Name: ______________________________

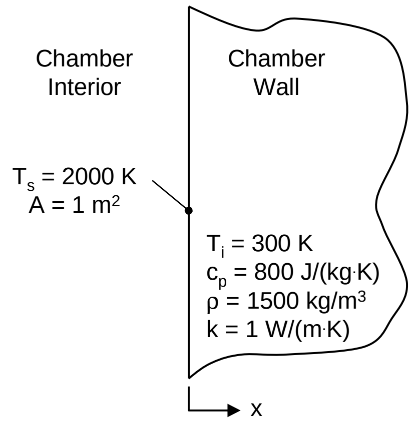
ME 315: Heat and Mass Transfer
Spring 2008
EXAM 2
Tuesday, 18 March 2008
7:00 to 8:00 PM

Instructions: This is an open-book exam. You may refer to your course textbook, your class notes and your graded homework assignments. Show all work clearly for maximum credit. **CIRCLE ALL ANSWERS**.

Problem	Score
1	/ 16
2	/ 22
3	/ 22
Total	/ 60

Name: ______________________________

1. (16 points) A **<u>very thick</u>** insulating wall of a large chemical reaction chamber is initially at 300 K and has a surface area of 1 m^2. Once the chemical reaction begins, the interior surface of the wall is suddenly exposed to a temperature of 2000 K for the duration of the reaction. Properties of the wall material are given in the figure.



   a. What is the temperature of the insulating wall at a depth of x = 5 cm from the surface at the instant of time that is 5 minutes from the start of the reaction?

   b. What is the rate of energy (W) transferred to the wall at the time instant, t = 5 min?

   c. Sketch how the temperature profile in the wall will look at various times and label all important features.

Problem 1

Given:

$T_i := (300)K$ $\qquad$ $t := 5min$ $\qquad$ $A := 1m^2$

$T_s := (2000)K$ $\qquad$ $x := 5cm$

$c_p := 800 \frac{J}{kg \cdot K}$ $\qquad$ $\rho := 1500 \frac{kg}{m^3}$ $\qquad$ $k := 1 \frac{W}{m \cdot K}$

(a)

Recognizing the wall as a semi-infinite medium, we can use equation 5.57 for the temperature:

$$\frac{T_x - T_s}{T_i - T_s} = \mathrm{erf}\left(\frac{x}{2 \cdot \sqrt{\alpha \cdot t}}\right)$$

Evaluating some of the terms:

$$\alpha := \frac{k}{\rho \cdot c_p} \qquad \alpha = 8.333 \times 10^{-7} \frac{m^2}{s}$$

$$\eta := \frac{x}{2 \cdot \sqrt{\alpha \cdot t}} \qquad \eta = 1.581$$

$$erf_\eta := \mathrm{erf}\left(\frac{x}{2 \cdot \sqrt{\alpha \cdot t}}\right) \qquad erf_\eta = 0.975$$

Solving for Tx:

$$T_x := T_s + \left(T_i - T_s\right) \cdot \left(\mathrm{erf}\left(\frac{x}{2 \cdot \sqrt{\alpha \cdot t}}\right)\right) \qquad T_x = 343.09\,K$$

(b)

Using equation 5.58 for the heat flux at t = 5 min:

$$q_s := \frac{k \cdot A \cdot \left(T_s - T_i\right)}{\sqrt{\pi \cdot \alpha \cdot t}} \qquad q_s = 6.066 \times 10^4\,W$$

(c)

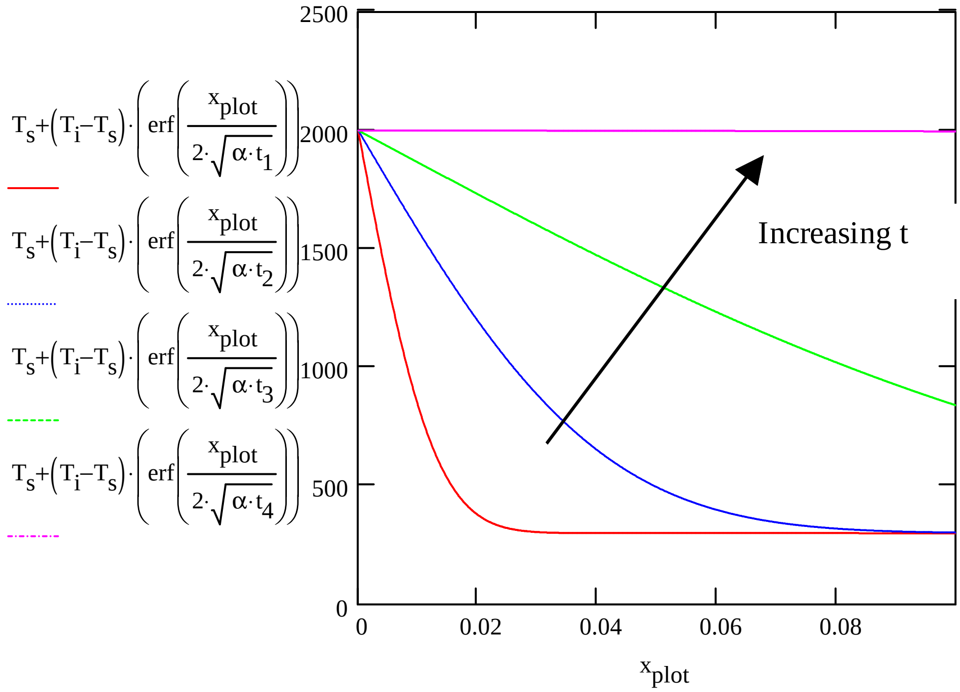
$t_1 := 1\text{min}$

$t_2 := 10\text{min}$

$t_3 := 100\text{min}$

$t_4 := 10000000\text{min}$

The plot should look something like this with the surface temperature at x=0 at 2000 K and exponentially decay to 300 K. As time increases, the exponential decrease occurs over a longer distance.



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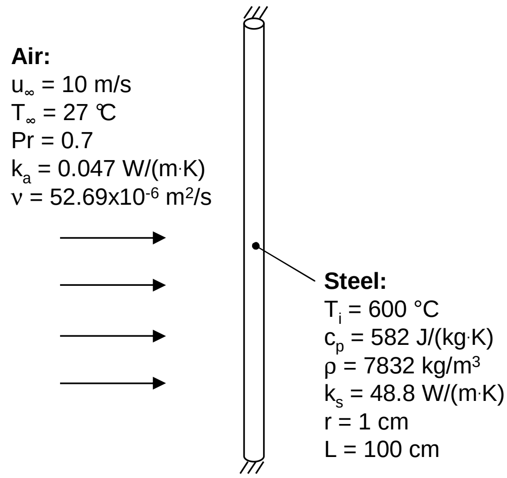
2. (22 points) Long steel rods (L = 100 cm, r = 1 cm) are annealed in a furnace and then allowed to cool in a crossflow of air at a velocity of u_∞ = 10 m/s and a temperature of T_∞ = 27 °C. The rods come out of the oven at 600 °C and may be considered safe to touch when the surface of the rods is at a temperature of 60 °C. You may assume the ends of the rods are well insulated. Properties for the cooling air (at an appropriate film temperature) and for the steel rods are provided in the figure to the right. Note that radiation is neglected in this problem.



a. How long does it take for the rods to reach the safe-to-touch temperature?

b. How much energy is lost from a single steel rod to the surrounding air if you let it cool to the safe-to-touch temperature?

c. What is the initial rate of heat transfer to the air at time t = 0 ?

Problem 2

Known convective quantities; properties for Air at assumed T_film = 600 K

$u := 10\frac{m}{s}$ $\nu := 52.69\cdot 10^{-6}\frac{m^2}{s}$

$T_{air} := (27 + 273)K$ $Pr := 0.7$

$k_{air} := 0.047\frac{W}{m\cdot K}$

Known steel (AISI 1010) properties (at 600 K ~ 330 C)

$r := 1cm$ $T_i := (600 + 273)K$ $k_s := 48.8\frac{W}{m\cdot K}$ $\rho := 7832\frac{kg}{m^3}$

$D := 2r$ $T_f := (60 + 273)K$

$L := 100cm$ $c_p := 582\frac{J}{kg\cdot K}$

(a)

Calculate the Reynolds number

$Re_D := \frac{u\cdot D}{\nu}$ $Re_D = 3.796\times 10^3$ Therefore, the flow is Laminar since Re < 5x10^5

Using equation 7.52 to solve for h

$C_{air} := 0.683$

$m_{air} := 0.466$

$$Nu_{D1} := C_{air}\cdot Re_D^{m_{air}}\cdot Pr^{\frac{1}{3}}$$

$Nu_{D1} = 28.232$

$$h_1 := \frac{k_{air}\cdot Nu_{D1}}{D}$$

$h_1 = 66.345\frac{W}{m^2\cdot K}$

$$Biot := \frac{h_1\cdot r}{2\cdot k_s}$$

$Biot = 6.798\times 10^{-3}$

Using equation 7.54 to solve for h

$$Nu_{D2} := 0.3 + \frac{\left(0.62\cdot Re_D^{\frac{1}{2}}\cdot Pr^{\frac{1}{3}}\right)}{\left[1 + \left(\frac{0.4}{Pr}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}}\cdot\left[1 + \left(\frac{Re_D}{282000}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$$

$Nu_{D2} = 31.654$

$$h_2 := \frac{k_{air}\cdot Nu_{D2}}{D}$$

$h_2 = 74.386\frac{W}{m^2\cdot K}$

$$Biot := \frac{h_2\cdot r}{2\cdot k_s}$$

$Biot = 7.622\times 10^{-3}$

With Bi<0.1, the lumped capacitance analysis is valid

Since the ends of the rod are insulated, the surface area only includes the cylinder sides

$A_s := \pi \cdot D \cdot L \qquad A_s = 0.063\,m^2$

$V_{vol} := \pi \cdot r^2 \cdot L \qquad V_{vol} = 3.142 \times 10^{-4}\,m^3$

Using equations 5.4 and 5.5 for both h's

$$t_1 := \left(\frac{\rho \cdot V_{vol} \cdot c_p}{h_1 \cdot A_s}\right) \cdot \ln\left(\frac{T_i - T_{air}}{T_f - T_{air}}\right)$$

$t_1 = 980.546\,s$

$t_1 = 16.342\,min$

$$t_2 := \left(\frac{\rho \cdot V_{vol} \cdot c_p}{h_2 \cdot A_s}\right) \cdot \ln\left(\frac{T_i - T_{air}}{T_f - T_{air}}\right)$$

$t_2 = 874.55\,s$

$t_2 = 14.576\,min$

(b)

The total energy lost by the cylinder to the air when it reaches safe-to-touch temp:

$E_{out} := -E_{st}$

$$E_{out} := \rho \cdot c_p \cdot V_{vol} \cdot \int_{T_i}^{T_f} 1\,dT$$

$E_{out} := -\rho \cdot V_{vol} \cdot c_p \cdot \left(T_f - T_i\right)$

$E_{out} = 7.733 \times 10^5\,J$

(c)

The initial rate of heat transfer to the surrounding air

$q_1 := h_1 \cdot A_s \cdot \left(T_i - T_{air}\right)$

$q_1 = 2.389 \times 10^3\,W$

$q_2 := h_2 \cdot A_s \cdot \left(T_i - T_{air}\right)$

$q_2 = 2.678 \times 10^3\,W$

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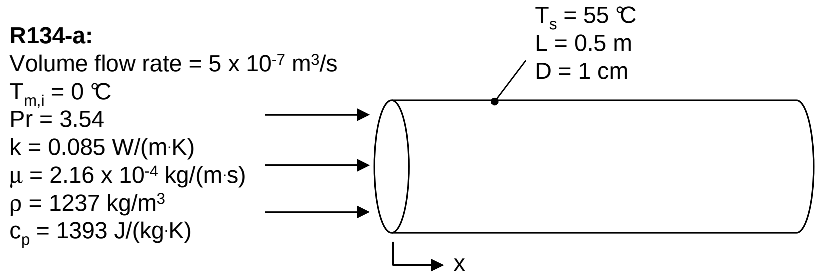
3. (22 points) Refrigerant R-134a at 0 °C is used to cool a hot circular tube such that its surface temperature is maintained constant at 55 °C. The tube has a diameter D = 1 cm and length L = 0.5 m. Fluid properties (at an appropriate temperature) are given in the figure.



a. Assuming that the flow is hydrodynamically fully developed as it enters the heated tubing, what is the mean outlet temperature of the R-134a?

b. What is the total energy input to the fluid?

c. If the flow had been both thermally and hydrodynamically fully developed throughout the tube length, would we calculate a higher or lower mean outlet temperature? Explain your answer from physical considerations, but do not provide any formal calculations.

Problem 3

Given fluid information:

$\rho := 1237\frac{kg}{m^3}$ $\quad c_p := 1393\frac{J}{kg\cdot K}$ $\quad \mu := 0.000216\frac{kg}{m\cdot s}$ $\quad k := .085\frac{W}{m\cdot K}$

$Pr := \frac{c_p\cdot\mu}{k}$ $\quad Pr = 3.54$ $\quad T_{mi} := (0 + 273)K$

$Q := 30\frac{mL}{min}$ $\quad Q = 5\times 10^{-7}\frac{m^3}{s}$

Given tube information:

$D := 1cm$ $\quad T_s := (55 + 273)K$

$L_t := 0.5m$ $\quad T_s = 328\,K$

$P := \pi\cdot D$

(a)

$m_{dot} := \rho\cdot Q$ $\quad m_{dot} = 6.185\times 10^{-4}\frac{kg}{s}$

$Re_D := \frac{4\cdot m_{dot}}{\pi\cdot D\cdot\mu}$ $\quad Re_D = 364.583$ $\quad$ Therefore the flow is laminar since Re<2300

Equation 8.23 for thermal entrance length in laminar flow

$x_{fdt} := 0.05\cdot Re_D\cdot Pr\cdot D$ $\quad x_{fdt} = 0.645\,m$

Therefore the problem is thermally developing since x_fdt > L_t

For a laminar, constant surface temperature condition with an assumed fully developed velocity profile and thermally developing flow, we can use equation 8.56

$$Nu_{Davg} := 3.66 + \frac{0.0668\left(\frac{D}{L_t}\right)\cdot Re_D\cdot Pr}{1 + 0.04\cdot\left[\left(\frac{D}{L_t}\right)\cdot Re_D\cdot Pr\right]^{\frac{2}{3}}} \qquad Nu_{Davg} = 4.938$$

$$h_{avg} := \frac{\left(k\cdot Nu_{Davg}\right)}{D} \qquad h_{avg} = 41.971\frac{W}{m^2\cdot K}$$

Using equation 8.41, we can arrive at the mean outlet temperature

$$\frac{T_s - T_{mo}}{T_s - T_{mi}} := e^{\left(\frac{-P\cdot L\cdot h_{avg}}{m_{dot}\cdot c_p}\right)}$$

Rearranging,

$$T_{mo} := T_s - (T_s - T_{mi}) \cdot e^{\left(\frac{-P \cdot L_t \cdot h_{avg}}{m_{dot} \cdot c_p}\right)} \qquad T_{mo} = 302.412\,K \qquad \text{(or T\_mo = 29.4 C)}$$

(b)

The total energy put into the liquid is:

$$q_{input} := m_{dot} \cdot c_p \cdot (T_{mo} - T_{mi}) \qquad q_{input} = 25.341\,W$$

(c)

For fully developed flow (both thermally and hydrodynamically) and constant wall temperature:

$$Nu_D := 3.66$$

Since the fully developed Nu is smaller than the Nu for thermally developing flow, the fully developed assumption would result in a lower heat transfer coefficient and lower mean outlet temperature according to equation 8.41. In other words, if convection decreased (since Nu and h decrease) the rate at which the fluid picks up energy is lower and therefore would exit at a lower temperature.