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Circle one:

Div. 1	**Div. 2**	**Div. 3**	**Div.4**
9:30am	**12:30 pm**	**2:30 pm**	**4:30 pm**
Marconnet	**Pan**	**Naik**	**Ghosh**

ME315 Heat and Mass Transfer
School of Mechanical Engineering
Purdue University

Exam #1

October 1, 2014

Instructions:

- Write your name on each page
- One equation sheet is allowed, no books, notes, and other materials
- Write on one side of the page only
- Keep all the pages in order
- You are asked to write your assumptions and answers to sub-problems in designated areas. Only the work in its designated area will be graded.

Performance

1	**35**	
2	**30**	
3	**35**	
Total	**100**	

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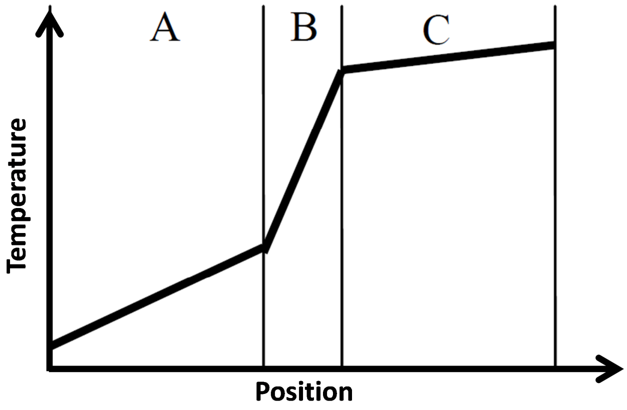
Problem 1 [35 points]

(a) [6 pts] The temperature profile of a three-layer composite plane wall is shown in the figure to the right. Assume steady state and no internal heat generation. Based on the temperature profile, rank the thermal conductivities (k_A, k_B and k_C) and the thermal resistances of three layers (R''_A, R''_B and R''_C) from low to high by filling below subscripts with "A", "B" or "C".



+3 $k_{(\ C\ )} > k_{(\ A\ )} > k_{(\ B\ )}$

+3 $R''_{(\ B\ )} > R''_{(\ A\ )} > R''_{(\ C\ )}$

(b) [4 pts] The heat diffusion equation is constructed based on two physical laws, which are

conservation of energy and Fourier's law of heat conduction.

+2 +2

(c) [6 pts] In general, boundary conditions are required to solve the heat diffusion equation for the steady-state conduction. There are three common types of linear boundary conditions, which include specified temperature, specified heat flux, and specified convection. +3

Circle the correct answer in the following sentence: It is physically possible / impossible to specify more than one type of boundary condition at a single boundary at the same time. +3

(d) [6 pts] Consider a fin with a uniform cross section and an adiabatic tip, subject to a constant base temperature, which is higher than the ambient temperature. **As the length of the fin increases,**

	Circle the correct answer			
the *fin efficiency* will	(i) increase	(ii) remain constant	(iii) decrease	+2
the *fin effectiveness* will	(i) increase	(ii) remain constant	(iii) decrease	+2
the *total heat removal rate* from the fin base will	(i) increase	(ii) remain constant	(iii) decrease	+2

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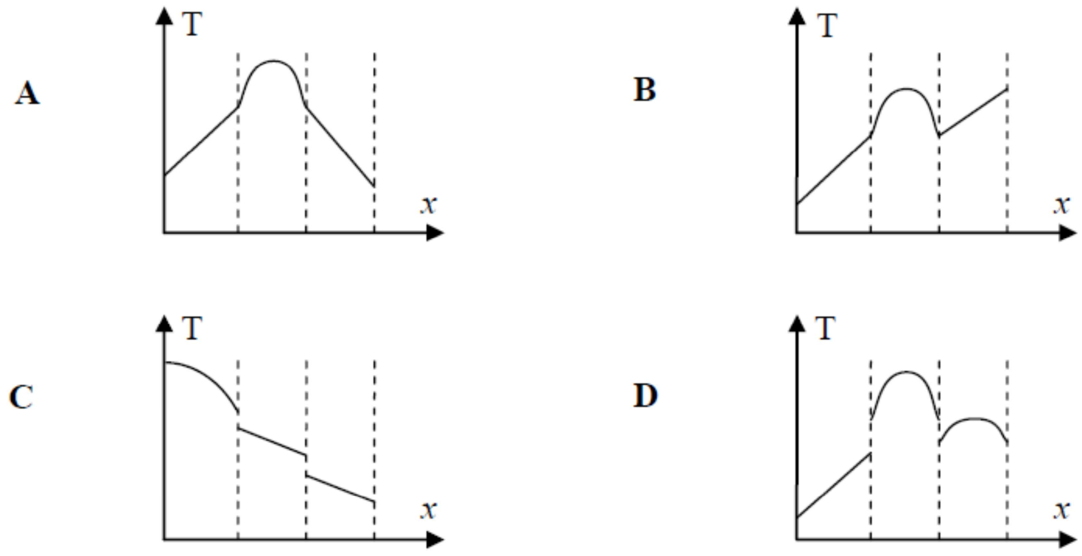
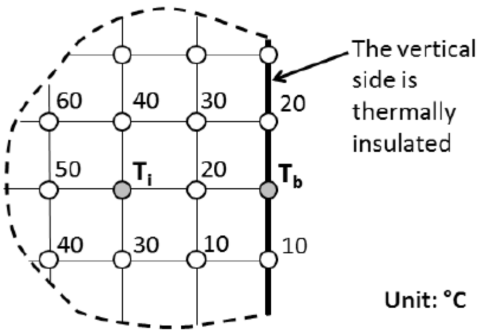
(e) [4 pts] **Circle** the temperature profile(s) below that is(are) possible for 1-D steady-state heat conduction through a three-layer composite wall made of uniform materials. **There may be uniform heat generation inside one or more layers and thermal contact resistance(s) at one or both interface(s)**.



A and C possible +4

(f) [6 pts] Consider the heat conduction in a uniform 2-D plate without heat generation. Some of the nodal temperatures calculated when numerically solving this problem using the Energy Balance Method are shown in the right figure. The nodes are evenly spaced along *x* and *y* directions.



Based on these known nodal temperatures,

the temperature (T_i) of the internal node is 35°C +3

and the temperature (T_b) of the boundary node is 17.5°C. +3

(g) [3 pts] Two cubes of different materials (material A and material B) have identical geometries and are both subject to the same initial temperatures and convection conditions (h and T_∞) on all exposed surfaces. The Biot numbers associated are the same. Based on above information, the relationship between their thermal time constants is:

(i) $\tau_A > \tau_B$ (ii) $\tau_A = \tau_B$ (iii) $\tau_A < \tau_B$ (iv) need more information. +3

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Problem 2 [30 points]

Consider a copper ball of diameter 20 cm ($A_s = 0.1257\ \text{m}^2$ and $V = 0.004189\ \text{m}^2$) undergoing a thermal treatment process. It is heated uniformly to 500 K and dropped in a large pool of water at 280 K. The convection heat transfer coefficient at the surface of the sphere is 200 W/m^2-K.

For Copper: Thermal conductivity: 400 W/m-K; density: 9000 kg/m^3; specific heat: 385 J/kg-K

(a) [10 pts] How long does it take for the surface temperature of the copper ball to reach 300 K?

+2 $$Bi = \frac{h_{conv}(r_o/3)}{k_{solid}} = \frac{200\frac{\text{W}}{\text{m}^2\text{-K}} \times \frac{10\times10^{-2}}{3}\text{m}}{400\frac{\text{W}}{\text{m-K}}} = 0.0166 \Rightarrow Bi < 0.1 \Rightarrow \text{lumped system}$$ +2

+2

Thermal time constant: $$\tau_t = \frac{\rho V C_p}{h_{conv} A} = \frac{9000\frac{\text{kg}}{\text{m}^3}\times 0.004189\ \text{m}^3 \times 385\frac{\text{J}}{\text{kg-K}}}{200\frac{\text{W}}{\text{m}^2\text{-K}}\times 0.1257\ \text{m}^2} = 577.4\ \text{seconds}$$

+2 $$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{t}{\tau_t}\right) \Rightarrow \frac{300-280}{500-280} = \exp\left(-\frac{t}{577.4}\right) \Rightarrow \underline{\boldsymbol{t = 1384.5\ \textbf{seconds}}}$$ +2

(b) [5 pts] What is the temperature (K) at the center of the copper ball at that time? Briefly justify your answer.

+3 Since the copper ball is lumped, the conductive resistance through the solid is negligibly small compared to convective resistance at the surface i.e. temperature copper ball is nearly uniform throughout its volume $\Rightarrow$ $\underline{\boldsymbol{T_o \cong T_s = 300\ \textbf{K}}}$ +2

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Now consider a bismuth ball of same dimension going through the same treatment. It is heated uniformly to 500 K and dropped in a large pool of water at 280 K. The convection heat transfer coefficient at the surface of the sphere is 200 W/m^2-K.

For Bismuth: Thermal conductivity: 8 W/m K; density: 9800 kg/m^3; specific heat: 122 J/kg-K

(c) [6 pts] How long does it take for the surface temperature of the bismuth ball to reach 300 K?

$$Bi = \frac{h_{conv}\left(r_o/3\right)}{k_{solid}} = \frac{200\,\frac{\text{W}}{\text{m}^2\text{-K}} \times \frac{10\times10^{-2}}{3}\,\text{m}}{8\,\frac{\text{W}}{\text{m-K}}} = 0.8333 \Rightarrow Bi > 0.1 \Rightarrow \text{non-lumped system} \quad \boxed{+1}$$

At the surface: $r^* = \dfrac{r}{r_o} = 1$; $\theta^* = \dfrac{T_s - T_\infty}{T_i - T_\infty} = \dfrac{300-280}{500-280} = 0.0909$

$$Bi = \frac{h_{conv} r_0}{k_{solid}} = \frac{200\,\frac{\text{W}}{\text{m}^2\text{-K}} \times 10\times10^{-2}\,\text{m}}{8\,\frac{\text{W}}{\text{m-K}}} = 2.5 \quad \boxed{+2}$$

Interpolating in Table 5.1: $C_1 = 1.551$; $\zeta_1 = 2.15885$

Assuming $Fo > 0.2$, for a sphere: $\theta^* = \dfrac{T - T_\infty}{T_i - T_\infty} = C_1 \exp\left(-\zeta_1^2 Fo\right)\dfrac{\sin\left(\zeta_1 r^*\right)}{\zeta_1 r^*} \Rightarrow Fo = 0.4040$ $\boxed{+2}$

Thermal diffusivity: $\alpha = \dfrac{k}{\rho C_p} = \dfrac{8\,\frac{\text{W}}{\text{m-K}}}{9800\,\frac{\text{kg}}{\text{m}^3} \times 122\,\frac{\text{J}}{\text{kg-K}}} = 6.7\times10^{-6}\,\dfrac{\text{m}^2}{\text{s}}$

$Fo = \dfrac{\alpha t}{r_o^2} = 0.4040 \Rightarrow \boldsymbol{t = 603.8\ \textbf{seconds}}$ $\boxed{+1}$

(d) [4 pts] What is the temperature (K) at the center of the bismuth ball at that time?

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = C_1 \exp\left(-\zeta_1^2 Fo\right) \Rightarrow \frac{T_o - 280}{500-280} = 1.551\times\exp\left(-\left(2.15885\right)^2 \times 0.4040\right) \quad \boxed{+3}$$

$\Rightarrow \underline{\boldsymbol{T_o = 331.9\ \text{K}}}$ $\boxed{+1}$

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(e) [5 pts] Which ball (copper or bismuth) releases more energy in the time it takes to cool the surface of each ball to 300 K? Qualitatively justify your answer with a few phrases, equations, and/or diagrams.

$$Q = mC_p\left(T_f - T_i\right) = \rho V C_p\left(T_f - T_i\right)$$

$$\left(\rho V C_p\right)_{copper} = 14515\,\frac{\text{J}}{\text{K}} \quad \text{and} \quad \left(\rho V C_p\right)_{bismuth} = 5008\,\frac{\text{J}}{\text{K}}$$ +3

In addition, the copper ball cools down nearly uniformly (lumped) while the bismuth ball has more significant spatial temperature gradient (non-lumped)

$\Rightarrow$ For the same initial temperature difference, **copper ball releases more energy** in the time it takes to cool the surface of each ball to 300 K

+2

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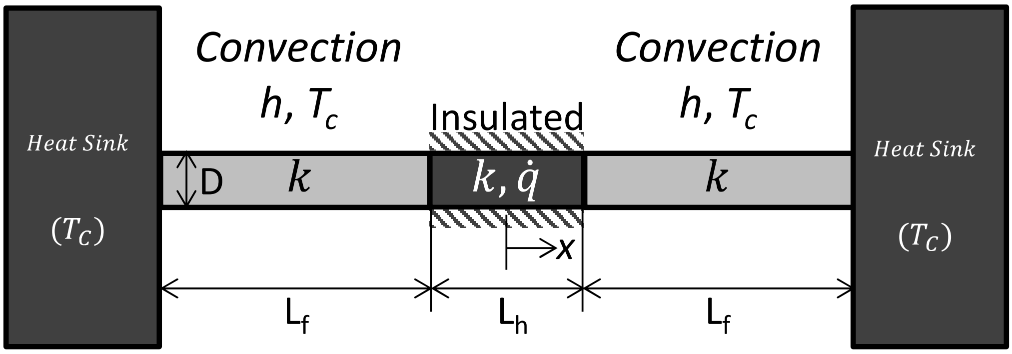
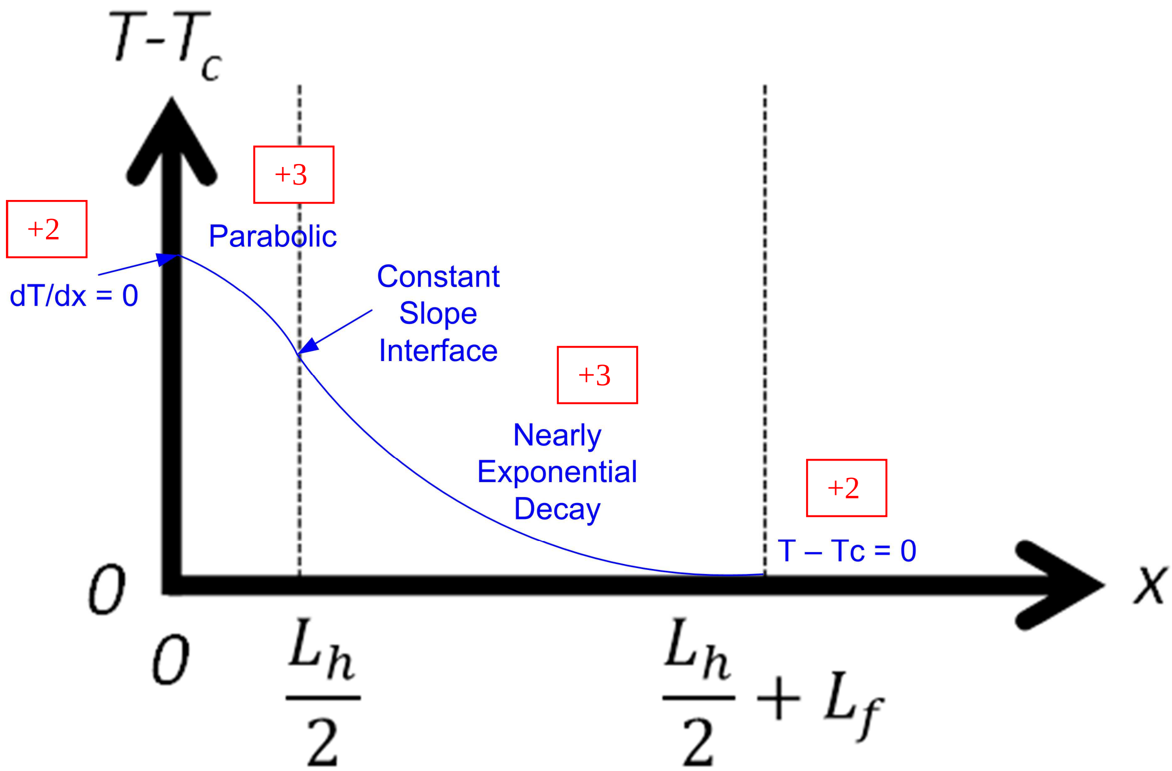
Problem 3 [35 points]

A long metal rod (of diameter D and thermal conductivity k) spans the distance between two heat sinks kept at temperature T_C. The center portion of the rod is well insulated and subject to uniform volumetric heat generation ($\dot{q}$). Outside of this region, the rods are exposed to convection with a convection coefficient h and free stream temperature T_C (equal to the heat sink temperature). The center of the rod (x = 0) must reach a temperature of T_M for the heat treatment process to complete. Assume 1-D heat transfer approximation is valid and neglect radiation and contact resistances. (Hint: The problem is symmetric about x = 0).



(a) [10 pts] <u>On the axes below sketch the temperature profile as function of *x* from the center to one heat sink.</u> Be sure all the boundary and interface conditions are correctly shown.



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(b) [5 pts] You would like to assume each of the exposed portion of the bar can be modeled as an infinite fin. The infinite fin approximation is valid if the fin heat transfer rate with appropriate boundary conditions is within 1% of the heat transfer rate for an infinitely long fin. <u>Using this criteria, what is the minimum length of the exposed regions of the rod (L_f) in order for the infinite fin approximation to be valid?</u> Express in terms of the parameters given in the problem statement.

Considering adiabatic tip in Table 3.4: $\frac{q_{fin,adibatic}}{q_{fin,long}}=\frac{M\tanh(mL)}{M}=\tanh(mL)\geq 0.99$ $\boxed{+2}$

$\Rightarrow mL\geq 2.65 \Rightarrow \boldsymbol{L\geq\frac{2.65}{m}}$ $\boxed{+1}$

$m=\left(\frac{hP}{kA_c}\right)^{1/2}=\frac{h\times\pi D}{k\times\frac{\pi}{4}D^2}\Rightarrow \boldsymbol{m=\left(\frac{4h}{kD}\right)^{1/2}}$ $\boxed{+2}$

(c) [20 pts] Assume the length of the exposed rod is sufficient for the infinite fin approximation to be valid. <u>Derive an expression for the required heat generation rate in the center portion of the rod to maintain the center temperature T(x = 0) = T_M.</u> Your expression should be in terms of the given material and geometrical parameters and specified temperatures (T_M and T_C).
(Hint: You should derive the general temperature profile in the center region of the rod using appropriate boundary conditions at the center (x = 0) and the edge of the heat region (x = L_h/2). Note that the correct boundary condition at x = 0 is not T(x = 0) = T_M.)

Considering heat diffusion equation in rectangular coordinates:

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial T}{\partial x}\right)+\cancel{\frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right)}+\cancel{\frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right)}+\dot{q}=\cancel{\rho C_p\frac{\partial T}{\partial t}}\Rightarrow\frac{d^2T}{dx^2}+\frac{\dot{q}}{k}=0$$

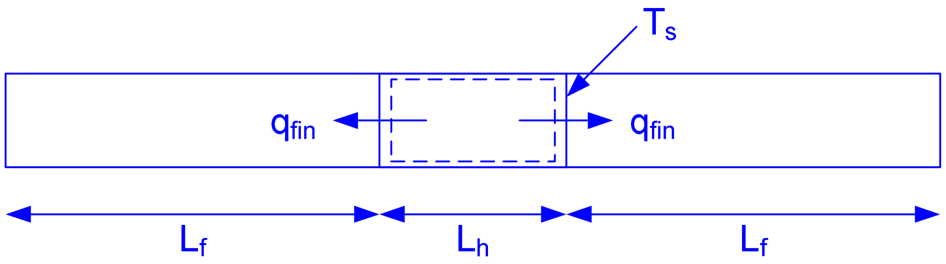
Integrate: $\frac{dT}{dx}=-\frac{\dot{q}}{k}x+c_1$

$\boxed{+6}$

Integrating again: $T(x)=-\frac{\dot{q}}{2k}x^2+c_1x+c_2$

Boundary Conditions:

For x = 0, $\frac{dT}{dx}=0$ $\boxed{+2}$



Considering CV around the central portion of the rod: $\cancel{\dot{E}_{in}}-\dot{E}_{out}+\dot{E}_{gen}=\cancel{\dot{E}_{st}}$

$2q_{fin}=\dot{q}V=\dot{q}\frac{\pi}{4}D^2L_h\Rightarrow q_{fin}=\dot{q}\frac{\pi}{8}D^2L_h$

For an infinitely long fin in Table 3.4: $q_{fin}=M=\sqrt{hPkA_c}\left(T_s-(T_\infty=T_C)\right)$

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Combining: $\sqrt{hPkA_c}\left(T_s - T_c\right) = \dot{q}\frac{\pi}{8}D^2 L_h$

For x = $L_h/2$, $T_s = T_C + \dfrac{\dot{q}\frac{\pi}{8}D^2 L_h}{\sqrt{hPkA_c}}$ +8

$$\frac{dT}{dx} = 0 = -\frac{\dot{q}}{k}\left(x=0\right) + c_1 \Rightarrow c_1 = 0$$

$$T\left(x = \frac{L_h}{2}\right) = T_s = c_2 - \frac{\dot{q}}{2k}\left(\frac{L_h}{2}\right)^2 \Rightarrow c_2 = T_s + \frac{\dot{q}L_h^2}{8k} = T_C + \frac{\dot{q}\frac{\pi}{8}D^2 L_h}{\sqrt{hPkA_c}} + \frac{\dot{q}L_h^2}{8k}$$

$$T_M = c_2 = T_C + \frac{\dot{q}\frac{\pi}{8}D^2 L_h}{\sqrt{hPkA_c}} + \frac{\dot{q}L_h^2}{8k}$$ +2

Required heat generation rate in the central portion of the rod:

$$\boldsymbol{\dot{q} = \frac{T_M - T_C}{\dfrac{\frac{\pi}{8}D^2 L_h}{\sqrt{hPkA_c}} + \dfrac{L_h^2}{8k}}; \; P = \pi D \text{ and } A_c = \frac{\pi}{4}D^2}$$ +2