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ME315 Heat and Mass Transfer

School of Mechanical Engineering, Purdue University

Exam #1
8 – 9:30 PM
February 25, 2016

Circle Your Division:

Div. 1 (Prof. Choi) **9:30 – 10:20 AM**

Div. 2 (Mr. Wang) **12:30 – 1:20 PM**

Instructions:

- This is a closed-book, closed-notes examination. *There is a formula sheet provided at the end of this exam.*
- You must turn off all communications devices before starting this exam, and leave them off for the entire exam.
- Please write legibly and show all work for receiving partial credit. Please show your final answers in the boxes provided.
- State all your assumptions.
- Please arrange all your sheets in the correct order.

Performance

1	15	
2	15	
3	15	
Total	45	

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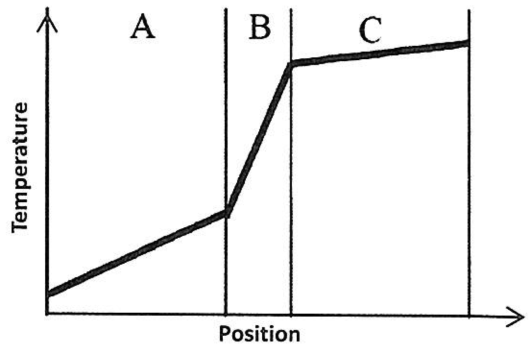
Problem 1 (15 points)

Answer the following questions with your reasoning.

(a) The temperature profile of a three-layer composite plane wall is shown in the figure on the right. You may assume 1-D steady state conduction with no internal heat generation. Based on the temperature profile, rank the thermal conductivities (k_A, k_B, and k_C) of the three layers from high to low by filling "A", "B", or "C" in the subscripts.

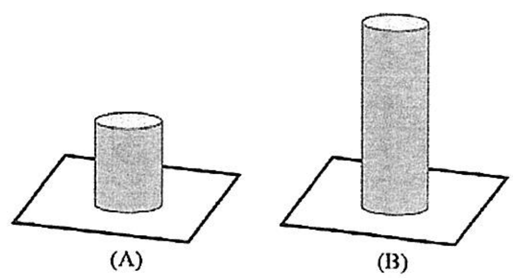
$k_{(\ )} > k_{(\ )} > k_{(\ )}$



(b) Consider two fins (**A** and **B** shown on the right) made of the same material, with the same circular cross-section area. These fins are attached to plates with the same base temperature (T_b), and subjected to the same convection condition (h and T_∞). Given that fin **B** is longer than fin **A**, compare the following quantities associated with the two fins by filling ">", "=", or "<" in the blanks.

(1) Total fin heat transfer rate: $q_{f,A}$ _____ $q_{f,B}$

(2) Fin effectiveness: $\varepsilon_{f,A}$ _____ $\varepsilon_{f,B}$



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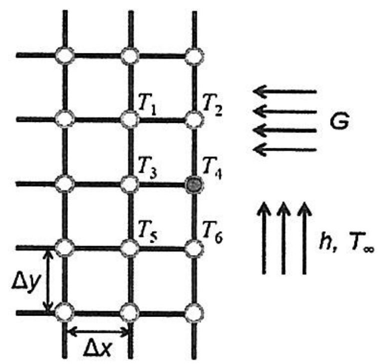
(c) A long, rectangular bar of thermal conductivity k is subjected to convection with a heat transfer coefficient h and an ambient air temperature T_∞ as well as radiation with a heat flux G. Part of the cross section is shown in the figure on the right. Write the 2-D nodal (finite-difference) equation for steady-state heat transfer with constant properties and no internal heat generation. You may assume that the surface absorptivity of $\alpha = 1$, and radiation from the surface can be neglected. You are also given that $\Delta x = \Delta y$. Using the energy balance method, solve for T_4 in terms of the given symbols.



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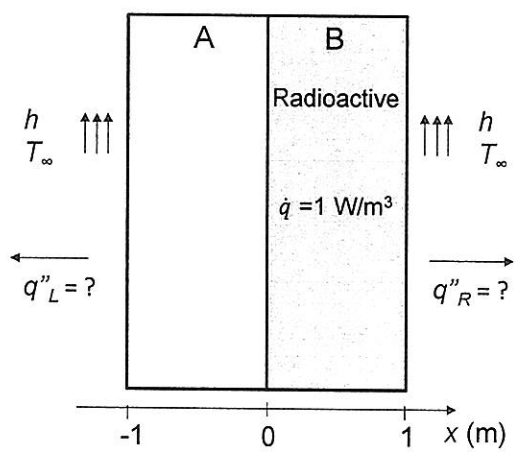
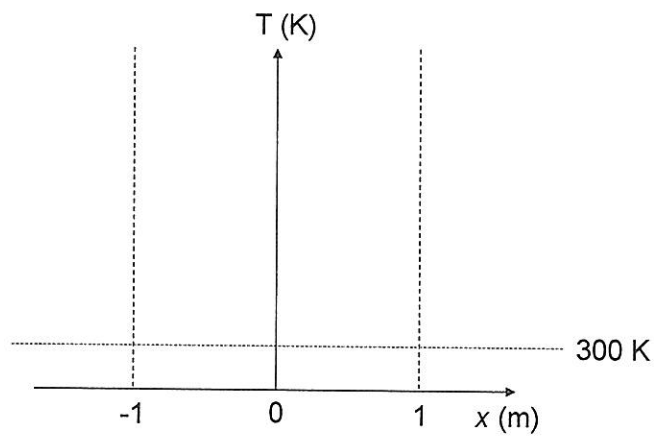
Problem 2 (15 points)

Consider a composite wall consisting of two layers (**A** and **B**). Both layers have the same thickness of $L_A = L_B = 1$ m and the identical thermal conductivity of $k_A = k_B = 0.5$ W/m K. Layer **B** is radioactive and provides a uniform volumetric heat generation rate $\dot{q} = 1$ W/m^3. The left ($x = -1$ m) and right ($x = 1$ m) surfaces of the composite wall are subject to convection with a heat transfer coefficient $h = 1$ W/m^2 K and an ambient temperature $T_\infty = 300$ K. The contact resistance between layers **A** and **B** is negligible. You may assume steady state and neglect any edge effects.



(a) Qualitatively sketch the temperature profile below.



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(b) Calculate the heat fluxes (q_L'' and q_R'') out of the left and right surfaces of the composite wall.

(c) Calculate the temperature profiles in the layers **A** and **B** of the composite wall.

(d) Determine the maximum temperature in the composite wall and the location.

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Problem 3 (15 points)

In an industrial atomization process, spherical droplets ($d = 2$ cm) of molten metal are formed, initially at $T_i = 1500$ K. An engineer is asked to investigate the cooling process of the droplets when exposed to convection at $T_\infty = 300$ K with a heat transfer coefficient of $h = 6000$ W/m^{2}K. Radiation losses may be neglected as a first approximation. You are given the following properties of molten metal: thermal conductivity $k = 60$ W/m K; heat capacity $C_p = 400$ J/kg K; density $\rho = 9000$ kg/m^3.

(a) Assuming the lumped thermal capacitance model, determine the thermal time constant, τ, of the droplet and the temperature at $t = 2\tau$.

(b) Calculate the total energy (J) lost by the droplet during this time ($0 - 2\tau$).

(c) Evaluate the validity of the uniform temperature assumption. Provide your reasoning.

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(d) If your answer in (c) is valid, stop here and proceed to other problems. If it is invalid, determine the temperature at the surface of the sphere at $t = 2\tau$.

ME 315 Exam 1
BASIC EQUATION SHEET

Conservation Laws

Control Volume Energy Balance: $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}$; $\dot{E}_{st} = mC_p \, dT/dt$; $\dot{E}_{gen} = \dot{q}V$

Surface Energy Balance: $\dot{E}_{in} - \dot{E}_{out} = 0$

Conduction

Fourier's Law: $q''_{cond,x} = -k\frac{\partial T}{\partial x}$; $q''_{cond,n} = -k\frac{\partial T}{\partial n}$; $q_{cond} = q''_{cond}A$

Heat Diffusion Equation:

Cartesian Coordinates: $\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q} = \rho C_p\frac{\partial T}{\partial t}$

Cylindrical Coordinates: $\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q} = \rho C_p\frac{\partial T}{\partial t}$

Spherical Coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\dot{q} = \rho C_p\frac{\partial T}{\partial t}$$

Thermal Resistance:

Conduction Resistance: Plane wall: $R_{t,cond} = \frac{L}{kA}$; Cylindrical shell: $R_{t,cond} = \frac{\ln(r_o/r_i)}{2\pi lk}$;

Spherical shell: $R_{t,cond} = \frac{(1/r_i)-(1/r_o)}{4\pi k}$

Convection Resistance: $R_{t,conv} = \frac{1}{h_{conv}A}$

Radiation Resistance: $R_{t,rad} = \frac{1}{h_{rad}A}$

Thermal Energy Generation:

Plane wall with uniform energy generation, given temperature T_s at both surfaces, width = $2L$, and x=0 at the center of the wall:

$$T(x) - T_s = \frac{\dot{q}L^2}{2k}\left(1-\frac{x^2}{L^2}\right)$$

General solution for plane wall with uniform energy generation:

$$T(x) = -\frac{\dot{q}x^2}{2k} + C_1x + C_2$$

Cylinder with uniform energy generation, surface temperature T_s:

$$T(r) - T_s = \frac{\dot{q} r_o^2}{4k}\left(1 - \frac{r^2}{r_o^2}\right)$$

General solution for a cylindrical shell, with uniform heat generation:

$$T(r) = -\frac{\dot{q}}{4k} r^2 + C_1 \ln r + C_2$$

Extended Surfaces/Fins:

Convective Tip: $\dfrac{\theta(x)}{\theta_b} = \dfrac{\cosh\left[m(L-x)\right] + (h/mk)\sinh\left[m(L-x)\right]}{\cosh(mL) + (h/mk)\sinh(mL)}$

$$q_{fin} = (hPkA_c)^{1/2}\,\theta_b \frac{\sinh(mL) + (h/mk)\cosh(mL)}{\cosh(mL) + (h/mk)\sinh(mL)}$$

Adiabatic Tip: $\dfrac{\theta(x)}{\theta_b} = \dfrac{\cosh\left[m(L-x)\right]}{\cosh(mL)}$; $q_{fin} = (hPkA_c)^{1/2}\,\theta_b \tanh(mL)$

Prescribed Tip Temperature: $\dfrac{\theta(x)}{\theta_b} = \dfrac{(\theta_L/\theta_b)\sinh(mx) + \sinh\left[m(L-x)\right]}{\sinh(mL)}$

$$q_{fin} = (hPkA_c)^{1/2}\,\theta_b \frac{\cosh(mL) - (\theta_L/\theta_b)}{\sinh(mL)}$$

Infinitely Long Fin: $\dfrac{\theta(x)}{\theta_b} = e^{-mx}$; $q_{fin} = (hPkA_c)^{1/2}\,\theta_b$

where $m^2 = \dfrac{hP}{kA_c}$; $\theta_b = T_b - T_\infty$; $q_{fin} = q_{conv,finsurface} + q_{conv,tip}$; $q_{conv,tip} = hA_c\theta_L$

$$\sinh x = \frac{e^x - e^{-x}}{2};\ \cosh x = \frac{e^x + e^{-x}}{2};\ \tanh x = \frac{\sinh x}{\cosh x}$$

Fin Effectiveness: $\varepsilon_{fin} = \dfrac{q_{fin}}{hA_{c,b}\theta_b}$; $\varepsilon_{fin} = \dfrac{R_{t,conv-base}}{R_{t,cond-fin}}$

Fin Efficiency: $\eta_{fin} = \dfrac{q_{fin}}{hA_{fin}\theta_b}$; for adiabatic tip, $\eta_{fin} = \dfrac{\tanh(mL)}{mL}$; an extended length of a fin using adiabatic tip $L_c = L + \dfrac{A_c}{P}$; $\eta_{fin} = \dfrac{\tanh(mL_c)}{mL_c}$

Fin array:

$$\eta_o = \frac{q_{total}}{hA_{total}\theta_b} = 1 - \frac{NA_{fin}}{A_{total}}(1 - \eta_{fin});\ R_{t,cond-fin} = \frac{1}{\eta_{fin} h A_{fin}};\ R_{t,cond-fin\,array} = \frac{1}{\eta_o h A_{total}}$$

Two Dimensional Steady Conduction:

Conduction Shape Factor: $R_{t,cond} = \dfrac{1}{Sk}$

Finite Volume Method: For uniform mesh, interior point, no energy generation, steady state,

$$T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} = 4T_{i,j}$$

Transient Conduction:

Lumped Capacitance Analysis: $Bi = \dfrac{R_{t-cond}}{R_{t-conv}} = \dfrac{h_{conv}L_c}{k_{solid}}$;

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{t}{\tau_t}\right);\ \tau_t = \frac{\rho V C_p}{h_{conv}A_s} = C_{t,solid}R_{t,conv}$$

$$Fo = \frac{\alpha t}{L_c^2}\ ;\ \frac{\theta}{\theta_i} = \exp\left[-\left(\frac{h_{conv}L_c}{k_{solid}}\right)\left(\frac{\alpha t}{L_c^2}\right)\right] = \exp\left[-(Bi)(Fo)\right]$$

Thermal diffusivity $\alpha = \dfrac{k}{\rho C_p}$

To use Bi to estimate the approximation of the Lumped Capacitance Method, for plane slab, $L_c = L$; for cylinder, $L_c = r_o/2$; for sphere, $L_c = r_o/3$.

Analytical Solutions for Transient Conduction with Spatial Effect:

$$\theta^* = \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty};\ x^* = \frac{x}{L};\ r^* = \frac{r}{r_o};\ t^* = \frac{\alpha t}{L^2};\ Q_o = \rho C_p V (T_i - T_\infty)$$

Plane Wall: $\theta^* \cong C_1 \exp\left(-\zeta_1^2 Fo\right)\cos\left(\zeta_1 x^*\right);\ Fo = t^* = \dfrac{\alpha t}{L^2};\ Bi = \dfrac{hL}{k}$

Center temperature at x=0: $\theta_o^* = C_1 \exp\left(-\zeta_1^2 Fo\right);\ \dfrac{Q}{Q_o} = 1 - \theta_o^* \dfrac{\sin(\zeta_1)}{\zeta_1}$

Long Cylinder: $\theta^* \cong C_1 \exp\left(-\zeta_1^2 Fo\right) J_0\left(\zeta_1 r^*\right);\ Fo = t^* = \dfrac{\alpha t}{r_o^2};\ Bi = \dfrac{hr_o}{k}$

Center temperature at r =0: $\theta_o^* = C_1 \exp\left(-\zeta_1^2 Fo\right);\ \dfrac{Q}{Q_o} = 1 - 2\theta_o^* \dfrac{J_1(\zeta_1)}{\zeta_1}$

Sphere: $\theta^* \cong C_1 \exp\left(-\zeta_1^2 Fo\right)\dfrac{\sin\left(\zeta_1 r^*\right)}{\zeta_1 r^*};\ Fo = t^* = \dfrac{\alpha t}{r_o^2},\ Bi = \dfrac{hr_o}{k}$

Center temperature at r =0: $\theta_o^* = C_1 \exp\left(-\zeta_1^2 Fo\right);\ \dfrac{Q}{Q_o} = 1 - 3\theta_o^* \dfrac{\left[\sin(\zeta_1) - \zeta_1 \cos(\zeta_1)\right]}{\zeta_1^3}$

$J_0(x)$ and $J_1(x)$ are the 0th and 1st order Bessel functions of the first kind and their values will be provided, if needed.

Semi-Infinite Solid: Similarity variable $\eta = \frac{x}{\sqrt{4\alpha t}}; \frac{d^2T}{d\eta^2} = -2\eta\frac{dT}{d\eta}$

Constant Surface Temperature (T_s): $\frac{T(\mathrm{x,t}) - T_s}{T_i - T_s} = erf\left(\frac{x}{\sqrt{4\alpha t}}\right); q_s''(t) = \frac{k(T_s - T_i)}{\sqrt{\pi\alpha t}}$

Constant Surface Heat Flux (q_o''): $T(\mathrm{x,t}) - T_i = \frac{2q_o''}{k}\sqrt{\frac{\alpha t}{\pi}}\exp\left(\frac{-x^2}{4\alpha t}\right) - \frac{q_o'' x}{k} erfc\left(\frac{x}{\sqrt{4\alpha t}}\right);$

Surface Convection ($-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = h\left[T_\infty - T(0,t)\right]$):

$$\frac{T(\mathrm{x,t}) - T_i}{T_\infty - T_i} = erfc\left(\frac{x}{\sqrt{4\alpha t}}\right) - \left[\exp\left(\frac{hx}{k} + \frac{h^2\alpha t}{k}\right)\right]\left[erfc\left(\frac{x}{\sqrt{4\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right]$$

Convection

Newton's Law of Cooling: $q_{conv}'' = h_{conv}(T_s - T_\infty)$; $q_{conv} = q_{conv}'' A$

Radiation

Emissive power = $E = \varepsilon\sigma T_s^4$

Irradiation received by surface from large surroundings: $G = \sigma T_{surr}^4$

Irradiation absorbed by surface: αG

Irradiation reflected by surface: ρG

Gray surface: $\varepsilon = \alpha$

Opaque surface: $\alpha + \rho = 1$

Radiative heat flux from a gray surface at T_s to a large surroundings at T_{surr}:

$$q_{rad}'' = \varepsilon\sigma\left(T_s^4 - T_{surr}^4\right) = h_{rad}\left(T_s - T_{surr}\right)$$

$$h_{rad} = \varepsilon\sigma\left(T_s^2 + T_{surr}^2\right)\left(T_s + T_{surr}\right)$$

Useful Constants

σ = Stefan-Boltzmann's Constant = $5.67\times10^{-8}\ \frac{\mathrm{W}}{\mathrm{m^2K^4}}$

Geometry

Cylinder: $A = 2\pi r l$; $V = \pi r^2 l$

Sphere: $A = 4\pi r^2$; $V = \frac{4}{3}\pi r^3$

Triangle: $A = bh/2$ b: base h: height

TABLE 5.1 Coefficients used in the one-term approximation to the series solutions for transient one-dimensional conduction

	Plane Wall		Infinite Cylinde		Sphere	
Bi^a	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.03	0.1723	1.0049	0.2440	1.0075	0.2991	1.0090
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.05	0.2218	1.0082	0.3143	1.0124	0.3854	1.0149
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.07	0.2615	1.0114	0.3709	1.0173	0.4551	1.0209
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.09	0.2956	1.0145	0.4195	1.0222	0.5150	1.0268
0.10	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.15	0.3779	1.0237	0.5376	1.0365	0.6609	1.0445
0.20	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.25	0.4801	1.0382	0.6856	1.0598	0.8447	1.0737
0.30	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0932	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0919	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5994	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2402	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	1.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2881	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2733	2.4050	1.6018	3.1415	2.0000

[a] $Bi = hL/k$ for the plane wall and hr_o/k for the infinite cylinder and sphere. See Figure 5.6.