Name: _ Circle your division: 1 2 3 4

Division Bumsoo Han

Div. 1	Div. 2	Div. 3	Div. 4
Prof. Pan	Prof. Han	Prof. Marconnet	Prof. Ameen
7:30 am	10:30 am	1:30 pm	4:30 pm

Fall 2013 – ME 315 Heat and Mass Transfer

School of Mechanical Engineering, Purdue University

Final Exam – Monday Dec 9, 2013

Total Exam Time: 120 Minutes

Instructions:

- Closed-book and closed-notes. There is a sheet for essential formulas, tables and figures at the back. Personal equation sheets up to 3 pages are allowed (double-sided is OK).
- Turn off all communication devices before starts, and leave them off for the entire exam.
- Write your **full name** and **circle your division** on each page.
- **Staple and turn in each problem separately.** Keep this cover sheet with problem 1.
- No need to turn in equation sheets.
- Show all work clearly on front side of the page only and keep all the pages in order.
- You are asked to write your assumptions and answers to sub-problems in designated areas. Only the work in its designated area will be graded.

Performance		
1	30	21
2	30	
3	30	
4	30	
Total	120	

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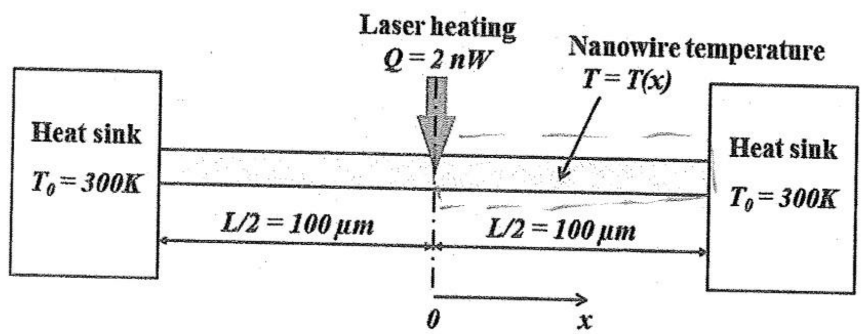
Problem 1 [30 points]

A technique for measuring the thermal conductivity of nanowires involves suspending the nanowire between two heat sinks (which maintain the ends of the nanowire at T_o = 300 K), heating the center point of the nanowire with a laser, and simultaneously measuring the centerline temperature. Consider a nanowire with a square cross-section of 100 nm × 100 nm and length L = 200 μm, heated with laser at the centerline (x=0), as indicated in the diagram below. The laser generated a heat rate of 2 nW at the centerline which is transferred along the nanowire via conduction. A vacuum pump maintains low vacuum pressure such that convection effects may be neglected in part (a).

Assume: (1) Radiation heat losses may be neglected. (2) The temperature is only a function of x. In other words, it doesn't vary within the cross-section (y or z directions). (3) No contact thermal resistance between nanowire and the heat sinks.

Note: 1 μm = 10^{-6} m; 1 nm = 10^{-9} m; 1 nW = 10^{-9} W.



(a) Determine the thermal conductivity of the nanowire if the measured steady-state temperature at the centerline is T(x=0) = 310 K when Q = 2 nW of power are applied to the nanowire. (10 Points)

$$q = -kA\frac{dT}{dx} = -kA\frac{(T_0 - T(x=0))}{\left(\frac{L}{2}\right)}$$

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

+1

+3 $\dot{E}_{in} = \dot{E}_{out}$

$$\frac{q\left(\frac{L}{2}\right)}{A(T(x=0) - T_0)} = k = \frac{(2e-9)\left(\frac{100e-6}{1}\right)}{(100e-9 \times 100e-9)(310-300)}$$

+2

$\boxed{k = 2\ W/mK}$ X

+2

+8

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(b) During a second experiment with a different nanowire of the same geometry and T_o, but with a known thermal conductivity of $k = 2$ W/(m K), the vacuum pump fails and the nanowire is exposed to convection with a heat transfer coefficient of $h = 100$ W/(m^{2}K) and $T_\infty = 300$ K.

If the length of the fin is more than 2.65 times of the thermal decay length $L_{decay} \triangleq 1/m$, where $m \triangleq \sqrt{hP/kA_C}$), one half of nanowire (100 μm long) can be modeled as an infinite fin. Confirm whether the infinite fin assumption is valid here. On the axes below, graph the non-dimensionalized temperature profile $\theta/\theta_b \triangleq (T - T_\infty)/(T_b - T_\infty)$ along the length of one half of the nanowire. Clearly indicate the thermal decay length and the non-dimensionalized temperatures at $x = 0$, $x = L_{decay}$, and $x = 100$ μm on your graph. (10 points)

$k = 2\ W/mK \quad h = 100\ W/m^2K \quad T_\infty = 300\ K$

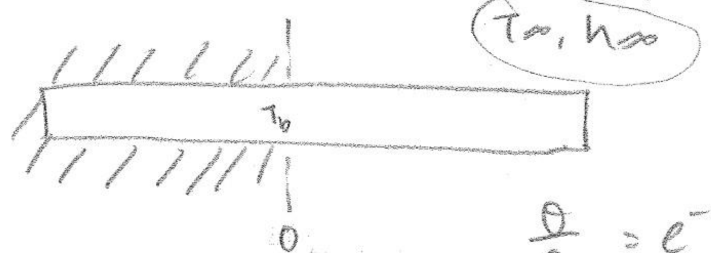
$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{100(100e-9)(4)}{2(100e-9)^2}} = 44721.36 \quad \text{units?}$$

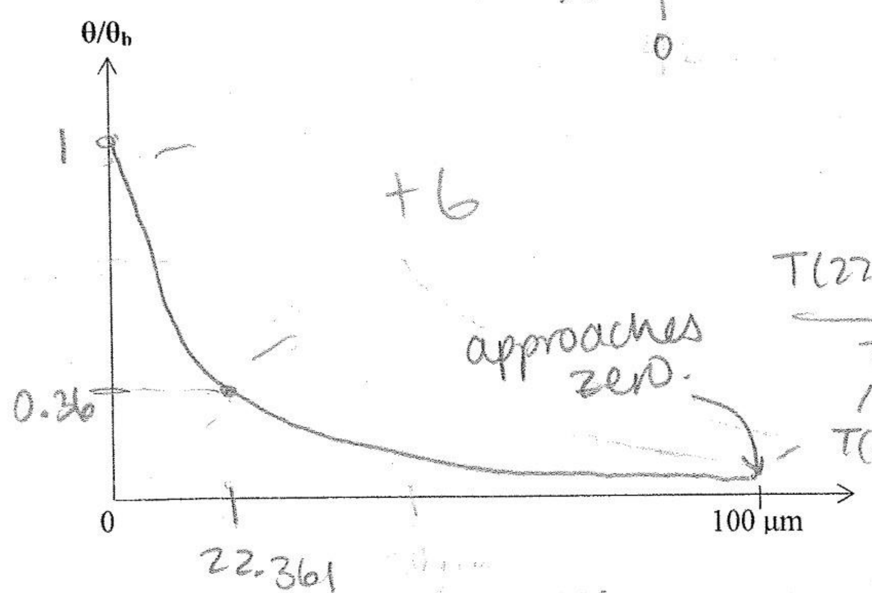
$L_{decay} = 0.0000\cancel{22361}\ m = 22.361\ \mu m$

$L_{fin} = 200\ \mu m > 2.65(22.36) = 59.2558\ \mu m$

$\boxed{\text{Valid}}$

+3





+6

$$\frac{\theta}{\theta_b} = e^{-mx}$$

$$m = \sqrt{\frac{hP}{kA_c}} = 44721.36$$

$$\frac{T(22.361\ \mu m) - T_\infty}{T_b - T_\infty} = e^{-44721.36(22.361)}$$

$T(0)$

$$T(x = 22.361) = e^{-44721.36(22.361\mu m)}(310 - 300) + 300$$

$T = 303.6\ K$

Bumsoo Han

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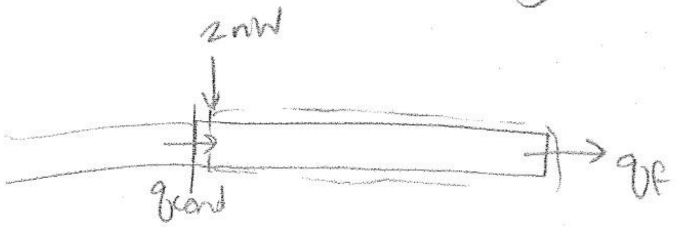
(c) Continue from part (b), determine the <u>steady-state temperature at the centerline</u> T(x=0) of this nanowire when Q = 2 nW of power is applied to the centerline of nanowire. (10 points)

$$\frac{\theta}{\theta_b} = 1$$

~~T(x=0) = 310 K? Rough?~~



$$q_f = \sqrt{hPkA_c}\,\theta_b \quad +4$$

$$q_{cond} + Q = q_f$$

$$q_f = \sqrt{10(4)(100e\text{-}9)(2)(100e\text{-}9)^2}\,(T_b - 300)$$
$$= 8.944e\text{-}10\,(T_b - 300)$$

$$q_{cond} = \frac{kA(T_b - T_0)}{\frac{L}{2}} = \frac{2(100e\text{-}9)^2(T_b - 300)}{100} \quad \times$$

$$2e\text{-}16(T_b - 300) = 8.944e\text{-}10(T_b - 300) \quad \times$$

$$2e\text{-}16\,T_b - 6e\text{-}14 = 8.944e\text{-}10\,T_b - 268e\text{-}9$$

$$(2e\text{-}16 - 8.944e\text{-}10)\,T_b = -268e\text{-}9 + 6e\text{-}14$$

$$\boxed{T_b = 299.64\text{ K}} \quad \times$$

Division 2 Bumsoo Han

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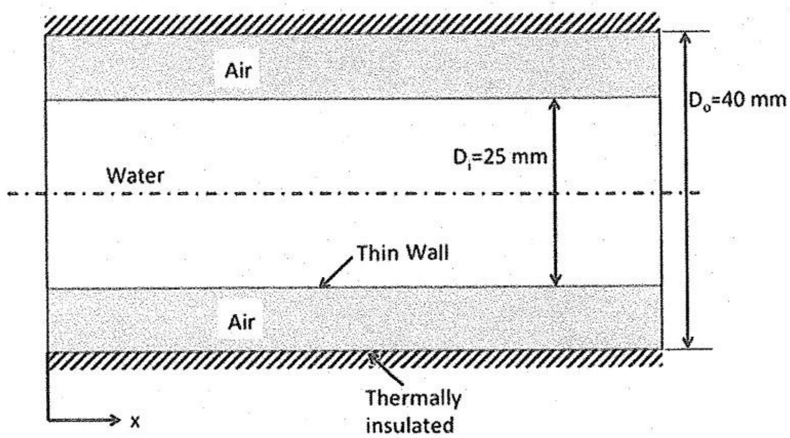
30

Problem 2 [30 points]

A single-pass concentric tube water to air heat exchanger is used to cool hot water. The exterior surface of the heat exchanger is thermally insulated. Water at an inlet temperature $T_{h,i} = 75$ °C enters the heat exchanger and leaves the heat exchanger at an outlet temperature of $T_{h,o} = 55$ °C. Air enters the heat exchanger at an inlet temperature of $T_{c,i} = 25$ °C. The water flows through the inner tube and the air flows through the outer tube. Assume that the tube walls are thin and have no surface fouling. The diameter of the inner tube is 25 mm, and that of the outer tube is 40 mm. The heat capacity rate of hot water is $C_h = 6{,}000$ W/K and that of air is $C_c = 3{,}000$ W/K.



(a) Determine the outlet temperature of air, $T_{c,o}$. Is the heat exchanger operating in a parallel flow, or a counterflow configuration? (12 Points)

$q = \dot{m} c_p \Delta T$

$q_h = C_h(T_{h_i} - T_{h_o}) = 6000(75-55) = 6000(20) = 120000$ W

$q_c = C_c(T_{c_o} - T_{c_i})$

$\frac{q_c}{C_c} + T_{c_i} = T_{c_o} = \frac{120000}{3000} + 25 = \boxed{65°C}$

$T_{c,o} > T_{h,o}$ counterflow

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(b) Determine the effectiveness of the heat exchanger, ε. (6 Points)

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_c (T_{c,o} - T_{c,i})}{C_{min}(T_{h,i} - T_{c,i})} = \frac{3000(65-25)}{3000(75-25)} = \boxed{0.8}$$

(c) If the convective heat transfer coefficients between the thin tube walls and the inner and outer tube flows are both constants and given by $h_{inner} = 200\ W/m^2K$ and $h_{outer} = 50\ W/m^2K$ respectively. What is the required tube length, L, to achieve this heat exchange? (12 Points)

$$\cancel{C_r = \frac{C_{min}}{C_{max}} = \frac{3000}{6000} = 0.5 \quad NTU = 2.1}$$

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{200} + \frac{1}{50}} = 40$$

$C_r = 0.5$ NTU = 2.1 from chart

$$\frac{NTU\, C_{min}}{U} = A = \frac{2.1(3000)}{40} = 157.5\,m^2$$

$$A = \pi D_i L \qquad L = \frac{A}{\pi D_i} = \frac{157.5}{\pi .025} = \boxed{2005.35m}$$

Dinsion Bumsoo Han

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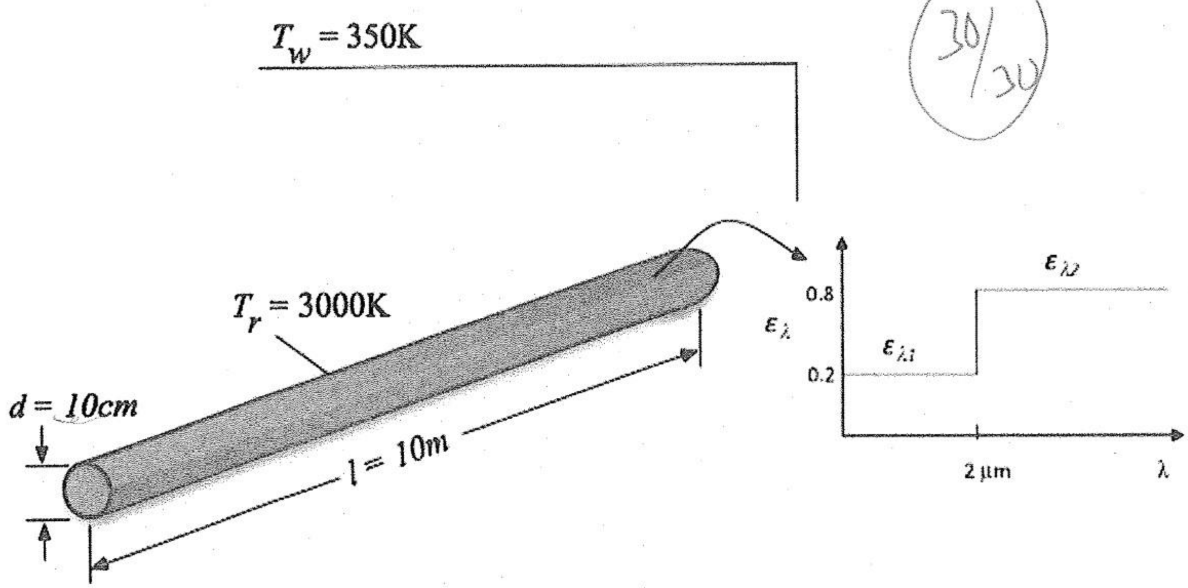
Problem 3 [30 points]

30/30

A long nuclear fuel rod ($l >> d$) is placed in a large isothermal enclosure. The surface temperature of the rod is T_r = 3000 K and the enclosure wall is T_w = 350 K. The rod surface is diffuse, and has a spectral emissivity as shown below.



(a) Determine the total emissivity of the fuel rod surface. (10 Points)

$$\varepsilon(T) = \frac{\int_0^\infty \varepsilon_\lambda(\lambda) E_{\lambda,b}(\lambda,T)\,d\lambda}{E_b(T) = \sigma T^4}$$

$$\varepsilon = \varepsilon_{\lambda_1} F_{0-2} + \varepsilon_{\lambda_2} F_{2-\infty} = \varepsilon_{\lambda_1} F_{0-2} + \varepsilon_{\lambda_2}(1 - F_{0-2})$$

$\lambda T_r = 2(3000) = 6000$ μmK 10

$F_{0-2} = 0.737818$

$\varepsilon = 0.2(.737818) + 0.8(1 - .737818)$

$\boxed{\varepsilon = 0.357}$

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(b) Determine the <u>total absorptivity</u> of the fuel rod surface. (10 points)

$\lambda T_{wall} = 2(350) = 700\ \mu m K$

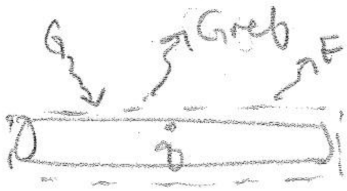
$F_{0-2} = 0.000008$

$\alpha = \alpha_{\lambda_1} F_{0-2} + \alpha_{\lambda_2}(1 - F_{0-2})$

10

$\alpha = .2(.000008) + .8(1 - .000008) = \boxed{0.799995}$

(c) If the system is in steady state, determine the <u>volumetric heat generation rate</u> (W/m^3) in the rod. (10 points)



Assuming
no convection
surface is opaque

$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \cancel{\dot{E}_{st}}$

$\alpha G - \varepsilon A \sigma T_s^4 - (1-\alpha)G + q_{gen} = 0$

10

$q_{gen} = \varepsilon A \sigma T_s^4 - \alpha G = 0.357(\pi)(.1)(10)5.67e{-8}(3000)^4$

$- .799995(5.67e{-8})(\pi)(.1)(10)(350)^4$

$- (1 - .799995)(5.67e{-8})(\pi)(.1)(10)(350^4)$

$= 5407974.18\ W$

$\dot{q}_{gen} = \frac{q_{gen}}{Volume} = \frac{5407974.18}{\frac{\pi}{4}(.1)^2(10)} = \boxed{68856465.83\ W/m^3}$

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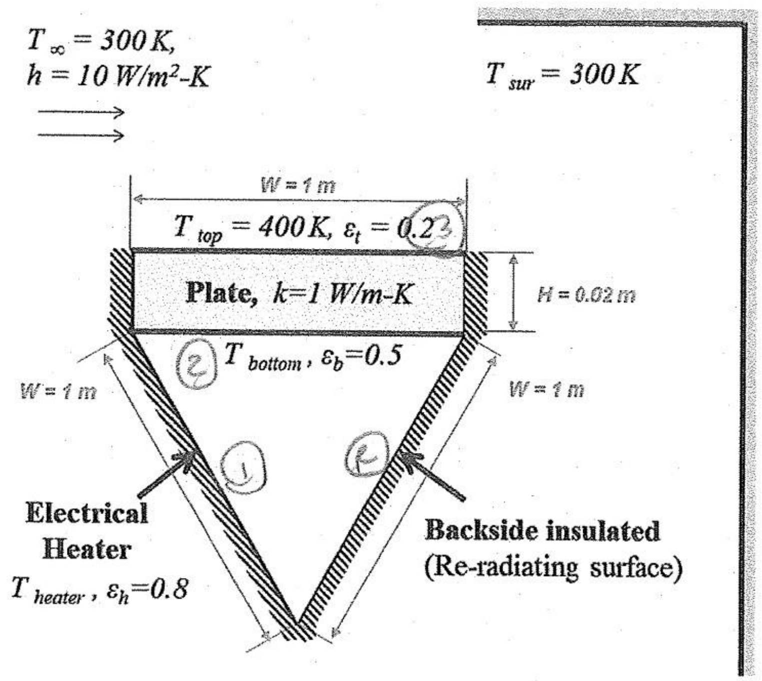
Division Bumsoo Han

Problem 4 [30 points]

30

A very long hot plate design is to be evaluated as shown below. The bottom of the plate is heated by an oven, whose cross-section is an equilateral triangle (consisting of the bottom surface of the hot plate, a flat heater and a re-radiating surface). All external surfaces are thermally isolated except the plate's top surface. Assume steady state condition, all three internal surfaces are isothermal, diffuse and gray, and ignore convections inside the oven. Parameters and symbols are given in figure below. The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/(m^2 K^4).



(a) Consider a unit depth of the oven enclosure (i.e., one meter along the direction into the paper) and use the values and symbols provided in the problem. Draw a radiation network for the enclosure of oven's three internal surfaces, and clearly label the resistances, nodal potentials and heat fluxes. Calculate the value of the resistances. (15 Points)

15

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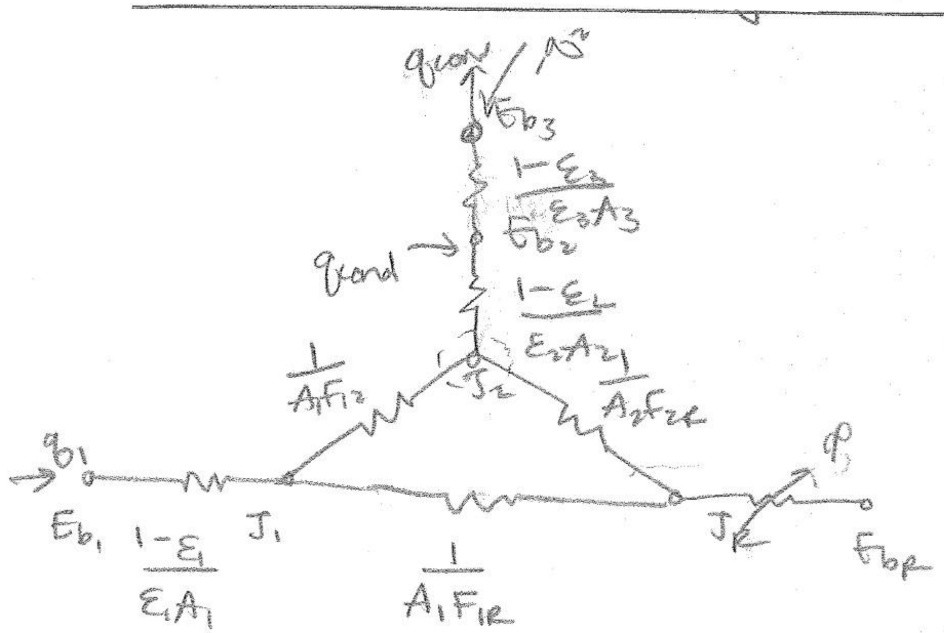

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = \frac{1-0.8}{0.8(1)} = \boxed{0.25}$$

$F_{12} = 0.5$ $\quad F_{12} = 1 - \sin\left(\frac{60}{2}\right) = 0.5$

$$\frac{1}{A_1 F_{12}} = \frac{1}{(1)(0.5)} = \boxed{2}$$

$F_{1R} = 0.5$

$$\frac{1}{A_1 F_{1R}} = \boxed{2}$$

$$\frac{1}{A_2 F_{2R}} = \boxed{2}$$

$$\frac{1-\varepsilon_2}{\varepsilon_2 A_2} = \frac{1-0.5}{0.5(1)} = \boxed{1}$$

$$\frac{1-\varepsilon_3}{\varepsilon_3 A_3} = \frac{1-0.2}{.2(1)} = \boxed{4}$$

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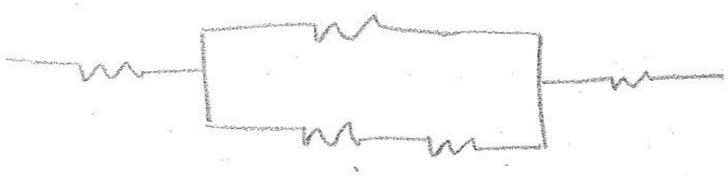
(b) Use the radiation network from part (a) to find <u>the symbolic expression of the net radiative heat transfer</u> to the unit depth of the plate bottom surface. The symbolic expression should only contain numbers, T_{heater} and T_{bottom}. (7 Points)

$$\left(\frac{1}{\frac{1}{A_1F_{1R}}+\frac{1}{A_2F_{2R}}}+\frac{1}{\frac{1}{A_1F_{12}}}\right)^{-1}+\frac{1-\varepsilon_1}{\varepsilon_1 A_1}+\frac{1-\varepsilon_2}{\varepsilon_2 A_2}=R_{tot}$$

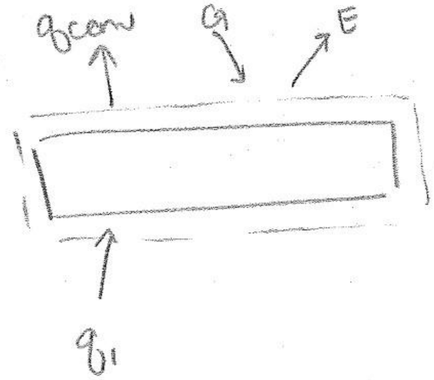


$$q_1=\frac{E_{b1}-E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1}+\left(\frac{1}{\frac{1}{A_1F_{1R}}+\frac{1}{A_2F_{2R}}}+\frac{1}{\frac{1}{A_1F_{12}}}\right)^{-1}+\frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

$$\boxed{q_1=\frac{\sigma\left(T_{heater}^4-T_{bottom}^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1}+\left(\frac{1}{\frac{1}{A_1F_{1R}}+\frac{1}{A_2F_{2R}}}+\frac{1}{\frac{1}{A_1F_{12}}}\right)^{-1}+\frac{1-\varepsilon_2}{\varepsilon_2 A_2}}}$$

Name: Han   Circle your division: 1 2 3 4

(c) Notice that the hot plate is in steady state and its top surface is subjected to multiple heat transfer modes. Calculate the temperature of the heater required to maintain the plate top surface at a constant temperature of $T_{top} = 400$ K. (8 points)



$\dot{E}_{in} - \dot{E}_{out} + \cancel{\dot{E}_g} = \cancel{\dot{E}_{st}}$ 8

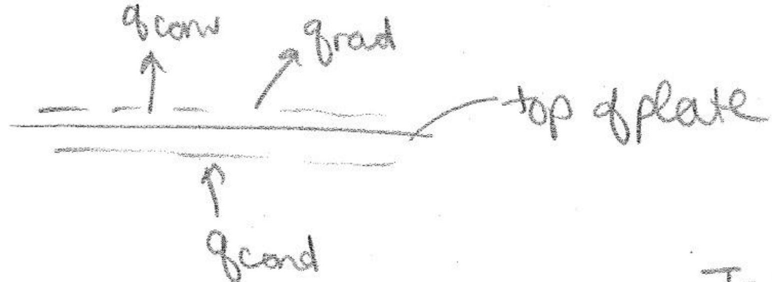
assume top of plate is gray/diffuse

$q_1 + \alpha G A - q_{conv} - EA = 0$

$q_1 + \sigma\alpha A T_{sur}^4 - hA(T_{top} - T_\infty) - \varepsilon\sigma A T_{top}^4 = 0$ $\alpha = \varepsilon$

$q_1 = \varepsilon A \sigma (T_{top}^4 - T_{sur}^4) + hA(T_{top} - T_\infty)$

$$\frac{\sigma(T_{heater}^4 - T_{bottom}^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \left(\frac{1}{\frac{1}{A_1F_{1R}} + \frac{1}{A_2F_{2R}}} + \frac{1}{\frac{1}{A_1F_{12}}}\right)^{-1} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}} = \varepsilon A \sigma (T_{top}^4 - T_{sur}^4) + hA(T_{top} - T_\infty)$$



$\dot{E}_{in} - \dot{E}_{out} + \cancel{\dot{E}_g} = \cancel{\dot{E}_{st}}$

$\dot{E}_{in} = \dot{E}_{out}$

$T_s = T_{top}$

$q_{cond} = q_{conv} + q_{rad}$

$kA\frac{dT}{dx} = hA(T_s - T_\infty) + \sigma\varepsilon A(T_s^4 - T_{sur}^4)$

$k\left(\frac{T_{bottom} - T_s}{H}\right) = h(T_s - T_\infty) + \sigma\varepsilon(T_s^4 - T_{sur}^4)$

$T_{bottom} = [h(T_s - T_\infty) + \sigma\varepsilon(T_s^4 - T_{sur}^4)]\frac{H}{k} + T_s$

$T_{bottom} = [10(400-300) + 5.67e{-8}(.2)(400^4 - 300^4)]\left(\frac{.02}{1}\right) + 400$

$T_{bottom} = 423.97$ K

problem 4c)

Heiny

$$T_{heater}^4 = \frac{\left[\varepsilon A\sigma(T_{top}^4 - T_{sur}^4) + hA(T_{top} - T_{\infty})\right]\left[\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \left(\frac{1}{\frac{1}{A_1F_{1R}} + \frac{1}{A_2F_{2R}}} + \frac{1}{\frac{1}{A_1F_{12}}}\right)^{-1} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}\right]}{\sigma}$$

$$+ T_{bottom}^4$$

$$T_{heater}^4 = \frac{\overbrace{\left[.2(1)(5.67e{-8})(400^4 - 300^4) + 10(1)(400-300)\right]}^{1198.45}\overbrace{\left[.25 + \left(\frac{1}{\frac{1}{.5} + \frac{1}{.5}} + \frac{1}{\frac{1}{.5}}\right)^{-1} + 1\right]}^{2.5833}}{5.67e{-8}}$$

$$+ 423.97^4$$

$$\left(\frac{1}{2+2} + \frac{1}{2}\right)^{-1}$$

$$= \left(\frac{1}{4} + \frac{1}{2}\right)^{-1}$$

$$T_{heater}^4 = 8.691266e10$$

$$\boxed{T_{heater} = 542.96\ K}$$