ME 309 – Fluid Mechanics
Fall 2016

LAST NAME: ____________________
FIRST NAME: ____________________

Neatly print your name: ____________________

Signature: ____________________
(Note that unsigned exams will be given a score of zero.)

Please note the following:

1. The exam is closed notes and closed book. You may use only the formula sheet provided with the exam, a pen/pencil/eraser, and a calculator fitting the policy stated in the course syllabus.
2. Show all of your work in order to receive credit. An answer without supporting work will not receive a full score. **Also, write neatly and organized and clearly box your answers.**
3. Clearly state your assumptions, draw control volumes and coordinates systems, and include other significant information in order to receive full credit.
4. Turn in all numbered pages. Do not turn in your formula sheets.
5. **The honor code is in effect.**
6. Work on the backside of each numbered page will not be graded.
7. Very clearly cross out work that you do not wish to be graded.
8. Only the first solution approach encountered when grading will be scored.

Write your name on all pages that are to be considered for grading.
If you do not write your name, that page will NOT be graded

SCORE:

1. (40 pts)	
2. (40 pts)	
3. (20 pts)	
TOTAL (out of 100):	

ME 309 – EXAM 3

SOLUTIONS

ME 309 – Fluid Mechanics
Fall 2016

LAST NAME: ____________________________
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Problem 1

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SCORE:

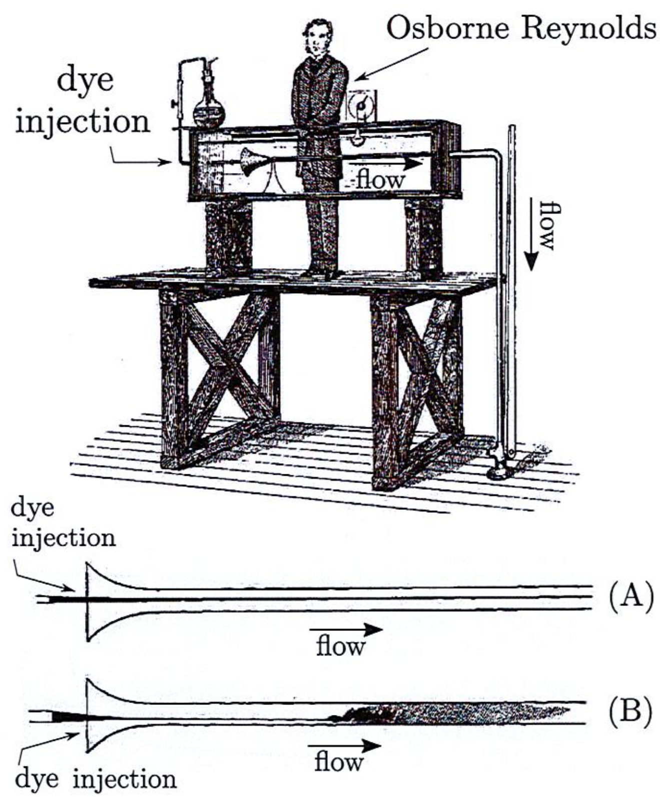
1. (40 pts)	

40 total: no partial credit on multiple choice questions

1) [5 pts total] In the early 1870s Osborne Reynolds conducted a famous series of experiments where he injected dye along the centerline of a glass tube through which a steady (but not fully developed) water flow was established. By testing various velocities $\bar{V}$, densities ρ, viscosities μ, and tube diameters, D, he realized that only one dimensionless number, named after him, was enough to describe the resulting flow regime.

Write the expression of the Reynolds number [1 pts] and circle the only correct statement regarding scenarios A and B [4 pts].

$$Re = \frac{\rho \bar{V} D}{\mu}$$



(a) Both scenarios depict a laminar flow.
(b) The tube diameter in A *must* be larger than in B.
(c) The fluid viscosity in A *must* be larger than in B.
(d) The flow in B transitions from laminar to turbulent; flow in A is laminar;
(e) The Reynolds number in B *must* be below 2300.

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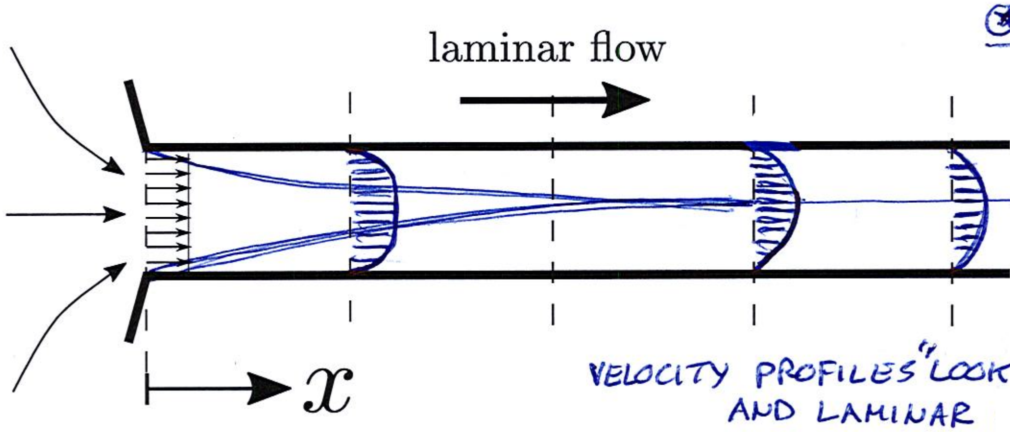
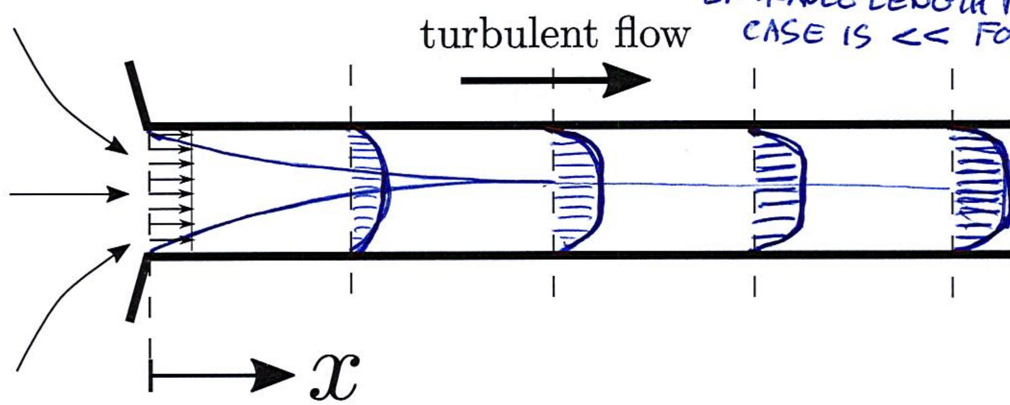
2) [6 pts] Illustrate the two-dimensional steady developing flow in the two scenarios below by drawing velocity profiles at the x –locations indicated with vertical dashed lines. Assume zero boundary layer thickness at $x = 0$, i.e. flat streamwise velocity profile at the entrance of the duct (already drawn in figure).

⊛ IGNORE POSSIBLE MASS CONSERVATION ISSUES

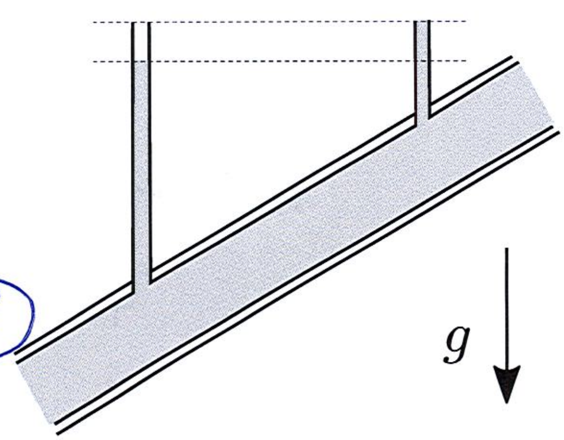


VELOCITY PROFILES "LOOK" TURBULENT AND LAMINAR

ENTRANCE LENGTH FOR TURBULENT CASE IS $<<$ FOR LAMINAR



3) [4 pts] Consider water flowing in an inclined pipe equipped with vertical water manometers. The manometers are attached to the pipe via very small orifices that allow pressure to be measured by the manometer. No flow is present in the vertical manometers. Circle the most correct answer based on the observable water levels in the figure.



(a) Flow is inviscid
(b) Water flows upwards in the inclined pipe
(c) Water flow downwards in the pipe but viscosity can be neglected
(d) Water flows downwards in the pipe ~~but~~ AND ~~viscosity~~ VISCOUS effects are measurable
(e) Flow is quiescent everywhere and pressure increases with depth hydrostatically

4) [6 pts] Calculate the magnitude of the pressure loss (in Pascals) in a straight horizontal circular pipe over a length of $L = 10$ m with diameter $D = 0.1$ m, relative roughness $\epsilon/D = 0.0001$, cross-sectional average velocity $\bar{V} = 3$ m/s, fluid viscosity $\mu = 1.5x10^{-5}$ Pa.s and density $\rho = 1000$ kg/m^3. Explicitly indicate whether the flow is laminar or turbulent.

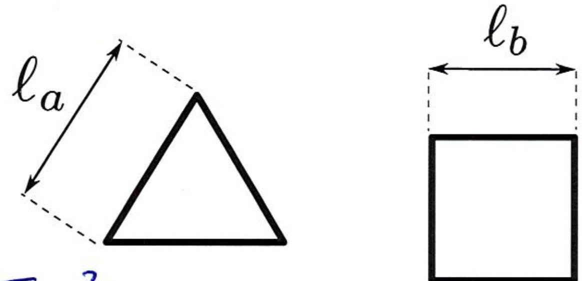
$$Re_D = \frac{\rho \bar{V} D}{\mu} = \frac{1000 \times 3 \times 0.1}{1.5 \times 10^{-5}} = 20{,}000{,}000 = 20 \times 10^{6} \; , \; \frac{\varepsilon}{D} = 0.0001$$

FROM MOODY $\Rightarrow f = 0.012$

$$\Delta p = \rho \, f\left(Re_D, \frac{\varepsilon}{D}\right) \frac{L}{D} \frac{\bar{V}^2}{2} = 1000 \times 0.012 \times \frac{10}{0.1} \frac{3^2}{2} = 5400 \, Pa$$

$$\boxed{\Delta p = 5400 \, Pa}$$

5) [4 pts] Calculate the edge lengths, ℓ_a and ℓ_b, of a triangular and squared cross-section pipe, respectively, that would result in the same pressure drop as a circular pipe of diameter D, assuming the pipe to be completely filled with water.



TRIANGLE :

$$D_h = \frac{4A}{P} = \frac{4 \frac{\sqrt{3}}{4} \ell_a^2}{3\ell} = \frac{\sqrt{3}}{3} \ell_a = D$$

SQUARE :

$$\frac{4\ell_b^2}{4\ell_b} = \ell_b = D$$

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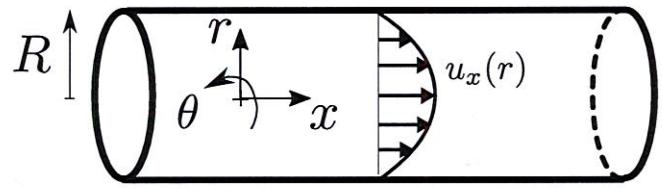
6) [6 pts] Consider a fully developed steady laminar viscous flow in a straight pipe of radius R, driven by a constant and uniform axial pressure gradient $\Delta p/\Delta x$. Solving the Navier-Stokes equations in cylindrical coordinates by neglecting velocity variations in the azimuthal, θ, and axial, x, direction yields the following analytical expression of the axial velocity profile as a function of the radial coordinate, r:

$$u_x(r) = -\frac{R^2}{4\mu}\left(\frac{\Delta p}{\Delta x}\right)\left[1-\left(\frac{r}{R}\right)^2\right]$$



where μ is the dynamic viscosity and r the independent radial coordinate. Find the analytical expression for the volumetric flow rate Q, as a function of R, $\Delta p/\Delta x$, and μ.

$$Q = \int_0^R u_x(r)\, 2\pi r\, dr = \int_0^R -\frac{R^2}{4\mu}\left(\frac{\Delta P}{\Delta x}\right)\left[1-\left(\frac{r}{R}\right)^2\right] 2\pi r\, dr$$

INTEGRATION LIMITS

SETTING UP INTEGRAL

$$= -\frac{R^2}{4\mu}\frac{\Delta P}{\Delta x} 2\pi \left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R = -\frac{R^2}{4\mu}\frac{\Delta P}{\Delta x}\frac{1}{2}\pi R^2$$

$$\boxed{Q = -\frac{\pi R^4}{8\mu}\frac{\Delta P}{\Delta x}}$$

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7) [9 pts] Consider a centrifugal pump operating at its best efficiency point delivering a head rise of water of H=10 m for a flow rate of Q =0.725 m^3/s. The pump has a diameter of D= 0.5 m, If the specific speed of the pump is 1.74 and the required input power is $\dot{W}_{in} = 120$ kW,

(a) determine the shut-off head, $H_0 = (\omega R)^2/g$, where R is the impeller radius.

(b) determine the best efficiency, η, assuming a quadratic relationship between head rise and flow rate, $H = H_0 - A\,Q^2$, where A is a constant to be determined.

$$N_s = \frac{\omega Q^{1/2}}{h^{3/4}} = \frac{\omega Q^{1/2}}{(gH)^{3/4}} \Rightarrow \omega = \frac{N_s (gH)^{3/4}}{Q^{1/2}}$$

$$\omega = 1.74 \frac{(9.81 \times 10)^{3/4}}{(0.725)^{1/2}} = 63.7 \text{ RAD/s}$$

$$H_0 = \frac{(\omega R)^2}{g} = \frac{(63.7 \times 0.25)^2}{9.81} = 25.9 \text{ m}$$

FIND A: $10 = 25.9 - A(.725)^2 \Rightarrow A = 30.25 \frac{s^2}{m^5}$

FIND η_{MAX}: $\frac{\partial \eta}{\partial Q} = 0 \Rightarrow \frac{\partial}{\partial Q}\left(Q H(Q)\right) = H_0 - 3AQ^2 = 0$

$$Q = \sqrt{H_0/3A} = 0.53 \text{ m}^3/\text{s}$$

$$\eta = \frac{\rho Q g H}{\dot{W}_{IN}}$$

$$\eta_{MAX} = 0.75$$

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Problem 2

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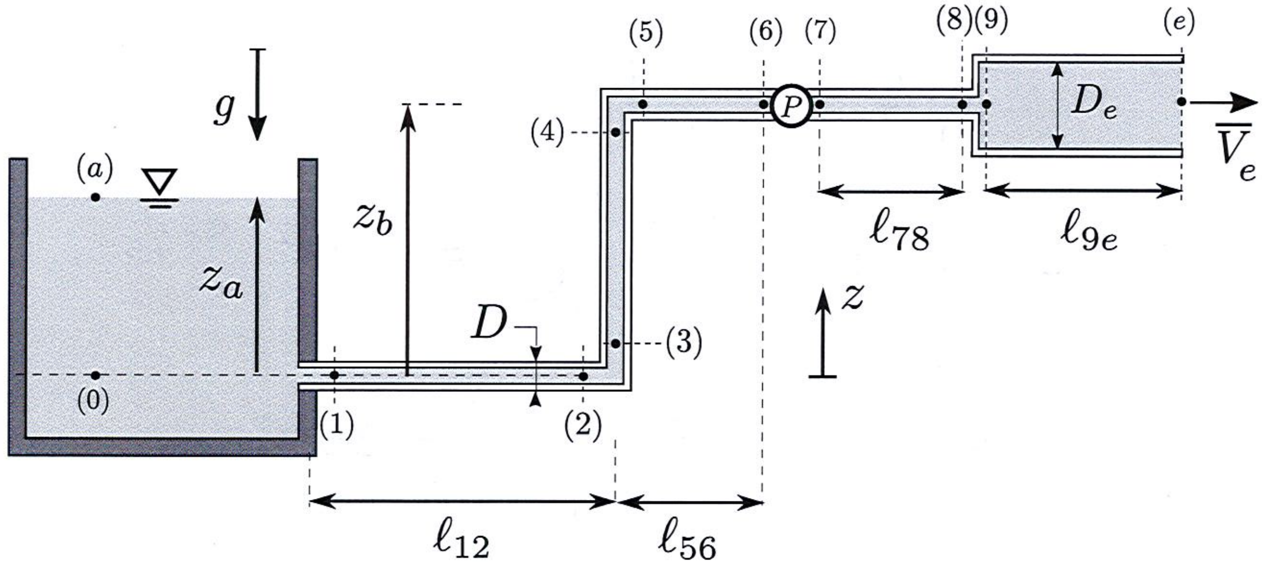
SCORE:

2. (40 pts)	



Water from a very large reservoir of finite depth is pumped out through the pipe system shown in the figure above. The pump is located between sections 6 and 7. Assume steady flow with exit cross-sectional average velocity $\overline{V}_e$, density ρ, and acceleration of gravity, g. The water elevation in the reservoir and exit height are z_a and z_b, respectively, measured with respect to the reference elevation, $z = 0$. Assuming laminar flow everywhere with given Reynolds numbers, Re_D and Re_{D_e}, THE PIPE SYSTEM TERMINATES WITH A LARGER DIAMETER $D_e > D_e$.

(a) [10 pts] find the sum of the major losses from points (a) to (e) assuming all aforementioned geometrical and flow parameters are given.

(b) [20 pts] identify the minor losses in the system and find the ~~corresponding~~ ASSOCIATED overall head loss from points (a) to (e) assuming all aforementioned geometrical and flow parameters, as well as the minor loss coefficients, to be known.

(c) [10 pts] find the value of the velocity $\overline{V}$, that would induce cavitation upstream of the pump assuming a minimum required NPSHR = $0.1\,p_v/(\rho g)$, where p_v is the given absolute vapor pressure.

(i) SECTIONS FROM (1) → (6) HAVE DIAMETER & VELOCITY D, $\bar{V} = \dfrac{D_e^2 \bar{V}_e}{D^2}$

SECTION FROM (7) → (8) HAVE DIAMETER & VELOCITY D_e, $\bar{V}_e$

$$h_\ell = f \, \frac{l_{12} + z_b + l_{56}}{D} \frac{\bar{V}^2}{2} + f_{78} \frac{l_{78}}{D_e} \frac{\bar{V}_e^2}{2}$$

WHERE $f = {}^{64}/_{Re_D}$, $f_{78} = {}^{64}/_{Re_{D_e}}$

(ii) SHARP EDGE ENTRANCE (0) → (1), K_{01}

90° BEND (2) → (3) / (4) → (5), K_{23}, K_{45}

SUDDEN EXPANSION (8) → (9), K_{89}

$$h_{\ell_m} = \left(K_{01} + K_{23} + K_{45} + K_{89}\right) \frac{\bar{V}^2}{2}$$

(iii) CAVITATION IS REACHED IN 6 IF P_6 IS SUCH THAT:

$$\left(\frac{P_6}{\rho g}+\frac{\bar{V}_6^2}{2g}\right)-\frac{P_v}{\rho g}=.1\,\frac{P_v}{\rho g}\Rightarrow P_6+\frac{1}{2}\rho\bar{V}_6^2=1.1\,P_v$$

$\bar{V}=\bar{V}_6$, WE NEED TO EXPRESS P_6 AS A FUNCTION OF $\bar{V}$

$$\left(\frac{P_a}{\rho}+gz_a\right)-\left(\frac{P_6}{\rho}+2\frac{\bar{V}^2}{2}+gz_e\right)=\underbrace{h_{\ell_m}+h_\ell}\propto\bar{V}$$

SUBTLE POINT: IF $\bar{V}$ IS UNKNOWN, THEN THIS RHS IS PROPORTIONAL TO $\bar{V}$

THIS YIELDS: $P_6=A\bar{V}^2+B\bar{V}+C$

WHERE A, B, C ARE GIVEN

$\bar{V}$ CAN BE FOUND BY SOLVING:

$$\left(\frac{1}{2}\rho+A\right)\bar{V}^2+B\bar{V}+\left(C-1.1\,P_v\right)=0$$

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Problem 3

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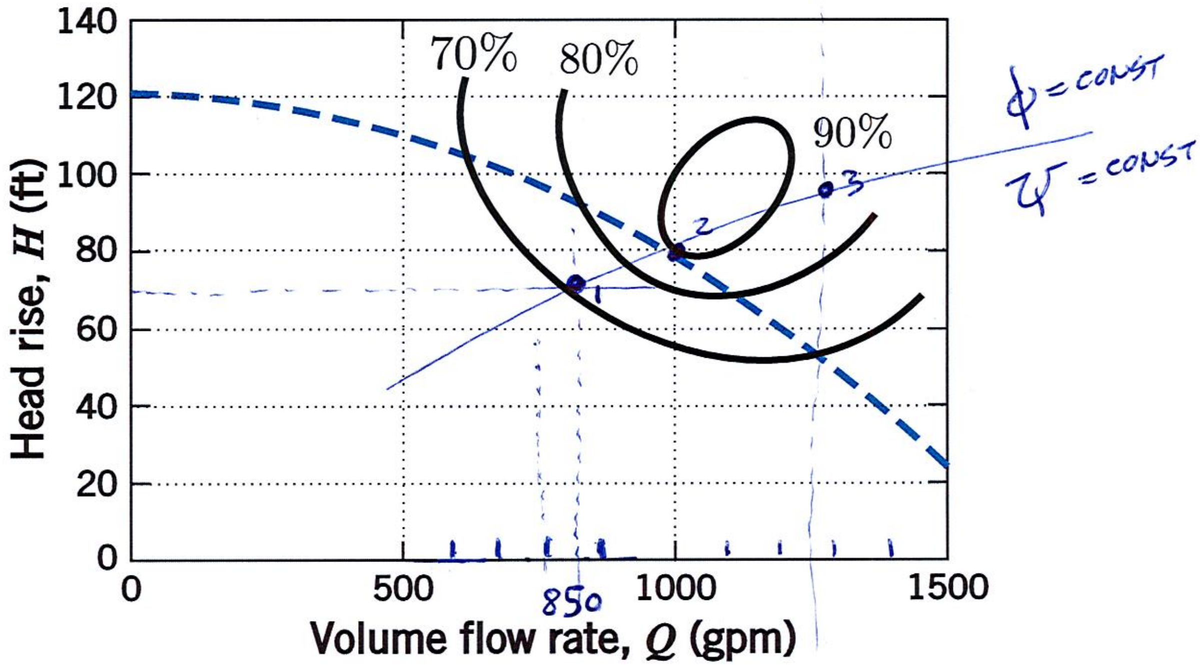
SCORE:

3. (20 pts)	

Consider the performance curve of a centrifugal pump (plotted below with a dashed line) for a given impeller diameter D_2 and angular speed ω. We want to predict the performance of the pump by keeping ω fixed but varying the impeller diameter.



Using the point of max efficiency at $D = D_2$ as a reference for dimensional scaling,

(a) [8 pts] predict the flow rate at the point of max efficiency for two other diameters, $D_1 = 0.9473 D_2$ and $D_3 = 1.0775\, D_2$.

(b) [8 pts] predict the head rise values at the point of max efficiency for the same set of diameters.

(c) [4 pts] draw two points on the graph corresponding to the (flow rate, head rise) pairs for diameters D_1 and D_3 and briefly discuss the accuracy of the adopted dimensional scaling technique in predicting the points of maximum efficiency

(i) @ $D = D_2 \Rightarrow H_2 = 80\,ft$, $Q_2 = 1000\ gpm$

$\eta_{MAX}\Big|_{D=D_2} = 90\%$

TO KEEP MAX EFFICIENCY : $\Phi = \dfrac{Q}{\omega D^3} = CONST \Rightarrow$

$$Q_1 D_1^{-3} = Q_2 D_2^{-3} \Rightarrow Q_1 = Q_2 \left(\frac{D_1}{D_2}\right)^3 = 1000 \times 0.9473^3$$

$$= 850\ gpm$$

$$Q_3 = Q_2 \left(\frac{D_3}{D_2}\right)^3 = 1000 \times (1.0775)^3 = 1250\ gpm$$

(ii) TO PREDICT H_1 AND H_3 WE USE $\Psi = CONST$

$$\Psi = \frac{gH}{\omega D^2} \Rightarrow H_1 = H_2 \frac{D_1^2}{D_2^2} = 80 \times (0.9473)^2 = 71.8\ ft$$

$$H_3 = H_2 \frac{D_3^2}{D_2^2} = 80 \times (1.0775)^2 = 92.9\ ft$$

(iii) SIMILARITY IS NOT ACCURATE FOR LARGER DIAMETERS IN THIS CASE