ME 315
Midterm Exam 1
6:30 - 7:30 PM
Thursday, February 10, 2011

- This is a closed-book, closed-notes examination. *There is a formula sheet provided at the end of this exam.*
- You must turn off all communications devices before starting this exam, and leave them off for the entire exam.
- Please write legibly and show all work for receiving partial credit. Please show your final answers in the boxes provided.
- State all assumptions.
- Please arrange all your sheets in the correct order. Make sure they are all included.

Name: ______________________________
Last **First**

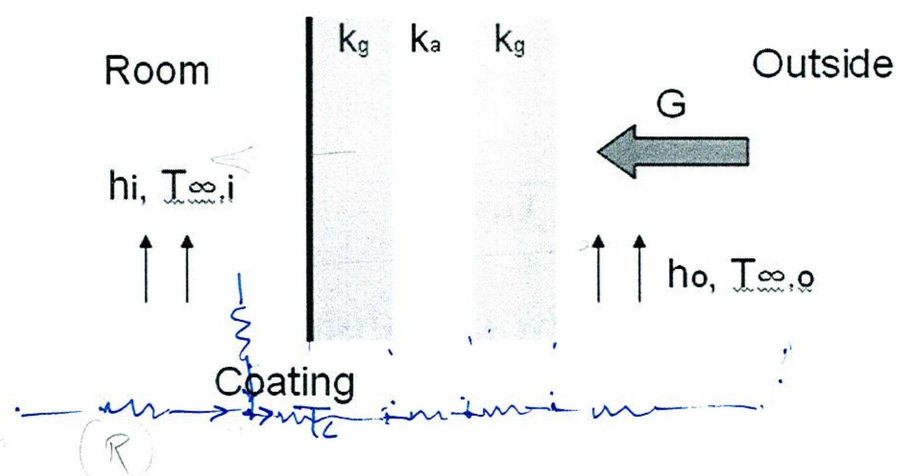
CIRCLE YOUR DIVISION

Div. 1 (9:30 am)
Prof. Choi

Div. 2 (12:30 pm)
Mr. Bao

Problem	Score
1 (25 Points)	
2 (40 Points)	
3 (35 Points)	
Total (100 Points)	

1. (25 points) A square glass window of width $W = 1\text{m}$ is composed of three layers, as shown in the figure. The interior layer and outer layer are glass with thickness $t_g = 5$ mm, while the intermediate layer is an air gap with thickness $t_a = 5$ mm. Heat transfer through air gap is only by conduction. Thermal conductivity of glass and air are $k_g = 1.4$ W/m·K and $k_a = 0.024$ W/m·K, respectively. A thin coating is deposited on the inner surface of the interior layer and has an absorptivity of $\alpha = 0.1$. The window is subjected to solar radiation flux $G = 1.4$ kW/m^2. The glass and air may be assumed to be perfectly transparent, so that solar energy is only absorbed by the coating. Any other radiation effect, such as the radiation from the room and emission of the coating, can be neglected. Both side of the glass is subjected to convective heat transfer. The inside air temperature is $T_{\infty,i} = 25$ °C, and the outside (ambient) has a temperature of $T_{\infty,o} = 10$ °C and convection coefficient $h_o = 20$ W/m^2·K. At steady state, it is found that the coating has a temperature $T_c = 40$ °C.



(a) Determine the heat transfer rate, q, *from the coating to the air outside.*

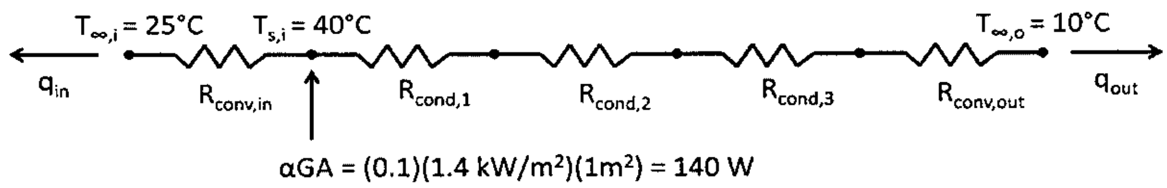
$$q = 113.004 \text{ W}$$

(b) Determine h_i, the convective heat transfer coefficient from the coating to the air in the room.

$$h_i = 1.8 \text{ W/m}^2\text{K}$$

<Solution>

(a)



$$A = W^2 = (1\,m)^2$$

$$R_{cond,1} = \frac{t_g}{k_g A} = \frac{0.005\,m}{(1.4\,W/m-K)(1\,m^2)} = 0.003571\,\frac{K}{W}$$

$$R_{cond,2} = \frac{t_a}{k_a A} = \frac{0.005\,m}{(0.024\,W/m-K)(1\,m^2)} = 0.208333\,\frac{K}{W}$$

$$R_{cond,3} = \frac{t_g}{k_g A} = \frac{0.005\,m}{(1.4\,W/m-K)(1\,m^2)} = 0.003571\,\frac{K}{W}$$

$$R_{conv,out} = \frac{1}{h_o A} = \frac{1}{(20\,W/m^2-K)(1\,m^2)} = 0.05\,\frac{K}{W}$$

$$R_{tot,out} = R_{cond,1} + R_{cond,2} + R_{cond,3} + R_{conv,out} = 0.265476\,\frac{K}{W}$$

$$q_{out} = \frac{T_{s,i} - T_{\infty,o}}{R_{tot,out}} = \frac{(40-10)°C}{0.265476\,\frac{K}{W}} = 113.004\,W$$

$$\therefore q_{out} = 113.004\,W$$

(b)

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$\dot{E}_{gen} = \dot{E}_{st} = 0$$

$$\therefore \dot{E}_{in} = \dot{E}_{out}$$

$$\dot{E}_{in} \Rightarrow \alpha\, G\, A = 140\ W$$

$$\dot{E}_{out} \Rightarrow q_{conv,in} + q_{conv,out}$$

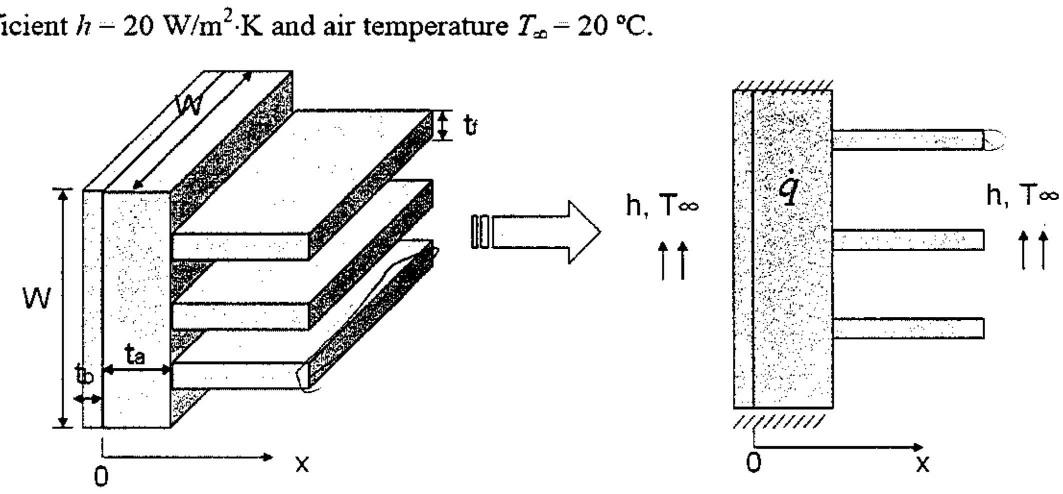
$$q_{conv,in} + q_{conv,out} = q_{conv,in} + 113.004\ W = 140\ W$$

$$q_{conv,in} = 140\ W - 113.004\ W = 26.996\ W$$
$$= h_i\,(T_{s,i} - T_{\infty,i})\,A = h_i\,(40-25)°C \cdot (1\,m^2)$$

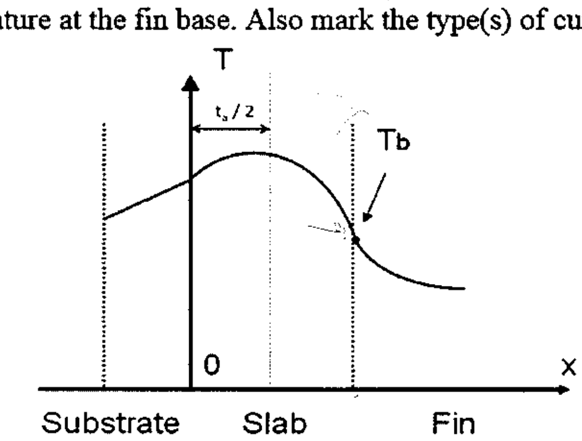
$$h_i = \frac{26.996\ W}{(40-25)°C \cdot (1\,m^2)} = 1.79973\ W/m^2 - K$$

$$\boxed{\therefore h_i = 1.79973\ W/m^2 - K}$$

2. (40 points) A rectangular metallic slab with thickness $t_a = 40$ mm is mounted on an electric insulating substrate with thickness $t_b = 10$ mm and thermal conductivity $k_b = 10$ W/m·K. The side surfaces of the slab and the substrate are adiabatic such that you may assume 1-D conduction heat transfer. Heat is generated in the metallic slab at a rate of $\dot{q} = 6\times10^4$ W/m^3. To dissipate the heat, three *very long* rectangular fins with thermal conductivity $k_f = 200$ W/m·K are attached to the other side of the slab. The slab and substrate are square and has a width $W = 50$ mm, as indicated in the sketch. The fins have the same width W as the slab, and the thickness is $t_f = 5$ mm. Both the substrate and the fin array are subjected to convective heat transfer with a coefficient $h = 20$ W/m^2·K and air temperature $T_\infty = 20$ °C.



(a) Sketch the temperature distribution in the substrate, the metallic slab, and along the fin. T_b indicates the temperature at the fin base. Also mark the type(s) of curves you are sketching.



(b) If the steady state temperature at $x = 0$ m (i.e., the interface between the metallic slab and the substrate) is $T_0 = 40$ °C, determine the temperature T_b at the fin base.

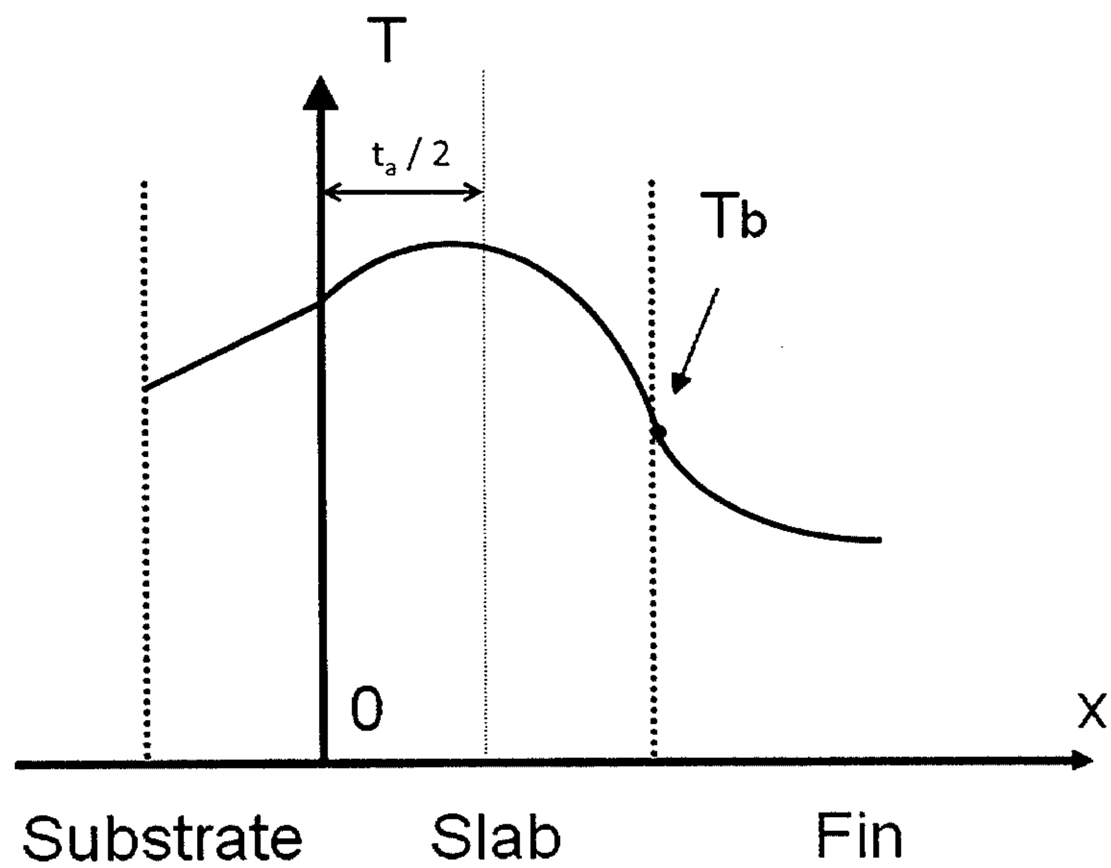
T_b = 24.87 °C

(c) Determine the thermal conductivity k_a of the metallic slab.

k_a = 2.136 W/mK

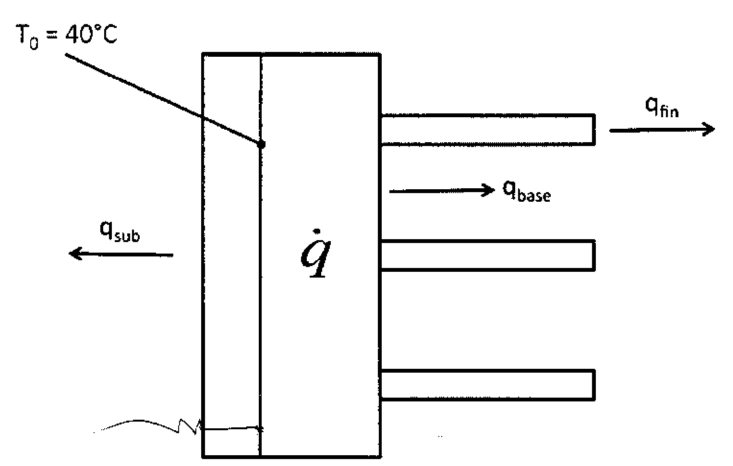
<Solution>

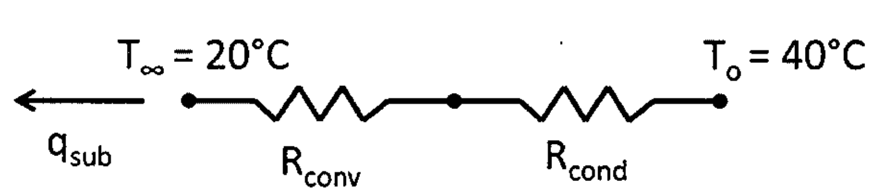
(a)



Location	Type of Curve
Substrate	Linear
Metallic Slab	Parabolic
Fins	Exponential

(b)





$$R_{conv} = \frac{1}{hA} = \frac{1}{hW^2} = \frac{1}{(20\ W/m^2-K)(0.05\ m)^2} = 20\ \frac{K}{W}$$

$$R_{cond} = \frac{t_b}{k_b A} = \frac{t_b}{k_b A} = \frac{0.010\ m}{(10\ W/m-K)(0.05\ m)^2} = 0.4\ \frac{K}{W}$$

$$q_{sub} = \frac{T_0 - T_\infty}{R_{conv} + R_{cond}} = \frac{(40-20)°C}{20.4\ \frac{K}{W}} = 0.980392\ W$$

$$q_{gen} = \dot{q} \cdot V = \dot{q} \cdot W^2 \cdot t_a = (6\times10^4\ W/m^3)(0.0025\ m^2)(0.040\ m) = 6.0\ W$$

$$q_{gen} = q_{sub} + 3 \cdot q_{fin} + q_{base} \Rightarrow 3 \cdot q_{fin} + q_{base} = q_{gen} - q_{sub} = 6.0\ W - 0.98\ W = 5.02\ W$$

$$q_{base} = h\,(T_b - T_\infty)\,A_b$$

$$A_b = A - 3 \cdot A_{c,fin} = W^2 - 3 \cdot W \cdot t_f = (0.0025\ m^2) - 3 \cdot (0.050\ m)(0.005\ m) = 0.00175\ m^2$$

Assuming an infinitely long fin,

$$q_f = M = \sqrt{h P k_f A_c}\,\theta_b \qquad ; \qquad \theta_b = T_b - T_\infty$$

$$h = 20\ W/m^2-K, \qquad P = 2\,(W + t_f) = 2\cdot(0.050 + 0.005)\ m = 0.11\ m$$
$$k_f = 200\ W/m-K, \qquad A_c = W\cdot t_f = (0.050\ m)\cdot(0.005\ m) = 0.00025\ m^2$$

$$\sqrt{h P k A_c} = \sqrt{(20\ W/m^2-K)(0.11\ m)(200\ W/m-K)(0.00025\ m^2)} = 0.331662\ \frac{W}{K}$$

$$\therefore q_{fin} = (0.331662\ \frac{W}{K})(T_b - T_\infty)K$$

$$3\cdot q_{fin} + q_{base} = 3\cdot(0.331662\ \frac{W}{K})(T_b - 20)K + (200\ W/m^2-K)(T_b - 20)\ (0.00175\ m^2) = 5.02\ W$$

$$\Rightarrow\ T_b = 24.87\ °C$$

$$\boxed{\therefore T_b = 24.87\ °C}$$

(c)

$$\frac{\partial}{\partial x}\left(k_a\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_a\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_a\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

1-D conduction, constant properties, and steady state $\Rightarrow k_a\frac{d^2T}{dx^2}+\dot{q}=0 \qquad \frac{d^2T}{dx^2}=-\frac{\dot{q}}{k_a}$

$\therefore T(x)=-2\frac{\dot{q}}{k_a}x^2+C_1x+C_2 \quad ; \quad BC's \Rightarrow T(x=0)=40\,°C \;\;\&\;\; T(x=t_a)=24.87\,°C$

$T(x=0)=C_2=40\,°C$

$T(x=t_a)=-2\frac{\dot{q}}{k_a}(0.04)^2+C_1\cdot(0.04)+40=24.87\,°C \Rightarrow C_1=\frac{\dot{q}}{2k_a}(0.04)-378.25$

$$\therefore T(x)=-\frac{\dot{q}}{2k_a}x^2+\left[\frac{\dot{q}}{2k_a}(0.04)-378.25\right]x+40$$

$\frac{dT}{dx}=-\frac{\dot{q}}{k_a}x+\frac{\dot{q}}{2k_a}(0.04)-378.25 \quad ; \quad \dot{q}=6\times10^4\ W/m^3$

$$-k_a\left.\frac{dT}{dx}\right|_{x=0}\cdot A=-k_a\left[\frac{\dot{q}}{2k_a}(0.04)-378.25\right](0.05\,m)^2=-0.98\,W$$

Solve for $k_a \Rightarrow k_a=2.136\,W/m-K$

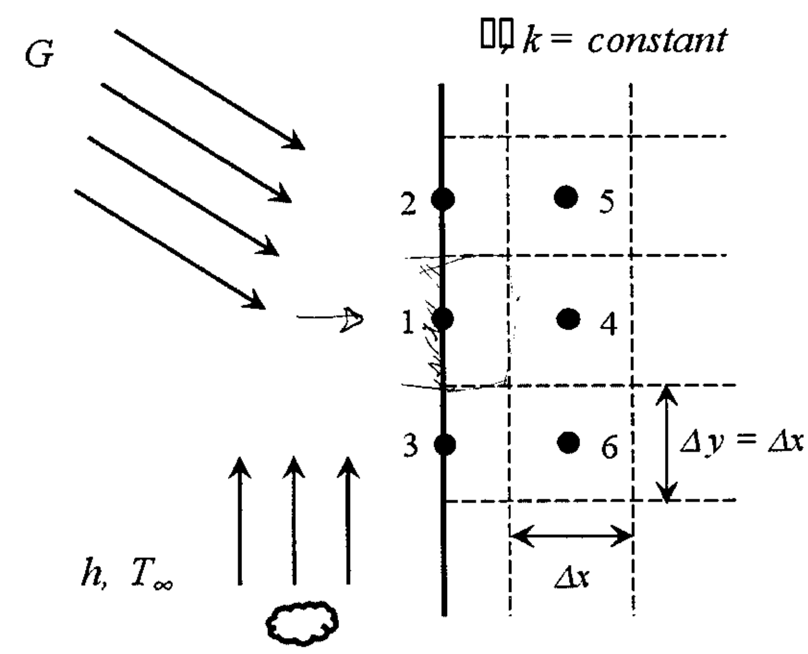
or

$$-k_a\left.\frac{dT}{dx}\right|_{x=t_a}\cdot A=-k_a\left[-\frac{\dot{q}}{k_a}(0.04)+\frac{\dot{q}}{2k_a}(0.04)-378.25\right](0.05\,m)^2=5.02\,W$$

Solve for $k_a \Rightarrow k_a=2.136\,W/m-K$

$$\boxed{\therefore k_a=2.136\,W/m-K}$$

3. (35 points) Develop a nodal (finite-difference) analysis for 2-D steady-state conduction heat transfer with volumetric heat generation and uniform irradiation G at a planar boundary. The surface has an absorptivity α, and is also exposed to convection as shown below. Radiation from the surface may be neglected.



(a) Use node 1 as the basis for the energy balance and solve for T_1 in terms of the other node temperatures and the variables indicated in the sketch.

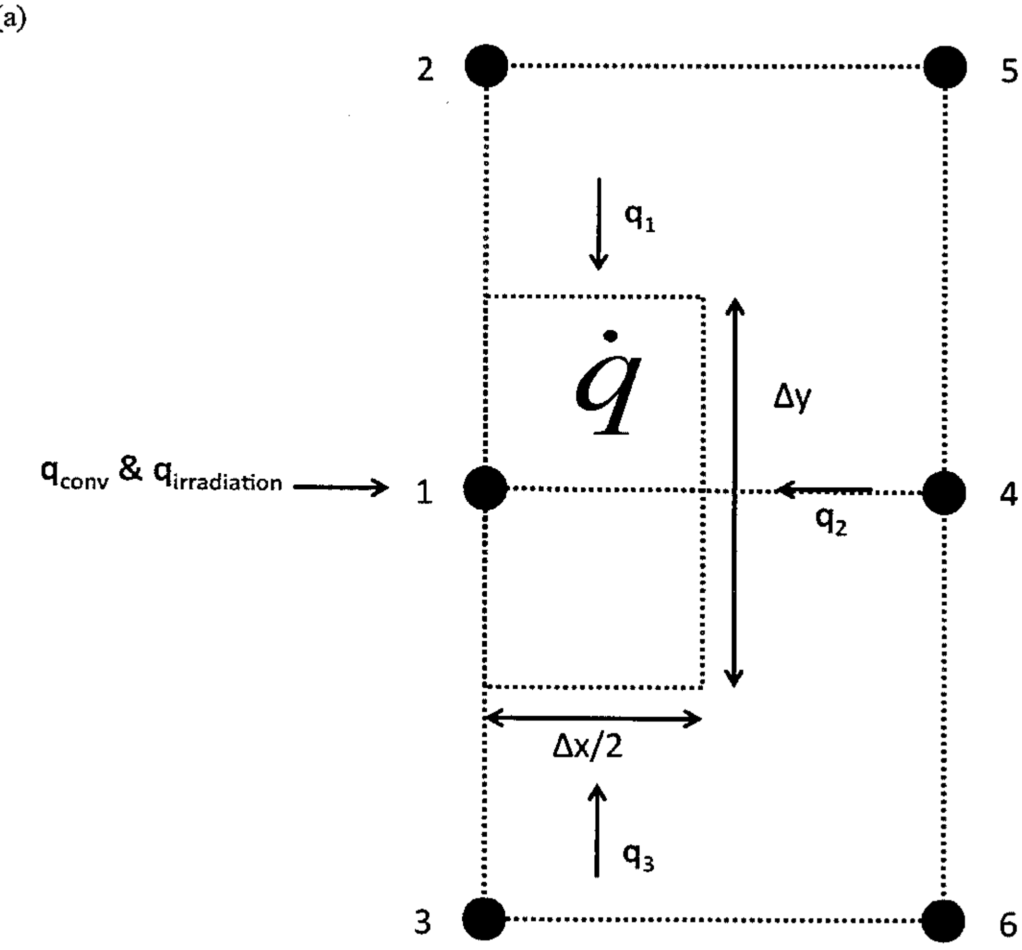
$$T_1 = \frac{1}{2\,h\,\Delta y + 4\,k}\left[2\left(h\,T_\infty + \alpha\,G\right)\Delta y + k\left(T_2 + T_3 + 2\,T_4\right) + \dot{q}\,\Delta x\,\Delta y\right]$$

(b) Now, determine T_1 for $T_2 = 300$ K, $T_3 = 360$ K, $T_4 = 400$ K, $T_5 = 330$ K, $T_6 = 430$ K, $G = 500\ \text{W/m}^2$, $\Delta x = \Delta y = 1$ m, $\alpha = 1$, and $k = 10$ W/mK, $= 5{,}000\ \text{W/m}^3$, $h = 50\ \text{W/m}^2\text{K}$, and $T_\infty = 280$ K.

$T_1 = 347.143$ K

<Solution>

(a)



$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st} \quad ; \ \dot{E}_{out} = \dot{E}_{st} = 0$$

$$\therefore \dot{E}_{in} + \dot{E}_{gen} = 0$$

$$q_{conv} = h\,(T_\infty - T_1)\,\Delta y$$

$$q_{irradiation} = \alpha\, G\, \Delta y$$

$$q_1 = k\,\frac{T_2 - T_1}{\Delta y}\,\frac{\Delta x}{2}$$

$$q_2 = k\,\frac{T_4 - T_1}{\Delta x}\,\Delta y$$

$$q_3 = k\,\frac{T_3 - T_1}{\Delta y}\,\frac{\Delta x}{2}$$

$$q_{gen} = \dot{q}\cdot V = \dot{q}\cdot\frac{\Delta x}{2}\cdot\Delta y$$

$$q_{conv} + q_{irradiation} + q_1 + q_2 + q_3 + q_{gen} = 0$$

$$\Rightarrow h\,(T_\infty - T_1)\,\Delta y + \alpha\, G\,\Delta y + \frac{k}{2}(T_2 - T_1) + k\,(T_4 - T_1) + \frac{k}{2}(T_3 - T_1) + \dot{q}\cdot\frac{\Delta x}{2}\cdot\Delta y = 0$$

$$T_1 = \frac{1}{2\,h\,\Delta y + 4\,k}\left[2\,(h\,T_\infty + \alpha\, G)\,\Delta y + k\,(T_2 + T_3 + 2\,T_4) + \dot{q}\,\Delta x\,\Delta y\right]$$

$$\boxed{\therefore T_1 = \frac{1}{2\,h\,\Delta y + 4\,k}\left[2\,(h\,T_\infty + \alpha\, G)\,\Delta y + k\,(T_2 + T_3 + 2\,T_4) + \dot{q}\,\Delta x\,\Delta y\right]}$$

(b)

$T_2 = 300\ K$

$T_3 = 360\ K$

$T_4 = 400\ K$

$T_\infty = 280\ K$

$G = 500\ W/m^2$

$\alpha = 1$

$k = 10\ W/m-K$

$\dot{q} = 5{,}000\ W/m^3$

$h = 50\ W/m^2-K$

Using the given values with the equation from part (a), $T_1 = 347.143\ K$

$$\therefore T_1 = 347.143\ K$$