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ME 315 Exam 3
July 29, 2015
4:30 p.m. to 6:30 p.m.

INSTRUCTIONS

1. This is a **closed book and closed notes examination**. You are provided with an equation sheet and any required tables.
2. Do not hesitate to ask the instructor if you do not understand a problem statement.
3. Start each problem on the same page as the problem statement. Write on only one side of the page. Materials on the back side of the page will not be graded.
4. Put only one problem on a page. Another problem on the same page will not be graded.
5. Show system/control volume and list relevant assumptions.
6. If you give multiple solutions, you will receive only a partial credit although one of the solutions might be correct. Delete the solution you do not want graded.
7. For your own benefit, please write clearly and legibly. Maximum credit for each problem is indicated below.
8. After you have completed the exam, at your seat put your papers in order. This may mean that you have to remove the staple and re-staple. Do not turn in loose pages.
9. Once time is called you will have three minutes to turn in your exam. Points will be subtracted for exams turned in after these three minutes.

Problem	Possible	Score
1	35	
2	35	
3	30	
Total	**100**	

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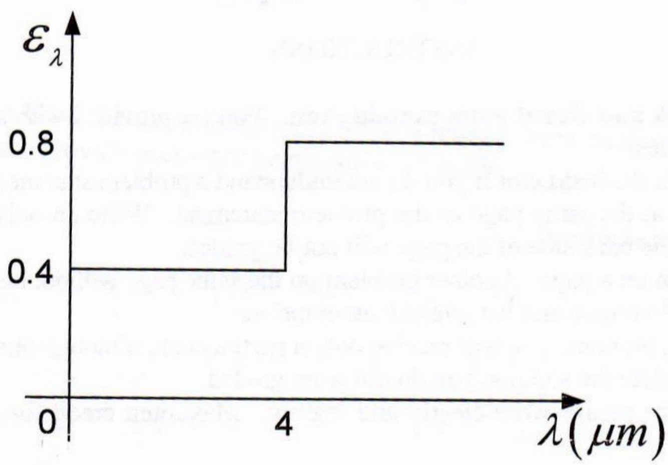
Problem 1 (35 points) Consider a thin coating, applied to a long cylindrical copper rod of diameter 10 mm, cured by placing horizontally in a large furnace whose walls are maintained at 1300 K. Initially the rods are at 300 K. The furnace has air at 1300 K with convective heat transfer coefficient of 15 W/m^2-K on surface of the copper rods. The surface of the rods is opaque and the spectral emissivity of the coating varies with wavelength as shown below.

Density of copper = 8900 m^3/kg; Specific heat of copper = 385 J/kg-K



(a) Calculate the total emissivity of the coated rod surface.

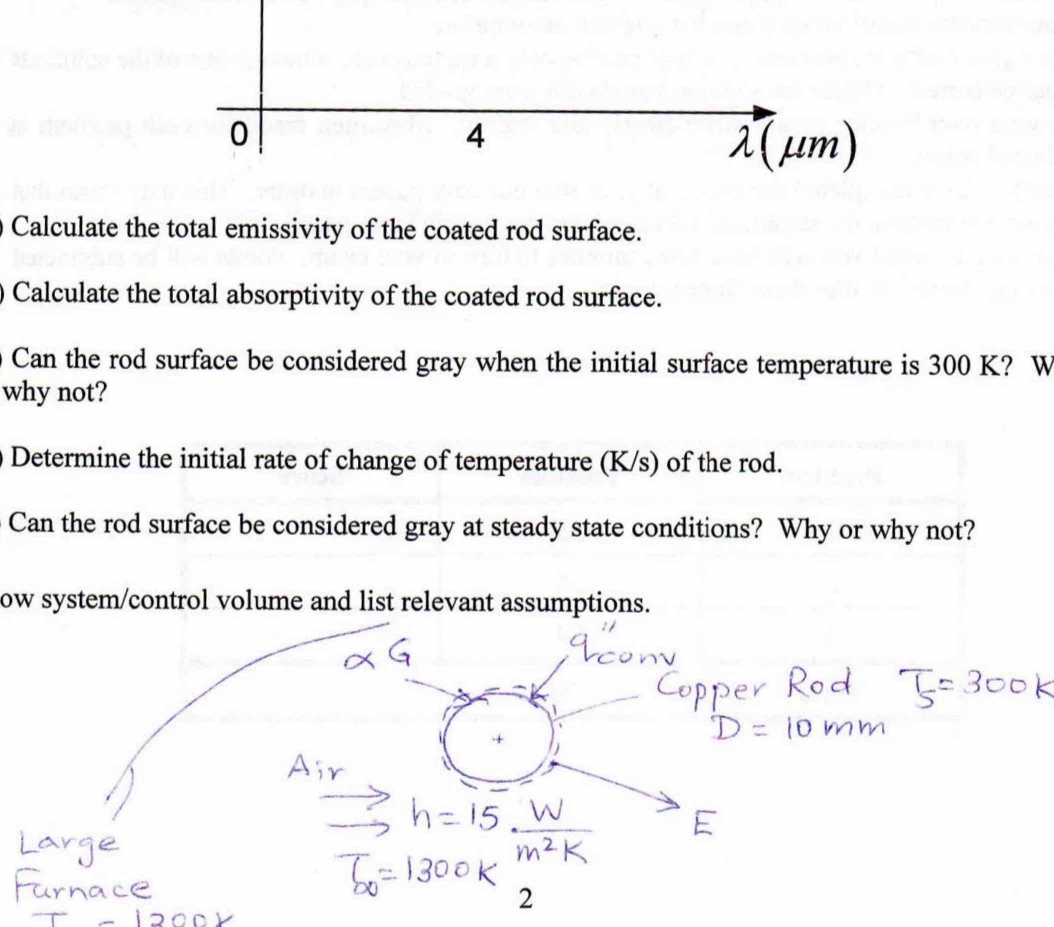
(b) Calculate the total absorptivity of the coated rod surface.

(c) Can the rod surface be considered gray when the initial surface temperature is 300 K? Why or why not?

(d) Determine the initial rate of change of temperature (K/s) of the rod.

(e) Can the rod surface be considered gray at steady state conditions? Why or why not?

Show system/control volume and list relevant assumptions.



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Problem 1 (continued)

Assumptions

Constant properties

Uniform convection on the entire surface 2

Negligible conduction within the rod

Diffuse $\Rightarrow \varepsilon_\lambda = \alpha_\lambda$, Opaque $\Rightarrow \tau = 0$

Solution

(a) $$\varepsilon = \frac{E}{E_b} = \frac{\int_0^\infty E_\lambda \, d\lambda}{\int_0^\infty E_{\lambda,b} \, d\lambda} = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b} \, d\lambda}{\int_0^\infty E_{\lambda,b} \, d\lambda}$$ 2

$$= 0.4\, F(0 \to 4\,\mu m) + 0.8\, F(4\,\mu m \to \infty)$$ 2

at $\lambda T_s = 4\,\mu m \times 300 K = 1200\,\mu m\text{-}K$

Table 12.1: $F(0 \to 4\,\mu m) = 0.002134$ 2

$F(4\,\mu m \to \infty) = 1 - F(0 \to 4\,\mu m) = 0.997866$

$\boxed{\varepsilon \simeq 0.8}$ 2

(b) $$\alpha = \frac{G_{absorbed}}{G} = \frac{\int_0^\infty \alpha_\lambda G_\lambda \, d\lambda}{\int_0^\infty G_\lambda \, d\lambda} = \frac{\int_0^\infty \alpha_\lambda E_{\lambda,b} \, d\lambda}{\int_0^\infty E_{\lambda,b} \, d\lambda}$$ 2 2

Since furnace is large, the irradiation falling on the surface from furnace walls is blackbody emission at T_w

$$\alpha = 0.4\, F(0 \to 4\,\mu m) + 0.8\, F(4\,\mu m \to \infty)$$

at $\lambda T_w = 4\,\mu m \times 1300 K = 5200 K$

Table 12.1: $F(0 \to 4\,\mu m) = 0.658970$

$F(4\,\mu m \to \infty) = 0.34103$ 2

$\boxed{\alpha = 0.536}$ 2

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Problem 1 (continued)

(c) 5 $\boxed{\varepsilon \neq \alpha \Rightarrow \text{surface is not gray at initial condition}}$

(d) Considering energy balance for the control volume, we have: $\dot{E}_{in} - \dot{E}_{out} + \cancel{\dot{E}_{gen}}^{\,0} = \dot{E}_{st}$ 2

$$(\alpha G + q''_{conv} - E)\, A_s = m C_p \frac{dT}{dt} = \rho \forall C_p \frac{dT}{dt}$$

$$G = E_b(T_w) = \sigma T_w^4 = 5.67 \times 10^{-8} \frac{W}{m^2 K^4} \times (1300)^4\, K^4$$

$$q''_{conv} = h(T_\infty - T_s) = 15 \frac{W}{m^2 K} \times (1300 - 300)\, K$$ 4

$$E = \varepsilon E_b(T_s) = \varepsilon \sigma T_s^4 = 0.8 \times 5.67 \times 10^{-8} \times (300)^4\, K^4$$

$$A_s = \pi D L\ ,\ \forall = \frac{\pi}{4} D^2 L$$

Rate of change of temperature:

$$\frac{dT}{dt} = \frac{(\alpha G + q''_{conv} - E)}{\rho C_p \frac{D}{4}} \Rightarrow \boxed{\frac{dT}{dt} = +11.8 \frac{K}{s}}$$ 2

(e) For steady-state: $T_s = T_w = T_\infty = 1300\,K$

4 $\boxed{\varepsilon = \alpha \Rightarrow \text{surface is gray at steady condition}}$

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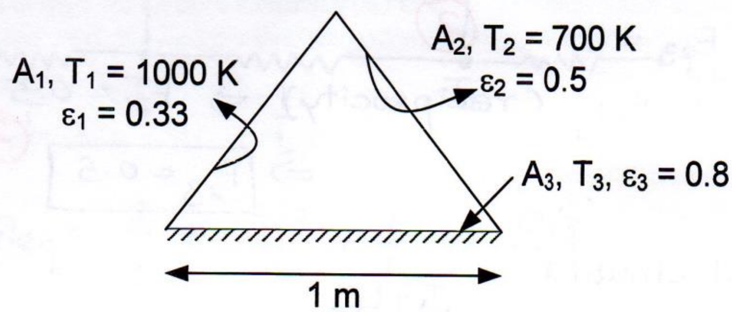
Problem 2 (35 points) Consider a long duct constructed with diffuse, gray walls each of which is 1 m wide as shown below. The bottom horizontal wall of the duct is well-insulated and convection inside the duct is negligibly small.



(a) Calculate F_{12}, F_{13}, and F_{23}.

(b) Draw a thermal circuit showing appropriate resistances.

(c) Calculate the rate of radiative heat transfer per unit length (W/m) for A_1.

Show system/control volume and list relevant assumptions.

Assumptions

Non-participating medium

Uniform radiosity for all surfaces

Diffuse and gray $\Rightarrow \varepsilon = \alpha$, Opaque $\Rightarrow \tau = 0$ 2

Inclined walls well-insulated with negligible convection

$\Rightarrow$ re-radiating

Solution

(a) $F_{11} + F_{12} + F_{13} = 1$ (summation) 2

$F_{11} = 0$ (flat surface)

$F_{12} + F_{13} = 1$

2 $F_{12} = F_{13}$ (symmetry) 2

$\Rightarrow \boxed{F_{12} = F_{13} = 0.5}$

Problem 2 (continued)

$F_{21} + F_{22} + F_{23} = 1$ (summation)

$F_{22} = 0$ (flat)

$F_{21} + F_{23} = 1$ 2

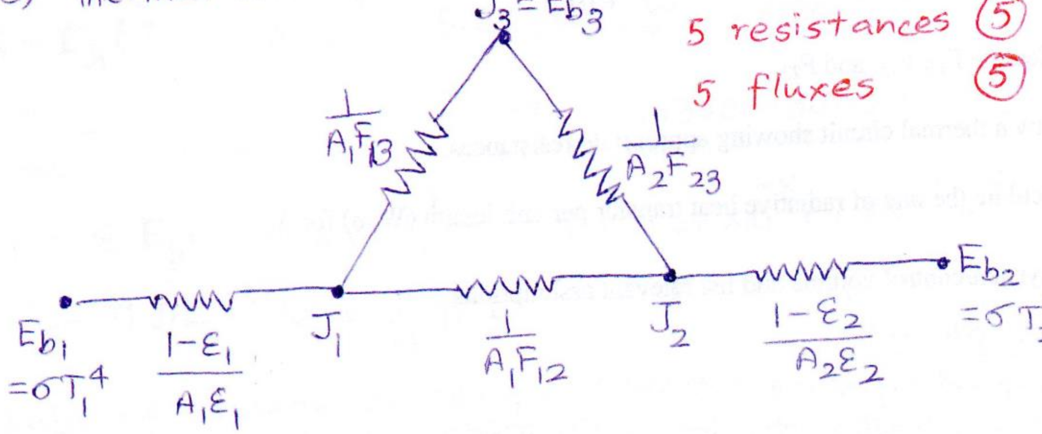
$A_1 F_{12} = A_2 F_{21}$ (reciprocity) $\Rightarrow F_{21} = 0.5$ 2

$\Rightarrow \boxed{F_{23} = 0.5}$

(b) Thermal circuit

5 resistances 5

5 fluxes 5

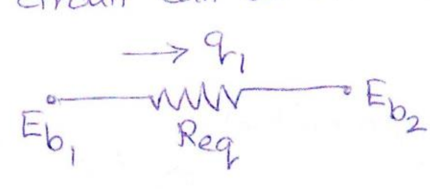


(c) $A_1 = A_2 = A_3 = (1 \times L)\ m^2$

$$\frac{1-\varepsilon_1}{A_1 \varepsilon_1} = \frac{2}{L}, \quad \frac{1-\varepsilon_2}{A_2 \varepsilon_2} = \frac{1}{L}$$

$$\frac{1}{A_1 F_{12}} = \frac{1}{A_1 F_{13}} = \frac{1}{A_2 F_{23}} = \frac{2}{L}$$

Thermal circuit can be shown as:



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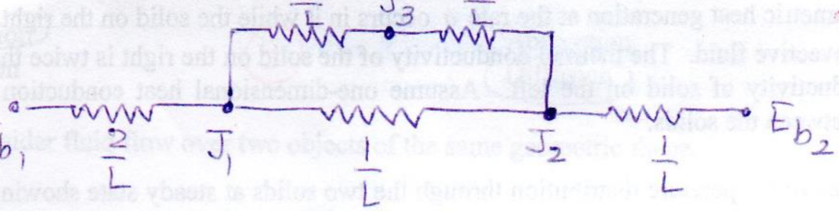
Problem 2 (continued)

Equivalent radiation resistance:



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$$R_{eq} = \frac{2}{L} + \frac{4/3}{L} + \frac{1}{L} = \frac{13/3}{L}$$

Rate of radiation heat transfer for A_1:

$$q_1 = \frac{E_{b_1} - E_{b_2}}{R_{eq}} = \frac{\sigma (T_1^4 - T_2^4)}{R_{eq}}$$ 5

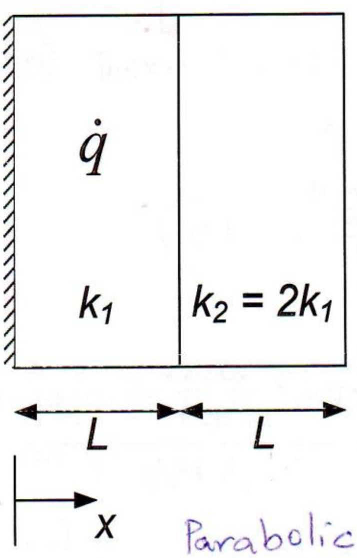
$$q'_{r_1} = \frac{q_{r_1}}{L} = \frac{\sigma (T_1^4 - T_2^4)}{13/3} = \frac{5.67\times10^{-8} \frac{W}{m^2K^4} \times [(1000)^4 - (700)^4] K^4}{13/3}$$

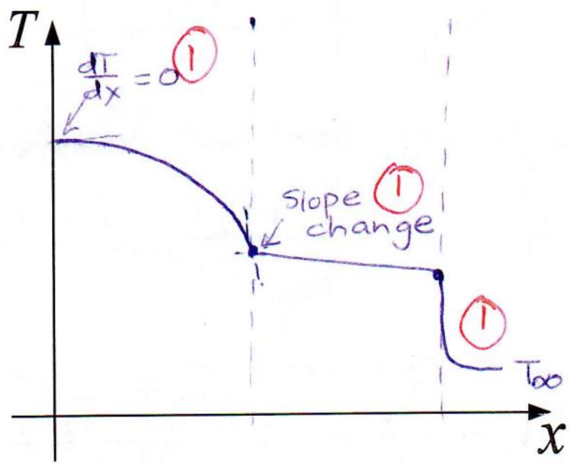
$$\boxed{q'_{r_1} = 9943 \frac{W}{m}}$$ 2

Problem 3 (30 points) Answer the following questions.

(a) Consider two solids each having thickness L. The solid on the left is insulated on the left side and uniform volumetric heat generation at the rate $\dot{q}$ occurs in it while the solid on the right side is exposed to convective fluid. The thermal conductivity of the solid on the right is twice that of the thermal conductivity of solid on the left. Assume one-dimensional heat conduction and perfect contact between the solids.

Sketch the qualitative temperature distribution through the two solids at steady state showing all important features. Provide necessary justification. (5 points)





2 Parabolic temperature distribution in the solid with heat generation; linear temperature distribution without heat generation

(b) What is the physical interpretation of the Biot number? (5 points)

$$Bi = \frac{hL_c}{k_{solid}} = \frac{L_c/k_{solid}}{1/h} = \frac{R_{t,cond}}{R_{t,conv}} \quad 5$$

i.e. ratio of conduction resistance through the solid and convection (plus radiation) resistance at the boundary between solid and fluid

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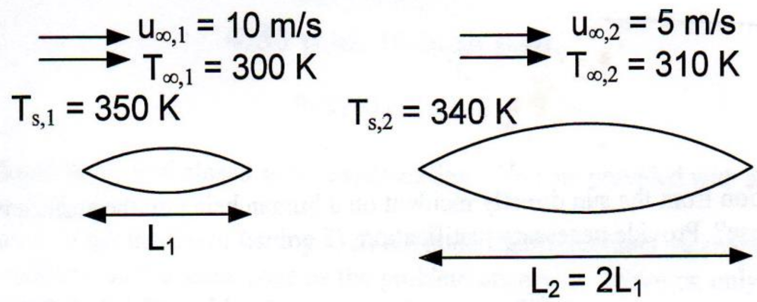
Problem 3 (continued)

(c) Which of the following processes are related to convection heat transfer? (5 points)

Advection 5 Absorption
Emission Diffusion

(d) Consider fluid flow over two objects of the same geometric shape.



Which of the following is true? Provide necessary justification. (5 points)

$h_1 = 2h_2$ 1 $h_2 = 2h_1$
$h_1 = h_2$ Insufficient Information

$T_{film,1} = T_{film,2} = 325\ K \Rightarrow$ boundary layer fluid properties $(\nu, \alpha, k, etc.)$ identical

2 $Re_{L_1} = \frac{u_{\infty,1} L_1}{\nu_1} = \frac{10 L_1}{\nu_2}$ and $Re_{L_2} = \frac{u_{\infty_2} L_2}{\nu_2} = \frac{10 L_1}{\nu_1}$

$Pr_1 = \frac{\nu_1}{\alpha_1}$ and $Pr_2 = \frac{\nu_2}{\alpha_2}$

Since objects have the same geometric shape, C, m, and n are the same $\overline{Nu_1} = C_1 Re_{L_1}^m Pr_1^n$, $\overline{Nu_2} = C_2 Re_{L_2}^m Pr_2^n$

2 $\Rightarrow \overline{Nu_1} = \overline{Nu_2}$

$\frac{\overline{h_1} L_1}{k_1} = \frac{\overline{h_2} L_2}{k_2} \Rightarrow \overline{h_1} = \frac{L_2}{L_1}\overline{h_2} \Rightarrow \boxed{\overline{h_1} = 2\overline{h_2}}$

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Problem 3 (continued)

(e) The surface of the sun can be considered as blackbody at a temperature of 5800 K. Calculate the wavelength corresponding to the maximum spectral emissive power for the sun. (5 points)

Wien's displacement law: $\lambda_{max} T = 2900\ \mu m\text{-}K$ 3

$\boxed{\lambda_{max} = 0.5\ \mu m}$ 2

(f) Can the irradiation from the sun directly incident on a human being on the surface of the earth be considered diffuse? Provide necessary justification. (5 points)

Yes 2 No Insufficient Information

Since ~~suns~~ the surface of the sun is very large compared to a normal ☺ human being, it is essentially

3 a blackbody which is a diffuse emitter

$\Rightarrow$ irradiation from the sun is diffuse