ln \left( \frac{p_2}{p_1} \right)
\]
\[
dE_{CV} = \dot{Q}_{CV} - \dot{W}_{CV} + \sum_{in} \left( h + \frac{v^2}{2} + g z \right)_{in} - \sum_{out} \left( h + \frac{v^2}{2} + g z \right)_{out}
\]
\[
dS_{CV} = \frac{\dot{Q}_i}{T_j} + \sum_{in} \frac{h_{in}}{T_{in}} - \sum_{out} \frac{h_{out}}{T_{out}} + \frac{\dot{S}}{T}
\]
Solution:
(a) From Table A-17, $h_1 = 295.17 \text{ KJ/kg}$  $P_1 = 1.3068$

$h_3 = 1137.89 \text{ KJ/kg}$  $P_3 = 155.2$

$P_{2} = P_{1} \left( \frac{P_{2}}{P_{1}} \right) = 1.3068 \left( \frac{5.69}{0.95} \right) = 7.827$

$h_{25} = 492.74 \frac{\text{KJ}}{\text{kg}}$

$P_{4} = P_{3} \left( \frac{P_{4}}{P_{3}} \right) = 155.2 \left( \frac{0.95}{5.69} \right) = 25.91$

$h_{45} = 691.82 \frac{\text{KJ}}{\text{kg}}$

$W_{\text{net}, s} = (h_3 - h_{45}) - (h_{25} - h_1)$

$= \left[ (1137.89 - 691.82) - (492.74 - 295.17) \right] \frac{\text{KJ}}{\text{kg}}$

$W_{\text{net}, s} = 248.5 \text{ KW/kg}$

(b) From $T_3$, $S_3^0 = 3.05608 \text{ KJ/kg K}$

From $P_{2}$, $S_{25} = 2.19876 \text{ KJ/kg K}$

$0 = \frac{Q_{\text{in}}}{T_{\text{source}}} + m (S_{25} - S_3) + \dot{\sigma}$

$\frac{\dot{\sigma}}{m} = (S_3^0 - S_{25} - R \ln \frac{P_3}{P_2}) - \frac{(h_3 - h_{25})}{T_{\text{source}}}$

$= (3.05608 - 2.19876) \frac{\text{KJ}}{\text{kg K}} - \frac{(1137.89 - 492.74)}{1100 \text{ K}} \frac{\text{KJ}}{\text{kg K}}$

$\frac{\dot{\sigma}}{m} = 0.27082 \frac{\text{KJ}}{\text{kg K}}$

$\frac{\dot{\sigma}}{m} > 0$ process possible
(Continued on Problem 3)

(c) \( \eta_c = \frac{h_{2S} - h_1}{h_2 - h_1} \)
\( \eta_t = \frac{h_5 - h_4}{h_5 - h_{4S}} \)

\( W_{net} = (h_3 - h_4) - (h_2 - h_1) \)
\( = \eta_t (h_3 - h_{4S}) - \left( \frac{h_{2S} - h_1}{\eta_c} \right) \)
\( = 0.85 (1137.89 - 691.82) \frac{kJ}{Kg} - \frac{(492.74 - 295.17) KJ}{Kg} \)

\( W_{net} = 138.2 \frac{kJ}{Kg} \)
The standard problem solving procedure is required for this problem.

Problem 4: (50/200 points)

Given: A vapor-compression heat pump system uses Refrigerant 134a as the working fluid. The refrigerant enters the compressor at 2.4 bar and 0°C. Compression is adiabatic to 9 bar, 60°C, and saturated liquid exits the condenser at 9 bar.

Find: (a). Complete the table on the next page.
(b). Draw the cycle in a T-s chart with a dome.
(c). The power input to the compressor, in kJ/kg.
(d). The heating capacity of the system, in kJ/kg.
(e). The coefficient of performance.

System sketch:

Assumptions:
- steady state, steady flow
- no pressure drop in condenser and evaporator
- \( \Delta P_E = 0, \Delta P_E = 0 \)

Basic Equations:

\[
COP_{HP} = \frac{Q_H}{W_{net.in}}
\]

\[
y = y_e + x (y_g - y_e)
\]

\[
\frac{dm_{cv}}{dt} = \sum_{in} m_{in} - \sum_{out} m_{out}
\]

\[
\frac{dE_{cv}}{dt} = Q_{cv} - W_{cv} + \sum_{in} m_{in} (h + \frac{V^2}{2} + g)_{in} - \sum_{out} m_{out} (h + \frac{V^2}{2} + g)_{out}
\]
(Continued on Problem 4)

Solution:
(a)

<table>
<thead>
<tr>
<th>State</th>
<th>$p$ (bar)</th>
<th>$T$ (°C)</th>
<th>$h$ (kJ/kg)</th>
<th>$s$ (kJ/kgK)</th>
<th>Quality or region</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.4</td>
<td>0</td>
<td>248.89</td>
<td>0.9399</td>
<td>SHV</td>
</tr>
<tr>
<td>2</td>
<td>9.0</td>
<td>60</td>
<td>293.21</td>
<td>0.9897</td>
<td>SHV</td>
</tr>
<tr>
<td>3</td>
<td>9.0</td>
<td>35.53</td>
<td>99.56</td>
<td>0.3656</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>2.4</td>
<td>-5.37</td>
<td>99.56</td>
<td>0.3821</td>
<td>0.281</td>
</tr>
</tbody>
</table>

For state 4, $h_4 = h_3 = 99.56 \frac{kJ}{kg}$

$h_f = 42.95 \frac{kJ}{kg}$  \hspace{1cm} \hspace{1cm} $h_g = 244.09 \frac{kJ}{kg}$

$s_f = 0.1710 \frac{kJ}{kgK}$  \hspace{1cm} \hspace{1cm} $s_g = 0.9222 \frac{kJ}{kgK}$

$x_4 = \frac{h_4 - h_f}{h_g - h_f} = \frac{99.56 - 42.95}{244.09 - 42.95} = 0.281$

$s_4 = s_f + x_4 (s_g - s_f) = 0.1710 + 0.281 (0.9222 - 0.1710)$

$s_4 = 0.3821 \frac{kJ}{kgK}$

(b)
(Continued from Problem 4)

(c) \( W_c = h_2 - h_1 \)
\[
= (293.21 - 248.89) \frac{\text{kJ}}{\text{kg}}
\]
\[
\boxed{W_c = 44.32 \frac{\text{kJ}}{\text{kg}}}
\]

(d) \( \theta_{out} = h_2 - h_3 \)
\[
= (293.21 - 99.56) \frac{\text{kJ}}{\text{kg}}
\]
\[
\boxed{\theta_{out} = 193.65 \frac{\text{kJ}}{\text{kg}}}
\]

(e) \( \text{COP}_{HP} = \frac{\theta_{out}}{W_c} = \frac{193.65}{44.32} = 4.37 \)