Name______________

Problem 1 (25 points)
Consider a tiny drop of water of diameter of D floating very slowly in air with a velocity u. The temperature of air is T_{air}. The relative humidity of air is ϕ.

It is known that the average Nusselt number is 2 for such a small spherical object. Assume that the Sherwood number also equals 2 according to the Reynolds analogy.

(a) Express the average heat transfer coefficient $\bar{h}$ in terms of given parameters and properties of water and/or air. **For each property used, clearly list it and state what it represents in the table below.** An example for the Prandtl number *Pr* is given in the table.

(b) Perform an energy balance analysis, and derive an expression for the steady-state temperature of the water droplet, and briefly discuss how the temperature of water droplet will be solved. Assume the evaporation rate is very small so the change of the diameter of the water droplet can be neglected. **List properties you used, what they represent in the same table below.**

Assumptions [2 pts] - **List assumptions here**

Steady state, radiation is negligible.

Table of the Properties used in Your Answer, identify it is air or water properties or both, and at what temperature they are evaluated [6 pts]

Pr	Prandtl number *of air*
k_f	Thermal conductivity of air
D_{AB}	Binary diffusion co. for water/air
h_{fg}	Latent heat of evaporation for water
$\rho_{A,SAT}(T)$	Saturation density of water vapor at $T_{droplet}$
$\rho_{A,SAT}(T_{air})$	Sat. density of water vapor at T_{air}

Start your answer to question (a) here [5 pts]:

$$\bar{Nu} = \frac{\bar{h} \cdot D}{k_f} = 2. \qquad \bar{h} = \frac{2 k_f}{D}.$$

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Problem 1. - cont'd

Start your answer to question (b) here [12 pts]:

Perform an energy balance on water droplet.

$$q''_{conv} = q''_{evap}.$$

$$\bar{h}\,(T_{air} - T) = \bar{h}_m \cdot h_{fg} \cdot [\rho_{A,SAT}(T) - \rho_{A,\infty}]$$

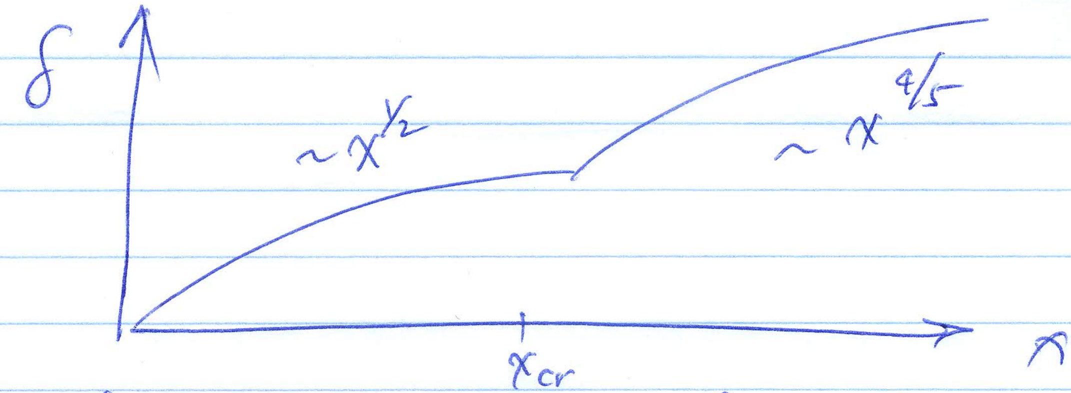
$$\bar{h} = \frac{2k_f}{D}, \quad \overline{Sh} = \frac{\bar{h}_m \cdot D}{D_{AB}}, \rightarrow \bar{h}_m = \frac{2 \cdot D_{AB}}{D}$$

$$\rho_{A,\infty} = \rho_{A,SAT}(T_{air}) \cdot \phi$$

$$\rightarrow k_f\,(T_{air} - T) = D_{AB} \cdot h_{fg}\,[\rho_{A,SAT}(T) - \rho_{A,SAT}(T_{air}) \cdot \phi\,].$$

Since $\rho_{A,SAT}(T)$ depends on unknown T, (also h_{fg}, and k_f, D_{AB}, are evaluated at film temperature), the equation above can't be solved explicitly. One needs to make an initial guess of T, then iterate till the equation is satisfied.

2. a)



laminar $\delta/x = 5/Re_x^{1/2} \Rightarrow \delta \sim x^{1/2}$

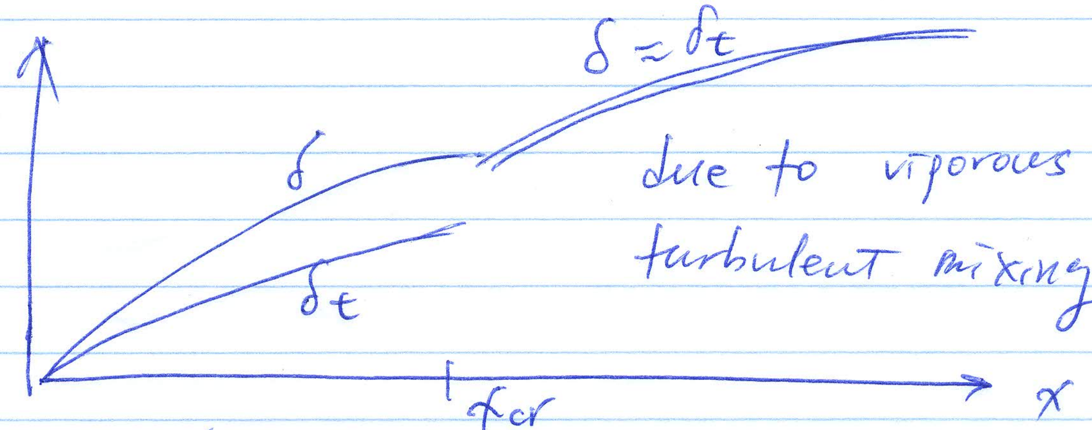
turbulent $\delta/x = 0.37/Re_x^{1/5} \Rightarrow \delta \sim x^{4/5}$

b)

$Pr = 4$

$Pr = \dfrac{\nu}{\alpha}$

$\dfrac{\delta}{\delta_t} \approx Pr^n$ for laminar

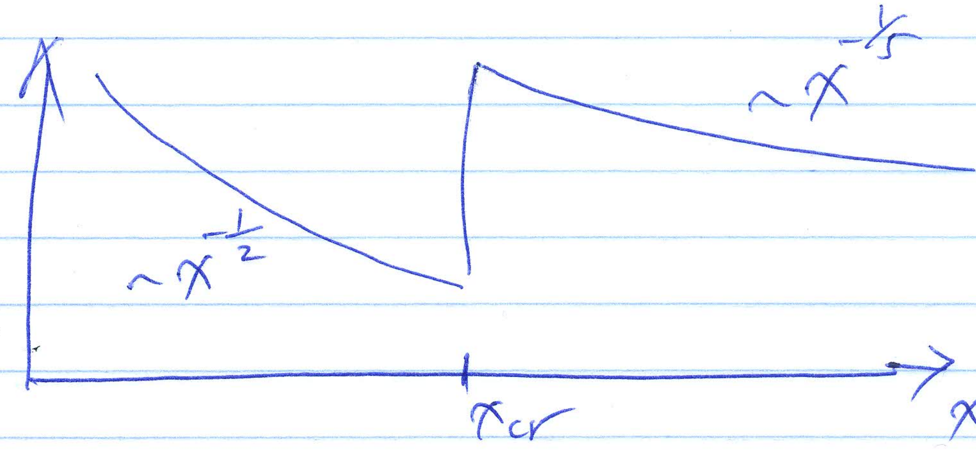


c) h, h_m

Heat/mass transfer analogy

$Nu = \dfrac{hx}{k}$

$Sh = \dfrac{h_m x}{D}$



laminar $Nu = 0.332 Re_x^{1/2} Pr^{1/3} \Rightarrow h \sim x^{-1/2}$

turbulent $Nu = 0.0296 Re_x^{4/5} Pr^{1/3} \Rightarrow h \sim x^{-1/5}$

P. 3 hot fluid $\rightarrow$ $\downarrow\downarrow$ air, cross flow, 25°C

(a) Air flow outside tube: (cylinder), $Re_{d,o} = \frac{\rho v d}{\mu} = 15729$; $Pr = 0.707$.

$$\overline{Nu_D} = 0.3 + \frac{0.62\, Re_D^{1/2}\, Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_D}{282000}\right)^{5/8}\right]^{4/5} = 69$$

$$\overline{h_o} = \frac{\overline{Nu_D}\, k_f}{d} = \boxed{181 \text{ W/m}^2\text{K}}$$

(b) Inside tube: $Re_{d,i} = \frac{4\dot{m}}{\pi d \mu} = 202 < 2300$ (Laminar)

$$Nu_d = 3.66 \text{ (constant Surface Temp)} = \frac{h_i D}{k_f} \Rightarrow \boxed{h_i = 98.8 \text{ W/m}^2\text{K}}$$

(c) Overall HT Coefficient: $\overline{U} = 63.92$ W/m²K

$$\frac{1}{\overline{U}} = \frac{1}{h_i} + \frac{1}{h_o}.$$

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{PL\overline{U}}{\dot{m} C_p}\right) \Rightarrow \boxed{T_{m,o} = 81.7°C}$$

(d)
$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(\frac{-PL h_i}{\dot{m} C_p}\right) \Rightarrow \boxed{T_s = 46.68°C}$$

or, $(\pi d L) h_o (T_s - T_\infty) = \dot{m}_a C_p (T_{m,i} - T_{m,o}) \Rightarrow \boxed{T_s = 46.5°C}$

(e)
$$Re_d = \frac{4\dot{m}}{\pi d \mu} = 202 \times 20 = 4040 > 2300 \text{ (Turbulent)}.$$

$$Nu_d = 0.023 Re_d^{4/5} Pr^{0.3} \text{ (Cooling)} = 21.7$$

$$h_i = \frac{k \times Nu_d}{d} = \boxed{586 \text{ W/m}^2\text{K}}$$

$$\frac{1}{\overline{U}} = \frac{1}{h_i} + \frac{1}{h_o} = \frac{1}{586} + \frac{1}{181} \Rightarrow \overline{U} = \boxed{138 \text{ W/m}^2\text{K}}$$

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{PL\overline{U}}{\dot{m} C_p}\right) \Rightarrow \boxed{T_{m,o} = 85.385°C}$$

$\hookrightarrow$ changed to 80 kg/hr.

4. Heat Exchanger Problem

$A = 100\,m^2$

Hot	Cold
$C_h = 4\ kW/K$	$C_c = 8\ kW/K$
$T_{hi} = 80\ °C$	$T_{ci} = 40\ °C$
$T_{ho} = ?$	$T_{co} = 58\ °C$

a) $$q = C_h (T_{hi} - T_{ho}) = C_c (T_{co} - T_{ci})$$

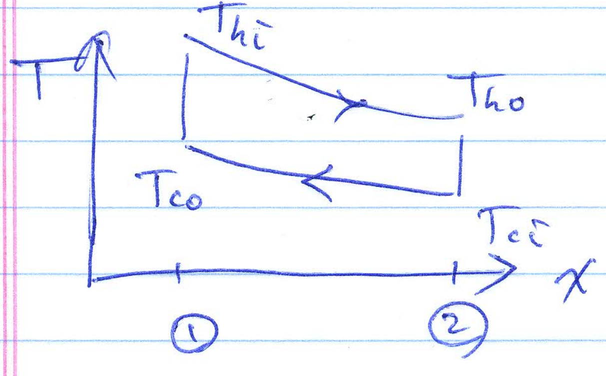
$$q = (8\ kW/K)(58 - 40\ °C) = 144\ kW.$$

b) $$144\ kW = (4\ kW/K)(80\ °C - T_{ho})$$

$$T_{ho} = 44\ °C$$

c) $T_{co} > T_{ho} \rightarrow$ counter-flow Heat Exch.

d) $$q = UA\,\Delta T_{lm} = UA \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$



$$\Delta T_1 = T_{hi} - T_{co} = 80 - 58 = 22\ °C$$

$$\Delta T_2 = T_{ho} - T_{ci} = 44 - 40 = 4\ °C$$

$$\Delta T_{lm} = 10.56\ °C$$

$$U = \frac{q}{A\,\Delta T_{lm}} = 136.4\ W/m^2 \cdot K$$