
(i) Determine the mobility using the Kutzbach criterion. Clearly number each link and label the lower pairs and the higher pairs on the given figure.
(ii) Define suitable vectors for a kinematic analysis. Label and show the direction of each vector clearly on the given figure.
(iii) Write the vector loop equations that are required for a kinematic analysis of the mechanism. Clearly identify suitable input(s) for the mechanism. List: (a) the known quantities; (b) the unknown variables; and (c) any constraints. If you identified constraints in part (b) then write the constraint equations.

Part II. (10 Points). Using trigonometry, determine the mechanical advantage of the four-bar linkage in the position shown in Fig. 1b. The input link $O_2A = 6 \text{ in}$, link $AB = 10 \text{ in}$, and output link $BO_4 = 4 \text{ in}$. 

Figure 1a. A planar mechanism.

Figure 1b. A four-bar linkage.
Problem 2 (25 Points). For the mechanism in the position shown in Figure 2, the angular velocity of the input gear 2 is $\omega_2 = 15 \text{ rad/s}$ counterclockwise. Gear 3 is rolling, without slip, on gear 2 at the point of contact E. The radius of gear 2 is $\rho_2 = 400 \text{ mm}$, the radius of gear 3 is $\rho_3 = 150 \text{ mm}$, and the distance from point C on link 3 to pin D connecting links 3 and 4 is $CD = 500 \text{ mm}$.

(i) Draw vectors clearly on the given figure that are suitable for a kinematic analysis of the mechanism. Then write the vector loop equation for the mechanism.

(ii) Using your vector loop equation determine the first-order kinematic coefficients for the mechanism.

(iii) Determine the angular velocities of links 3 and 4. Give the magnitudes and directions of the vectors.

Figure 2. A planar mechanism.
Problem 3 (25 Points). For the mechanism in the position shown in Figure 3, the input link 2 is moving downward with a constant velocity $V_2 = 90 \text{ mm/s}$.

(i) List the primary instant centers and the secondary instant centers for the mechanism.

(ii) Using the given Kennedy circle, show the location of all the instant centers on the given figure.

Using the location of the instant centers, determine:

(iii) The first-order kinematic coefficients of links 3, 4, and 5.

(iv) The magnitudes and the directions of the angular velocities of links 3, 4, and 5.

(v) The magnitude and direction of the velocity of point D which is fixed in link 4.

Figure 3. A planar mechanism. Scale: full size, that is, 1 mm = 1 mm.
Problem 4 (25 Points). For the mechanism in the position shown in Figure 4, the angular velocity of the input link 2 is $\omega_2 = 50 \text{ rad/s}$ counterclockwise. The link lengths are $O_2A = 500 \text{ mm}$, $AB = 400 \text{ mm}$, and the radius of the wheel which is rolling without slipping on link 1 is $\rho_4 = 140 \text{ mm}$.

(i) Write a vector loop equation that is suitable for a kinematic analysis of this mechanism. Draw your vectors clearly on the given figure.

(ii) Using your vector loop equation determine the first-order kinematic coefficients for the mechanism.

(iii) Determine the angular velocities of links 3 and 4. Give the magnitude and direction of each vector.

(iv) Determine the velocity of point B fixed in link 4. Give the magnitude and direction of this vector.

Figure 4. A planar mechanism.
Solution to Problem 1.

Part I. (i) 4 points. The links of the mechanism and the joints are as shown in Figure 1(a).

![Figure 1(a). The links and joints of the mechanism.](image1)

The Kutzbach mobility criterion can be written as

\[ M = 3(n - 1) - 2j_1 - 1j_2 \]  

(1)

The number of links, the number of lower pairs (or \( j_1 \) joints), and the number of higher pairs (or \( j_2 \) joints), respectively, are

\[ n = 7, \quad j_1 = 8, \quad \text{and} \quad j_2 = 1 \]  

(2)

Substituting Equation (2) into Equation (1), the mobility of the mechanism is

\[ M = 3(7 - 1) - 2(8) - 1 = 1 \]  

(3)

This is the correct answer for this mechanism, that is, for a single input there is a unique output.

(ii) 4 Points. Suitable vectors for a kinematic analysis of the mechanism are shown in Figure 1(b).

![Figure 1(b). Vectors for the mechanism.](image2)
(iii) **7 Points.** (a) The input link is chosen to be link 2 (see Figure 1a) and the input variable is the angle $\theta_2$. Note that link 4 would also be suitable as an input link since it pins to the ground link. The slider (link 7) would also be suitable as an input link. Other choices for the input link are not really practical.

(b) The six unknown variables are the four angles $\theta_3, \theta_4, \theta_5, \text{ and } \theta_6$ and the two distances $R_{36}$ and $R_7$.

(c) There are four constraints, the four constraint equations are

\[
\begin{align*}
\theta_3 &= \theta_1 - \phi \\
\theta_6 &= \theta_3 + (180^\circ - \alpha) \\
\theta_6 &= \theta_6 - \beta \\
\theta_3 &= \theta_3 + \gamma
\end{align*}
\]

Since there are 6 unknown variables then three independent vector loop equations are required. The three vector loop equations can be written as

\[
\begin{align*}
\text{Loop 1:} & \quad \frac{\sqrt{1}}{R_1} + \frac{\sqrt{1}}{R_2} + \frac{\sqrt{?}}{R_3} + \frac{\sqrt{?}}{R_4} = 0 \\
\text{Loop 2:} & \quad \frac{\sqrt{1}}{R_2} + \frac{\sqrt{?}}{R_3} + \frac{\sqrt{?}}{R_3} + \frac{\sqrt{?}}{R_6} - \frac{\sqrt{?}}{R_6} + \frac{\sqrt{?}}{R_1} + \frac{\sqrt{?}}{R_1} = 0 \\
\text{Loop 3:} & \quad \frac{\sqrt{1}}{R_2} + \frac{\sqrt{?}}{R_3} + \frac{\sqrt{?}}{R_3} + \frac{\sqrt{?}}{R_6} + \frac{\sqrt{?}}{R_5} + \frac{\sqrt{?}}{R_5} + \frac{\sqrt{?}}{R_4} + \frac{\sqrt{?}}{R_4} = 0
\end{align*}
\]

**Part II. 10 points.** The mechanical advantage of the four-bar linkage, see Chapters 1 and 3, is defined as

\[
MA = \frac{R_4 \sin \psi}{R_2 \sin \phi} = \frac{BO_4 \sin \psi}{AO_2 \sin \phi}
\]

where the angles $\psi$ and $\phi$ are as shown in Figure 1(c).
From the triangle $AO_2B$, the angles can be written as

$$\sin \psi = \frac{AO_2}{AB} \quad (2a)$$

and

$$\sin \phi = \frac{BO_2}{AB} \quad (2b)$$

Substituting Equations (2) into Equation (1), the mechanical advantage of the four-bar linkage can be written as

$$MA = \frac{BO_4}{AO_2} \frac{AO_2}{AB} = \frac{BO_4}{BO_2} \quad (3a)$$

Therefore, the mechanical advantage of the four-bar linkage can be written as

$$MA = \frac{BO_4}{BO_2} \quad (3b)$$

Notice that the triangle $AO_2B$ is a right-angled triangle with the angle $\angle AO_2B = 90^\circ$. Therefore, the length of $O_2B$ is

$$BO_2 = \sqrt{AB^2 - O_2A^2} = \sqrt{10^2 - 6^2} = 8 \text{ inches} \quad (4)$$

Substituting Equation (4) and the known length of the output link, that is, $BO_4 = 4$ in, into Equation (3), the mechanical advantage of the four-bar linkage is

$$MA = \frac{4}{8} = 0.5 \quad (5)$$
Solution to Problem 2.

(i) 7 Points. A suitable choice of vectors for the mechanism are as shown on Figure 2.

![Figure 2. A planar mechanism.](image)

The vector loop equation (VLE) for the mechanism can be written as

$$\sqrt{C_1} \sqrt{C_2} \sqrt{C_3} \sqrt{C_4} = R_7 - R_3 - R_4 + R_1 = 0$$  \hspace{1cm} (1)

where link 7 is the arm. The input of this mechanism is the angular position of gear 2.

(ii) 12 points. The rolling contact constraint between gear 2 and link 3 can be written as

$$\rho_2 \theta_2 - \rho_3 \theta_3 = \rho_2 \Delta \theta_2 - \rho_3 \Delta \theta_3 \pm \rho_2 \Delta \theta_3 - \rho_3 \Delta \theta_2$$  \hspace{1cm} (2)

The correct sign is positive because there is internal contact between the two gears.

Differentiating Equation (2) with respect to the input position $\theta_2$ gives

$$\frac{\rho_2}{\rho_3} = \frac{\Delta \theta_3 - \Delta \theta_2}{\Delta \theta_2 - \Delta \theta_3}$$  \hspace{1cm} (3a)

Rearranging Equation (3a) and substituting the kinematic coefficient of the input link $\theta'_2 = 1$ into the resulting equation, the first-order kinematic coefficient for the arm (that is, link 7) can be written as
\[ \theta_1' = \frac{\rho_3 \theta_3' - \rho_2 \theta_2}{\rho_3 - \rho_2} \]  

(3b)

Substituting the given dimensions into Equation (3b), the first-order kinematic coefficient for the arm can be written as

\[ \theta_1' = \frac{150 \theta_3' - 400}{150 - 400} \]  

(3c)

The X and Y components of the VLE, see Equation (1), are

\[ R_7 \cos \theta_7 - R_3 \cos \theta_3 - R_4 \cos \theta_4 + R_1 \cos \theta_1 = 0 \]  

(4a)

and

\[ R_7 \sin \theta_7 - R_3 \sin \theta_3 - R_4 \sin \theta_4 + R_1 \sin \theta_1 = 0 \]  

(4b)

Differentiating Equations (4) with respect to the input position \( \theta_2 \) gives

\[ -R_7 \sin \theta_7 \theta_7' + R_3 \sin \theta_3 \theta_3' + R_4 \sin \theta_4 \theta_4' = 0 \]  

(5a)

and

\[ +R_7 \cos \theta_7 \theta_7' - R_3 \cos \theta_3 \theta_3' - R_4 \cos \theta_4 \theta_4' = 0 \]  

(5b)

Then substituting Equation (3b) into Equations (5) gives

\[ -R_7 \sin \theta_7 \left( \frac{\rho_3 \theta_3' - \rho_2 \theta_2}{\rho_3 - \rho_2} \right) + R_3 \sin \theta_3 \theta_3' + R_4 \sin \theta_4 \theta_4' = 0 \]  

(6a)

and

\[ R_7 \cos \theta_7 \left( \frac{\rho_3 \theta_3' - \rho_2 \theta_2}{\rho_3 - \rho_2} \right) - R_3 \cos \theta_3 \theta_3' - R_4 \cos \theta_4 \theta_4' = 0 \]  

(6b)

Then writing Equations (6) in matrix form gives

\[
\begin{bmatrix}
-(\frac{\rho_3 R_7}{\rho_3 - \rho_2}) \sin \theta_7 + R_3 \sin \theta_3 + R_4 \sin \theta_4 \\
+(\frac{\rho_3 R_7}{\rho_3 - \rho_2}) \cos \theta_7 - R_3 \cos \theta_3 - R_4 \cos \theta_4
\end{bmatrix}
\begin{bmatrix}
\theta_3' \\
\theta_4'
\end{bmatrix}
= 
\begin{bmatrix}
-R_7 \sin \theta_7 \left( \frac{\rho_2}{\rho_3 - \rho_2} \right) \\
+R_7 \cos \theta_7 \left( \frac{\rho_2}{\rho_3 - \rho_2} \right)
\end{bmatrix}
\]

(7)

Substituting the known data into Equation (7) gives

\[
\begin{bmatrix}
+353.55 \text{ mm} \\
+203.55 \text{ mm}
\end{bmatrix}
\begin{bmatrix}
\theta_3' \\
\theta_4'
\end{bmatrix}
= 
\begin{bmatrix}
0 \text{ mm} \\
-400 \text{ mm}
\end{bmatrix}
\]

(8a)

The determinant of the coefficient matrix in Equation (8a) is

\[
\text{DET} = (353.55 \text{ mm})(-603.55 \text{ mm}) - (+348.46 \text{ mm})(+203.55 \text{ mm}) = -284314 \text{ mm}^2
\]

(8b)
Using Cramer’s rule, the first-order kinematic coefficient for link 3 from Equation (8a) is

\[
\theta'_3 = \frac{+139384 \text{ mm}^2}{-284314 \text{ mm}^2} = -0.4902 \text{ rad / rad} \tag{9}
\]

The negative sign indicates that link 3 is rotating clockwise as link 2 rotates counterclockwise. Also, the first-order kinematic coefficient for link 4 is

\[
\theta'_4 = \frac{-141420 \text{ mm}^2}{-284314 \text{ mm}^2} = +0.4974 \text{ rad / rad} \tag{10}
\]

The positive sign indicates that link 4 rotates counterclockwise as gear 2 rotates counterclockwise.

Substituting Equation (9) into Equation (3c), the first-order kinematic coefficient for the arm is

\[
\theta'_7 = \frac{150(-0.4902)-400}{150-400} = +1.89 \text{ rad/rad} \tag{11}
\]

The positive sign indicates that the arm is rotating counterclockwise as gear 2 rotates counterclockwise. **(iii) 6 Points.** The angular velocity of link 3 can be written as

\[
\omega_3 = \theta'_3 \omega_2 \tag{12a}
\]

Substituting Equation (9) and the angular velocity of the input link 2 into Equation (12a) gives

\[
\omega_3 = (-0.4902 \text{ rad})(+15 \text{ rad / s}) = -7.35 \text{ rad / s} \tag{12b}
\]

The negative sign indicates that link 3 is rotating clockwise.

The angular velocity of link 4 can be written as

\[
\omega_4 = \theta'_4 \omega_2 \tag{13a}
\]

Substituting Equation (10) and the angular velocity of the input link 2 into Equation (13a) gives

\[
\omega_4 = (+0.4974 \text{ rad})(+15 \text{ rad / s}) = +7.46 \text{ rad / s} \tag{14b}
\]

The positive sign indicates that link 4 is rotating counterclockwise.

Note that the angular velocity of the arm (link 7) can be written as

\[
\omega_7 = \theta'_7 \omega_2 \tag{15a}
\]

Substituting Equation (11) and the angular velocity of the input link 2 into Equation (15a) gives

\[
\omega_7 = (+1.89 \text{ rad})(+15 \text{ rad / s}) = +28.35 \text{ rad / s} \tag{15b}
\]

The positive sign indicates that the arm (link 7) is rotating counterclockwise.
Solution to Problem 3.

(i) 1 Point. The number of links in the mechanism is four, therefore, the total number of instant centers for this mechanism is six; i.e.,

$$N = \frac{n(n-1)}{2} = \frac{5 \times 4}{2} = 10$$

There are 6 primary instant centers; namely, I_{12}, I_{15}, I_{45}, I_{23}, I_{14}, and I_{34}. Therefore, there are 4 secondary instant centers; namely, I_{24}, I_{25}, I_{13}, and I_{35}. 

Note: The point of intersection of the line through I_{15}I_{45} and the line through the contact point and perpendicular to the sliding direction is the instant center I_{14}.

The procedure to locate the 4 secondary instant centers is:

(i) The point of intersection of the line through I_{23}I_{34} and the line through I_{12}I_{14} is the instant center I_{24}.
(ii) The point of intersection of the line through I_{15}I_{12} and the line through I_{45}I_{24} is the instant center I_{25}.
(iii) The point of intersection of the line through I_{14}I_{34} and the line through I_{12}I_{23} is the instant center I_{13}.
(iv) The point of intersection of the line through I_{45}I_{34} and the line through I_{25}I_{23} is the instant center I_{35}.

(ii) 10 Points. The locations of the 10 instant centers for this mechanism are shown on Figure 3.

Figure 3. The locations of the instant centers.
(iii) 6 Points. The velocity of point A fixed in link 3 is equal to the velocity of point A fixed in link 2, that is

$$V_{A_3} = V_{A_2} = V_2$$  \hspace{1cm} (1a)$$

which can be written as

$$(I_{13}I_{23})\omega_3 = \dot{R}_2$$  \hspace{1cm} (1b)$$

Rearranging this equation, the first-order kinematic coefficient of link 3 can be written as

$$\theta'_3 = \frac{\omega_3}{R_2} = \frac{1}{I_{13}I_{23}}$$  \hspace{1cm} (1c)$$

The distance $I_{13}I_{23}$ is measured as $I_{13}I_{23} = 33.3 \text{ mm}$. Therefore, the first-order kinematic coefficient of link 3 is

$$\theta'_3 = \frac{1}{33.3 \text{ mm}} = 0.030 \text{ rad/mm}$$  \hspace{1cm} (2a)$$

Note that the correct sign of this kinematic coefficient is positive because link 3 is rotating counterclockwise about the instant center $I_{13}$ as the length of the input vector is increasing, that is

$$\theta'_3 = + 0.030 \text{ rad/mm}$$  \hspace{1cm} (2b)$$

The velocity of link 2 can be written as

$$(I_{14}I_{24})\omega_4 = \dot{R}_2$$  \hspace{1cm} (3a)$$

Rearranging this equation gives the first-order kinematic coefficient of link 4, that is

$$\theta'_4 = \frac{1}{I_{14}I_{24}}$$  \hspace{1cm} (3b)$$

The distance $I_{14}I_{24}$ is measured as $I_{14}I_{24} = 10.7 \text{ mm}$. Therefore, the first-order kinematic coefficient of link 4 is

$$\theta'_4 = \frac{1}{10.7 \text{ mm}} = 0.0933 \text{ rad/mm}$$  \hspace{1cm} (4a)$$

Note that the correct sign is negative because link 4 is rotating clockwise about the instant center $I_{14}$ while the length of the input vector is increasing, that is

$$\theta'_4 = - 0.0933 \text{ rad/mm}$$  \hspace{1cm} (4b)$$

The velocity of link 2 can be written as

$$(I_{15}I_{25})\omega_5 = \dot{R}_2$$  \hspace{1cm} (5a)$$

Therefore, the first-order kinematic coefficient of link 5 can be written as

$$\theta'_5 = \frac{1}{I_{15}I_{25}}$$  \hspace{1cm} (5b)$$
The distance $I_{15}I_{25}$ is measured as $I_{15}I_{25} = 16.7$ mm. Therefore, the first-order kinematic coefficient is

$$\theta'_5 = \frac{1}{16.7 \text{ mm}} = 0.060 \text{ rad/mm} \quad (6a)$$

Note that the correct sign is negative because link 5 is rotating clockwise about the instant center $I_{15}$ while the length of the input vector is increasing, that is

$$\theta'_5 = -0.060 \text{ rad/mm} \quad (6b)$$

(iv) **6 Points.** The angular velocity of link 3 can be written as

$$\omega_3 = \theta'_5 \dot{R}_2 \quad (7)$$

The input vector $\dot{R}_2$ is defined here as downward, therefore, the vector is becoming longer for the given input velocity $\dot{V}_2 = -90 \hat{j} \text{ mm/s}$. Therefore, the time rate of change of the input vector is

$$\dot{R}_2 = +90 \text{ mm/s} \quad (8b)$$

Substituting Equations (2b) and (8b) into Equation (7) gives

$$\omega_3 = \theta'_5 \dot{R}_2 = (+0.030)(+90) = +2.7 \text{ rad/sec} \quad (9)$$

The positive sign implies that the direction of the angular velocity of link 3 is counterclockwise.

The angular velocity of link 4 can be written as

$$\omega_4 = \theta'_4 \dot{R}_2 \quad (10a)$$

Substituting Equations (4b) and (8b) into Equation (10a) gives

$$\omega_4 = (-0.0933)(+90) = -8.40 \text{ rad/sec} \quad (10b)$$

The negative sign implies that the direction of the angular velocity of link 4 is clockwise.

The angular velocity of link 5 can be written as

$$\omega_5 = \theta'_5 \dot{R}_2 \quad (11a)$$

Substituting Equations (6b) and (8b) into Equation (11a) gives

$$\omega_5 = (-0.06)(+90) = -5.4 \text{ rad/sec} \quad (11b)$$

The negative sign implies that the direction of the angular velocity of link 5 is clockwise.

(v) **2 Points.** The velocity of point D (where D is fixed in link 4) can be written as

$$V_D = (I_{14}D)\omega_4 = (I_{14}D)\theta'_4 \dot{R}_2 \quad (12a)$$

The distance $I_{14}D$ is measured as $I_{14}D = 95$ mm. Substituting this measurement and Equation (10b) into Equation (12a), the velocity of point D is

$$V_D = (95 \text{ mm})(-8.40 \text{ rad/s}) = -798 \text{ mm/s} \quad (12b)$$

The negative sign indicates that the velocity of point D is directed to the left.
Solution to Problem 4.
(i) 8 Points. A set of vectors for a kinematic analysis of the mechanism are shown in Fig. 4(a).

Figure 4(a). Suitable vectors for the mechanism.

The vector loop equation (VLE) can be written as

\[
\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_7} + \frac{1}{R_9} + \frac{1}{R_1} = 0
\]

where the vector \( R_7 \) is the arm and the vector \( R_9 \) follows the path of pin B.

(ii) 10 Points. The X and Y components of the VLE, see Equation (1), are

\[
R_2 \cos \theta_2 + R_3 \cos \theta_3 + R_7 \cos \theta_7 + R_9 \cos \theta_9 + R_1 \cos \theta_1 = 0
\]

and

\[
R_2 \sin \theta_2 + R_3 \sin \theta_3 + R_7 \sin \theta_7 + R_9 \sin \theta_9 + R_1 \sin \theta_1 = 0
\]

The angles for the given position of the mechanism are

\[
\theta_2 = 120^\circ, \quad \theta_3 = 210^\circ, \quad \theta_9 = 270^\circ, \quad \text{and} \quad \theta_7 = 180^\circ
\]

Differentiating Equations (2) with respect to the input position \( \theta_2 \) gives

\[
-R_2 \sin \theta_2 - R_3 \sin \theta_3 \theta_3' + R_9' \cos \theta_9 = 0
\]

and
Writing Equations (4) in matrix form gives

\[
\begin{bmatrix}
-R_3 \sin \theta_3 & + \cos \theta_3 \\
+ R_3 \cos \theta_3 & + \sin \theta_3
\end{bmatrix}
\begin{bmatrix}
\theta_3' \\
R_9'
\end{bmatrix}
= \begin{bmatrix}
+ R_2 \sin \theta_2 \\
- R_2 \cos \theta_2
\end{bmatrix}
\]

Substituting the known data into Equation (5) gives

\[
\begin{bmatrix}
-400 \sin(210^\circ) \text{ mm} & \cos(270^\circ) \text{ rad} \\
+ 400 \cos(210^\circ) \text{ mm} & \sin(270^\circ) \text{ rad}
\end{bmatrix}
\begin{bmatrix}
\theta_3' \\
R_9'
\end{bmatrix}
= \begin{bmatrix}
+ 500 \sin(120^\circ) \text{ mm} \\
- 500 \cos(120^\circ) \text{ mm}
\end{bmatrix}
\]

which simplifies to

\[
\begin{bmatrix}
+ 200 \text{ mm} & 0 \text{ rad} \\
-346.41 \text{ mm} & -1 \text{ rad}
\end{bmatrix}
\begin{bmatrix}
\theta_3' \\
R_9'
\end{bmatrix}
= \begin{bmatrix}
+ 433.01 \text{ mm} \\
+ 250 \text{ mm}
\end{bmatrix}
\]

The determinant of the coefficient matrix in Equation (6b) is

\[\text{DET} = (+ 200 \text{ mm})(-1 \text{ rad}) = -200 \text{ mm}\]

Using Cramer’s rule, the first-order kinematic coefficient for the angle of link 3 is

\[\theta_3' = \frac{-433.01 \text{ mm}}{-200 \text{ mm}} = +2.165 \text{ rad/rad}\]

The positive sign indicates that link 3 is rotating counterclockwise for a counterclockwise rotation of the input link 2.

From Cramer’s rule, the first-order kinematic coefficient for point B is

\[R_9' = \frac{+200 \times 250 + 433.01 \times 346.41 \text{ mm}^2}{-200 \text{ mm}} = -999.995 \text{ mm/rad}\]

Therefore, the first-order kinematic coefficient for point B is

\[R_9' = -1.0 \text{ m/rad}\]

The negative sign indicates that the length of the vector \(\vec{R}_9\) is becoming shorter for a counterclockwise rotation of the input link 2, that is point B is moving downward.

The rolling contact constraint between link 4 and the ground link 1 can be written as

\[\pm \Delta R_y = \rho_4(\Delta \theta_4 - \Delta \theta_i)\]

The correct sign is positive because as the wheel 4 rotates counterclockwise the magnitude of the vector \(\vec{R}_9\) becomes longer. Also, the change in the angular position of the vector \(\vec{R}_i\) is zero, that is, \(\Delta \theta_i = 0\), therefore, Equation (10) can be written as

\[+ \Delta R_y = \rho_4 \Delta \theta_4\]
Differentiating Equation (11) with respect to the input position $\theta_2$ gives

$$+ R'_9 = \rho_4 \theta'_4$$  \hspace{1cm} (12a)

Then rearranging this equation, the first-order kinematic coefficient for link 4 can be written as

$$\theta'_4 = + \frac{R'_9}{\rho_4}$$  \hspace{1cm} (12b)

Substituting Equation (9b) and the radius of link 4 into Equation (12b), the first-order kinematic coefficient for link 4 is

$$\theta'_4 = + \left( \frac{-999.995 \text{ mm}}{140 \text{ mm}} \right) = -7.143 \text{ rad/rad}$$  \hspace{1cm} (13)

The negative sign indicates that link 4 is rotating clockwise as the input link 2 rotates counterclockwise.

(iii) 4 Points. The angular velocity of link 3 can be written as

$$\omega_3 = \theta'_3 \omega_2$$  \hspace{1cm} (14a)

Substituting Equation (8) and the given angular velocity of the input link 2 into Equation (14a), the angular velocity of link 3 is

$$\omega_3 = (+2.165 \text{ rad/ rad})(50 \text{ rad/s}) = +108.25 \text{ rad/s}$$  \hspace{1cm} (14b)

The positive sign indicates that link 3 is rotating counterclockwise as the input link 2 rotates counterclockwise.

The angular velocity of link 4 can be written as

$$\omega_4 = \theta'_4 \omega_2$$  \hspace{1cm} (15a)

Substituting Equation (13b) and the given angular velocity of the input link 2 into Equation (15a), the angular velocity of link 3 is

$$\omega_4 = (-7.143 \text{ rad/ rad})(50 \text{ rad/s}) = -357.14 \text{ rad/s}$$  \hspace{1cm} (15b)

The negative sign indicates that link 4 is rotating clockwise as the input link 2 rotates counterclockwise.

(iv) 3 Points. The velocity of point B fixed in link 4 can be written as

$$V_B = R'_9 \omega_2$$  \hspace{1cm} (16a)

Substituting Equation (9b) and the given angular velocity of the input link 2 into Equation (16a), the velocity of point B is

$$V_B = (-1.0 \text{ m/ rad})(50 \text{ rad/s}) = -50 \text{ m/s}$$  \hspace{1cm} (16b)

The negative sign indicates that the velocity of point B is directed downward from point B as the input link 2 rotates counterclockwise, that is

$$V_B = -50 \text{ m/s}$$  \hspace{1cm} (17)