ME 200 Thermodynamics I
Spring 2014 Final Examination
Wed 5/7/14 8:00 – 10:00 AM
LILLY 1105, WTHR 200, LILLY G126

LAST NAME ______________________  FIRST NAME ______________________  Grade

Circle your division:
WTHR 200  WTHR 200  LILLY G126  LILLY 1105  LILLY 1105  WTHR 200
7:30 Joglekar  9:30 Wassgren  10:30 Gore  12:30 Chen  2:30 Woodlau  4:30 Sircar

Instructions:

1) This is a closed book/notes exam.
2) Write legibly.
3) You may use a TI-30XIIS calculator.
4) Write on only one side of the page. Work on the back side of the pages will not be graded.
5) Failure to show appropriate assumptions and basic equations for Problems 3 to 5 will result in loss of credit.
6) Providing a numerical answer without showing work will not receive credit.
7) If you give multiple solutions, you will receive only a partial credit although one of the solutions might be correct. Delete the solution(s) you do not want to be graded.
8) Place answers in the boxes provided in various parts of the questions.
9) Show units for all numerical answers.
10) If you are caught cheating you will get a zero for the exam and your name will be turned over to the Dean of Students.

<table>
<thead>
<tr>
<th>Question</th>
<th>TOTAL POSSIBLE SCORE</th>
<th>YOUR SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question 1</td>
<td>35 Points</td>
<td></td>
</tr>
<tr>
<td>Question 2</td>
<td>45 Points</td>
<td></td>
</tr>
<tr>
<td>Question 3</td>
<td>60 Points</td>
<td></td>
</tr>
<tr>
<td>Question 4</td>
<td>60 Points</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>200 Points</td>
<td></td>
</tr>
</tbody>
</table>
Question 1 (35 points): The standard solution format is not required for both parts of this question.

Question 1A (15 Points):

Helium (ideal gas with constant specific heat at temperatures of interest) in a piston-cylinder assembly undergoes two different processes for which the temperature rises from $T_1$ to $T_2$. The first process occurs at constant pressure and the second process occurs at constant volume. What is the ratio of the entropy change for the constant pressure process over that for the constant volume process?

\[
\Delta S = C_p \ln \frac{T_2}{T_1} - R \ln \frac{V_2}{V_1} \Rightarrow \frac{\Delta S_{\text{p}}}{\Delta S_{\text{v}}} = \frac{C_p}{C_v} = K
\]

\[
\frac{\Delta S_{\text{p}}}{\Delta S_{\text{v}}} = \frac{C_p}{C_v} = K
\]

Ratio = $C_p/C_v = K$

Question 1B (20 Points):

A fixed mass of CO$_2$ undergoes an isobaric process in a piston-cylinder assembly such that the temperature increases from 300 K to 500 K. How much heat in kJ/kmol is required during this process considering variable specific heat?

\[
m, T_1 = 300 \text{ K}, \quad T_2 = 500 \text{ K}
\]

\[
\Delta W = \Delta U
\]

\[
\Delta Q = \Delta U + \Delta W \Rightarrow \Delta Q = \Delta U + \nabla \Delta V = \Delta h - V \Delta P
\]

Since $\Delta P = 0$ for isobaric process

\[
\Delta Q = \Delta h = h_{500} - h_{300} = 17,678 - 9,431 = 8,247 \text{ kJ/kmol}
\]

\[
\Delta Q = 8,247 \text{ kJ/kmol}
\]
Question 2 (45 Points): The standard solution format is required for this question.

Given:
An inventor has developed a new process in which 0.5 lbₐ of air at 15 psi, and 80°F is compressed to 100 psi, and 440°F in an un-insulated piston-cylinder apparatus. She claims that the environment, which is maintained at 80°F, receives 10 Btu of heat during this process. Molar mass for air is 28.97 lbₐ/lbₐmol. The specific heats of air vary with temperature.

Find:
(a) Determine the work needed by the device, in Btu;
(b) Determine (b1) the change in entropy and (b2) the entropy produced during the process, both in Btu/°R;
(c) Is the claimed process feasible (circle Yes or No) and why (answer in one sentence)?

Assumptions:
- Negligible ΔKE, ΔPE
- Quasi-equilibrium process
- Air as an ideal gas

Basic equations:
\[ Q - W = \Delta E = \Delta U = m \Delta U \]
\[ \frac{Q}{T} = -m \Delta s = -s \]

Solution:

<table>
<thead>
<tr>
<th>Table A.22 - E</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10 - W = 0.5 (154.57 - 92.04)</td>
</tr>
<tr>
<td>- W = 31.265 + 10</td>
</tr>
<tr>
<td>= 41.265 Btu</td>
</tr>
</tbody>
</table>

(a) 41.265 Btu
\[ \Delta S = m \left( \frac{8^0 - 3^0}{2} - \frac{R}{M} \ln \frac{p_2}{p_1} \right) \\
= 0.5 \left( 0.72438 - 0.60078 - \frac{1.986}{28.97} \ln \frac{100}{15} \right) \\
= 0.5 \left[ (0.1236) - (0.06855)(1.8972) \right] \\
= 0.5 (0.1236 - 0.1305) \\
= -3.25 \left( 10^{-3} \right) \]

\[ -\frac{10}{540} + 0 = \Delta S = -3.25 \left( 10^{-3} \right) \]

\[ \sigma = \frac{10}{540} - 3.25 \left( 10^{-3} \right) \]

\[ = 0.01527 \geq 0 \]

\[ 0.01527 \text{ Btu/}^\circ\text{R} \]

(c) Why in one sentence?

\( \sigma > 0 \) is essential for a process to be sensible.
Question 3 (60 Points): The standard solution format is required for this question.

Given:
A cold region is maintained at 32°F by an R-134a vapor compression cycle shown in the sketch below. The evaporator discharges saturated vapor at a flow rate of 0.175 lbm/s to the inlet of the compressor at 15°F (State 1). The adiabatic compressor discharges vapor at 160 lbm/in² and 140°F (State 2a) to the condenser which in turn delivers saturated liquid (State 3) to the expansion valve. The expansion valve delivers a liquid-vapor mixture to the evaporator (State 4). The condenser rejects energy to the warm region at 80°F.

Find:
(a) Show the states and the processes on the T-s diagram below paying attention to the temperature scale; and list assumptions and basic equations;
(b) Complete the table of properties on the next page;
(c) Evaluate the Coefficient of Performance (COP) or \( \beta \) of the cycle;
(d) Calculate the isentropic efficiency of the compressor, and
(e) Evaluate the entropy production rate for the evaporator, in Btu/K-R-s.

(a) System sketch and T-s Diagram:

Assumptions:
1. Steady state, steady flow
2. Neglect \( \Delta E \) and \( \Delta PE \)
3. No heat transfer for compressor, expansion valve
4. No work done for condenser, evaporator

Basic Equations:
- \( m_e = m_i = m \)
- \( \dot{q} - \dot{w} = m_e(h_e + \frac{v_e^2}{2} + g_z) - m_i(h_i + \frac{v_i^2}{2} + g_z) \)
- \( \frac{\dot{q}}{T} + \dot{\omega}_{CV} = m_e s_e - m_i h_i \)
Question 3 (60 Points): The standard solution format is required for this question.

Given:
A cold region is maintained at 32°F by an R-134a vapor compression cycle shown in the sketch below. The evaporator discharges saturated vapor at a flow rate of 0.175 lbm/s to the inlet of the compressor at 15°F (State 1). The adiabatic compressor discharges vapor at 160 lbf/in² and 140°F (State 2a) to the condenser which in turn delivers saturated liquid (State 3) to the expansion valve. The expansion valve delivers a liquid-vapor mixture to the evaporator (State 4). The condenser rejects energy to the warm region at 80°F.

Find:
(a) Show the states and the processes on the T-s diagram below paying attention to the temperature scale; and list assumptions and basic equations;
(b) Complete the table of properties on the next page;
(c) Evaluate the Coefficient of Performance (COP) or β of the cycle,
(d) Calculate the isentropic efficiency of the compressor, and
(e) Evaluate the entropy production rate for the evaporator, in Btu/°R-s.

(a) System sketch and T-s Diagram:

Assumptions:
1. Steady state, steady flow
2. Neglect KE, PE
3. No heat transfer for compressor, expansion valve
4. No work done for condenser, evaporator

Basic Equations:
\[
\begin{align*}
\dot{m}_e &= \dot{m}_i = \dot{m} \\
\Delta \dot{w} &= \dot{m}_e (h_e + v_e^2 + g \Delta z) - \dot{m}_i (h_i + v_i^2 + g \Delta z) \\
\dot{q}_T &= \dot{q}_C = \dot{m}_e s_e - \dot{m}_i s_i
\end{align*}
\]
**Solution:**

<table>
<thead>
<tr>
<th>State</th>
<th>( h ), Btu/lbm</th>
<th>( s ), Btu/lbm °R</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>103.90</td>
<td>0.2209</td>
</tr>
<tr>
<td>2s</td>
<td>118.89</td>
<td>0.2209</td>
</tr>
<tr>
<td>2a</td>
<td>124.41</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>47.65</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>47.65</td>
<td>0.1024</td>
</tr>
</tbody>
</table>

\[
h_3 = h_4 = 47.65 \text{ Btu/lbm} \]

\[
\chi_4 = \frac{h_{54} + \chi_4 h_{5g4}}{16.20 + \chi_4} = \frac{87.71}{87.71} = 1.00
\]

\[
\chi_4 = \frac{47.65 + 16.20 - 0.3586}{87.71} = 0.5024 \text{ Btu/lbm}
\]

\[
\chi_4 = \frac{(-\chi_4) h_{54} + \chi_4 h_{5g4}}{h_{2a} - h_1} = \frac{(1-0.3586)0.0361 + 0.3586(0.2209)}{124.41 - 103.90} = 2.743
\]

\[
\text{COP} = \frac{Q_{in}}{W_{comp}} = \frac{h_1 - h_4}{h_{2a} - h_1} = \frac{103.9 - 47.65}{124.41 - 103.90} = 2.743
\]

\[
\frac{h_{2s} - h_1}{h_{2a} - h_1} = \frac{118.89 - 103.9}{124.41 - 103.90} = 0.731
\]

\[
\dot{Q}_1 = m (h_1 - h_4) = 0.175 (103.9 - 47.65) = 3.844 \text{ Btu}
\]

\[
\dot{Q}_{rap} = \frac{\chi_4 (8e - 3i) \dot{m}}{T} = \frac{-3.844 + 0.175(0.2209 - 0.1024)}{480 + 15} = -0.02072 + 0.02074 = 0.00002 \approx 0
\]
**Question 4 (60 Points):** The standard solution format is required for this question.

Given: A jet engine shown in the schematic below is flying under steady state steady flow conditions. Air enters the isentropic compressor of the jet engine at 300 K and 150 kPa and leaves at 500 K to enter the combustor. After the heat addition in the combustor, the air enters the isentropic turbine (designed to provide just enough power to run the compressor) at 1200 K. The specific heats are assumed to be constant \((c_p = 1.005 \text{ kJ/kg-K}, c_v = 0.718 \text{ kJ/kg-K})\), and \(k = 1.4\).

Find:
(a) Draw schematic diagrams with control volumes (CVs) for analysis of parts (d), (e), and (f) below; write basic equations and simplify these equations for the selected CVs.
(b) Combustor pressure, in kPa.
(c) Show the states and the processes on the T-s diagram by paying attention to the temperature scale.
(d) Work consumed by the compressor, in kJ/kg.
(e) Heat added in the combustor, in kJ/kg.
(f) Temperature at the exit of the turbine, in K.
(g) Entropy production in kJ/kg·K of the heat addition process if the source of heat can be approximated to be at a constant temperature of 1300 K.

(a) Control Volumes:

Asumptions:
(1) steady state, steady flow
(2) Negligible KE, PE for gas generator components
(3) No heat transfer compressor turbine
(4) No work done combustor
(5) work of compressor = work of turbine

Basic Equations:

\[
\dot{m}_c = \dot{m}_e
\]

\[
\dot{q}_c - \dot{w}_c = \dot{m}_e \left( h_e + \frac{v_e^2}{2} + g z_e \right) - \dot{m}_i \left( h_i + \frac{v_i^2}{2} + g z_i \right)
\]

\[
\frac{\dot{q}_c}{T_b} + \dot{\delta}_c = \dot{m}_c \delta_e - \dot{m}_i \delta_i
\]
Solution:
(b) Isentropic compression, ideal gas, $\frac{c_p}{c_v} = 1.4$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{c_v}{c_p}} = \frac{500}{300} \Rightarrow P_2 = \frac{150}{300} \cdot 896.5 \text{ kPa}$$

(c) 

(d) \( - \dot{W}_C = h_2 - h_1 = c_p (T_2 - T_1) = 1.005 (500 - 300) \)

\( \dot{Q}_{comb} = h_3 - h_2 = 1.005 (1200 - 500) \)

(e) \( \dot{Q}_{comb} = 703.5 \text{ kJ/kg} \)

(f) \( - \dot{W} = \dot{W}_C = -201 = c_p (T_3 - T_4) = 1.005 (1200 - T_4) \)

\( T_4 = \frac{-201}{1.005} + 1200 = 1000 \text{ K} \)

(g) \( \dot{Q}_{CV} \frac{\dot{m}}{m} + \dot{Q}_{CV} \frac{\dot{m}}{m} = h_3 - h_2 = c_p \ln \frac{T_3}{T_1} = 0.3387 \text{ kJ/kg} \)