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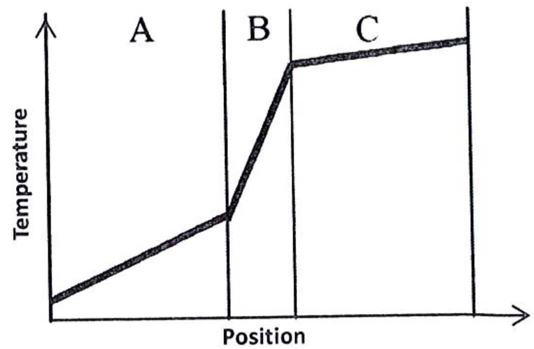
Problem 1 (15 points)

Answer the following questions with your reasoning.

(a) The temperature profile of a three-layer composite plane wall is shown in the figure on the right. You may assume 1-D steady state conduction with no internal heat generation. Based on the temperature profile, rank the thermal conductivities (k_A, k_B, and k_C) of the three layers from high to low by filling "A", "B", or "C" in the subscripts.

$k_{(C)} > k_{(A)} > k_{(B)}$



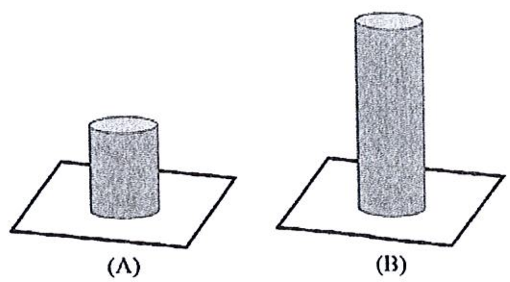
No heat generation : $q = \text{const}$

$$k\left|\frac{dT}{dx}\right| = \text{const}$$

$$k \propto 1 / \left|\frac{dT}{dx}\right|$$

$$\left|\frac{dT}{dx}\right| : B > A > C$$

(b) Consider two fins (**A** and **B** shown on the right) made of the same material, with the same circular cross-section area. These fins are attached to plates with the same base temperature (T_b), and subjected to the same convection condition (h and T_∞). Given that fin **B** is longer than fin **A**, compare the following quantities associated with the two fins by filling ">", "=", or "<" in the blanks.



(1) Total fin heat transfer rate: $q_{f,A}$ <u> < </u> $q_{f,B}$

(2) Fin effectiveness: $\varepsilon_{f,A}$ <u> < </u> $\varepsilon_{f,B}$

$\because$ same h. A has smaller area

$\therefore \quad q_A < q_B$

$\varepsilon_f = q_{fin} / q_{base}$ $\qquad q_{base}$ is the same

$\therefore \quad \varepsilon_{f,A} < \varepsilon_{f,B}$

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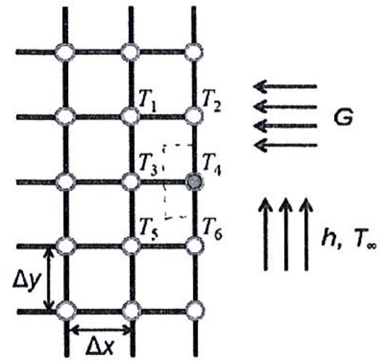
(c) A long, rectangular bar of thermal conductivity k is subjected to convection with a heat transfer coefficient h and an ambient air temperature T_∞ as well as radiation with a heat flux G. Part of the cross section is shown in the figure on the right. Write the 2-D nodal (finite-difference) equation for steady-state heat transfer with constant properties and no internal heat generation. You may assume that the surface absorptivity of $\alpha = 1$, and radiation from the surface can be neglected. You are also given that $\Delta x = \Delta y$. Using the energy balance method, solve for T_4 in terms of the given symbols.



Consider control volume around T_4

Energy balance:

$$q_{2\to4} + q_{3\to4} + q_{6\to4} + q_{conv} + q_{rad} = 0$$

$$k\,\frac{T_2 - T_4}{\Delta y}\left(\frac{\Delta x}{2}\cdot 1\right) + k\,\frac{T_3 - T_4}{\Delta x}(\Delta y \cdot 1) + k \cdot \frac{T_6 - T_4}{\Delta y}\left(\frac{\Delta x}{2}\cdot 1\right)$$

$$+ h(\Delta y \cdot 1)(T_\infty - T_4) + G(\Delta y \cdot 1) = 0$$

$\Delta x = \Delta y$

$$\therefore\ k(T_2 - T_4) + 2(T_3 - T_4) + k(T_6 - T_4) + 2h\Delta y(T_\infty - T_4) + 2G\Delta y = 0$$

$$\therefore\ T_4 = \frac{k(T_2 + 2T_3 + T_6) + 2h\Delta y T_\infty + 2G\Delta y}{4k + 2h\Delta y}$$

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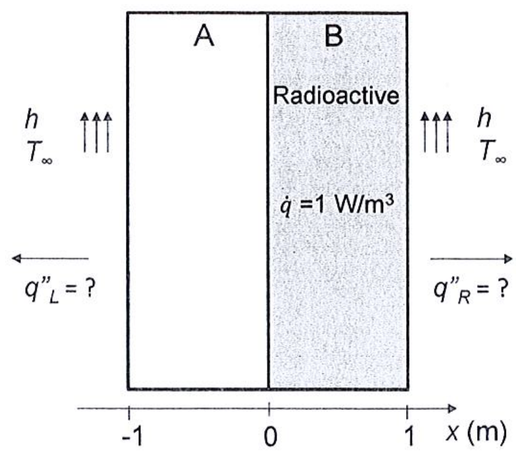
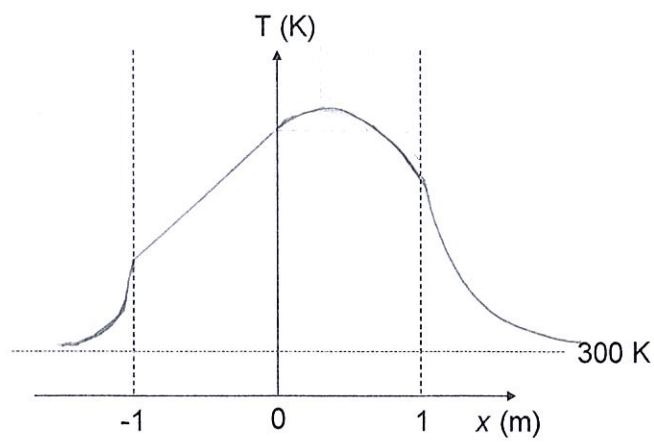
Problem 2 (15 points)

Consider a composite wall consisting of two layers (**A** and **B**). Both layers have the same thickness of $L_A = L_B = 1$ m and the identical thermal conductivity of $k_A = k_B = 0.5$ W/m K. Layer **B** is radioactive and provides a uniform volumetric heat generation rate $\dot{q} = 1$ W/m^3. The left (x = -1 m) and right (x = 1 m) surfaces of the composite wall are subject to convection with a heat transfer coefficient $h = 1$ W/m^2 K and an ambient temperature $T_\infty = 300$ K. The contact resistance between layers **A** and **B** is negligible. You may assume steady state and neglect any edge effects.



(a) Qualitatively sketch the temperature profile below.



(b) Calculate the heat fluxes (q_L'' and q_R'') out of the left and right surfaces of the composite wall.

constant flux in slab A

$$q_A'' = -q_L'' = q_x''\Big|_{x=0} = -k\frac{dT}{dx}\Big|_{x=0}$$

In slab:

$$k\frac{d^2T}{dx^2} + \dot{q} = 0$$

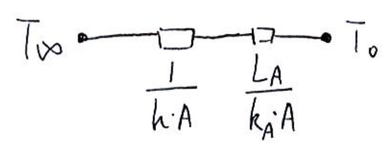
$$\frac{d^2T}{dx^2} = -\frac{\dot{q}}{k} = -2$$

$$\frac{dT}{dx} = -2x + C_1$$

$$T = -x^2 + C_1 x + C_2 \quad (\text{for } 0 \le x \le 1)$$

$$T_0 \equiv T\big|_{x=0} = C_2$$

$$T_1 \equiv T\big|_{x=1} = -1 + C_1 + C_2$$



$$q_L'' = k\frac{dT}{dx}\Big|_{x=0} = \frac{C_1}{2} \quad ①$$

$$q_R'' = -k\frac{dT}{dx}\Big|_{x=1} = \frac{2-C_1}{2} \quad ②$$

consider convective Boundary condition

$$q_L'' = \frac{T_0 - T_\infty}{\frac{1}{h} + \frac{L}{k}} = \frac{1}{3}(C_2 - T_\infty) \quad ③$$

$$q_R'' = \frac{T_1 - T_\infty}{\frac{1}{h}} = -1 + C_1 + C_2 - T_\infty \quad ④$$

From equation ① & ③:

$$\frac{C_1}{2} = \frac{1}{3}(C_2 - T_\infty)$$

From equation ② & ④:

$$\frac{2-C_1}{2} = -1 + C_1 + (C_2 - T_\infty)$$

solve the equation set:

$$\begin{cases} C_1 = \frac{2}{3} \\ C_2 = T_\infty + 1 = 301 \end{cases}$$

Therefore

$$\begin{cases} q_L'' = \frac{1}{3} \ W/m^2 \\ q_R'' = \frac{2}{3} \ W/m^2 \end{cases}$$

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(c) Calculate the temperature profiles in the layers **A** and **B** of the composite wall.

In A: $q_A'' = -q_L'' = -\frac{1}{3}$

$q_A'' = -k\frac{dT}{dx}$

$\therefore \quad -k\frac{dT}{dx} = -\frac{1}{3}$

$\frac{dT}{dx} = \frac{1}{3}/k = \frac{2}{3}$

Since $T_0 = C_2 = 301$ K

$T = \frac{2}{3}x + 301$ in A $(x \in [-1, 0])$

In B: $T = -x^2 + \frac{2}{3}x + 301$, as solved in (b)

(d) Determine the maximum temperature in the composite wall and the location.

maximum temperature should happen in B

$T(x) = -x^2 + \frac{2}{3}x + 301$

$= -x^2 + \frac{2}{3}x - (\frac{1}{3})^2 + (\frac{1}{3})^2 + 301$

$= -(x - \frac{1}{3})^2 + 301.11$

maximum occurs at $x = \frac{1}{3}$ m

$T_{max} = 301.11$ K

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Problem 3 (15 points)

In an industrial atomization process, spherical droplets ($d = 2$ cm) of molten metal are formed, initially at $T_i = 1500$ K. An engineer is asked to investigate the cooling process of the droplets when exposed to convection at $T_\infty = 300$ K with a heat transfer coefficient of $h = 6000$ W/m^{2}K. Radiation losses may be neglected as a first approximation. You are given the following properties of molten metal: thermal conductivity $k = 60$ W/m K; heat capacity $C_p = 400$ J/kg K; density $\rho = 9000$ kg/m^3.

(a) Assuming the lumped thermal capacitance model, determine the thermal time constant, τ, of the droplet and the temperature at $t = 2\tau$.

$$\tau = \rho V C_p / hA_s = \rho \cdot \tfrac{4}{3}\pi r_0^3 \cdot C_p / (h \cdot 4\pi r_0^2) = \frac{\rho \cdot C_p \cdot r_0}{3h} = 2s$$

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-t/\tau} = e^{-2\tau/\tau} = e^{-2}$$

$$\therefore T = (T_i - T_\infty) e^{-2} + T_\infty = 462.4 \text{ K}$$

(b) Calculate the total energy (J) lost by the droplet during this time ($0 - 2\tau$).

$$Q_{loss} = \rho V C_p (T_i - T) = \rho \cdot \tfrac{4}{3}\pi r_0^3 \cdot C_p (T_i - T) = 1.565 \times 10^4 \text{ J}$$

(c) Evaluate the validity of the uniform temperature assumption. Provide your reasoning.

$$Bi = \frac{hL_c}{k} = \frac{h \cdot \frac{r_0}{3}}{k} = 0.333 > 1, \quad \text{invalid}$$

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(d) If your answer in (c) is <u>valid</u>, stop here and proceed to other problems. If it is <u>invalid</u>, determine the temperature at the surface of the sphere at $t = 2\tau$.

$$F_0 = \frac{\alpha t}{r_0^2} = \frac{k/(\rho c_p) \cdot t}{r_0^2} = 0.667 > 0.2$$

use 1-term approximation

$$\frac{T - T_\infty}{T_i - T_\infty} = C_1 e^{-\zeta_1^2 F_0} \cdot \left[\frac{\sin(\zeta r_i^*)}{\zeta r^*} \right]$$

recalculate $Bi = \frac{h r_0}{k} = 1$

$\therefore \quad \zeta_1 = 1.5708$

$C_1 = 1.2732$

$$\frac{T - T_\infty}{T_i - T_\infty} = 0.1563$$

$T = 487.6 \ K$