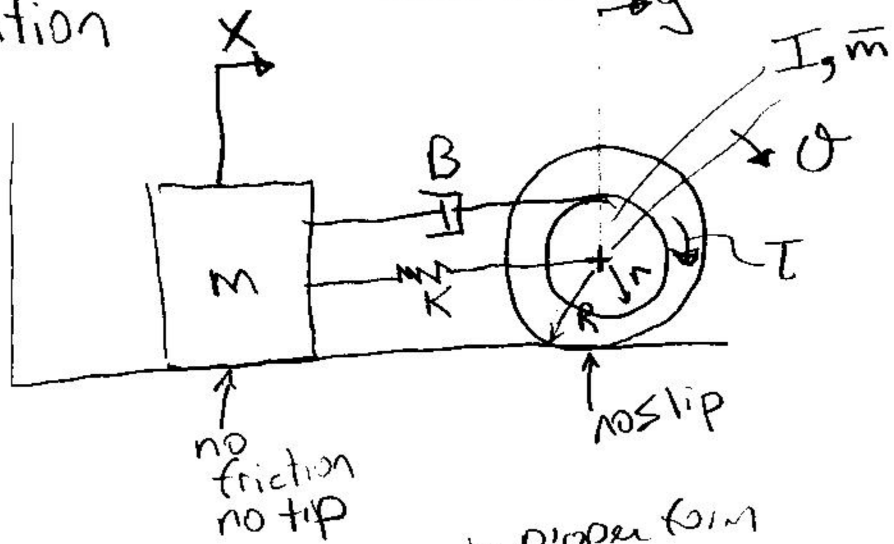
ME 375 Exam#1
10/2/2007
Solution

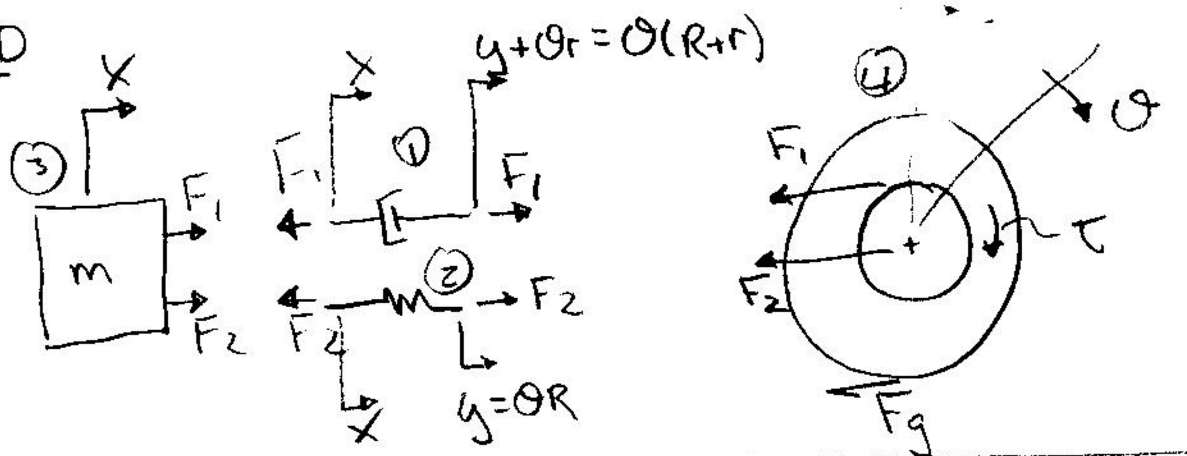
Problem 1

Shaver/Harwood



Find: Eqns of Motion in proper form in terms of $x, \dot x, \ddot x, \theta, \dot\theta, \ddot\theta, m, B, K, r, R, I, \bar m$ + input τ

EBD



(b) + (c)

(1) $F_1 = B(R+r)\dot\theta - B\dot x$ (2) $F_2 = KR\theta - Kx$

(3) $m\ddot x = F_1 + F_2 = B(R+r)\dot\theta + KR\theta - B\dot x - Kx$

$$\rightarrow \boxed{m\ddot x + B\dot x + Kx - B(R+r)\dot\theta - KR\theta = 0} \quad \boxed{1}$$

(4) $\bar m \underbrace{\ddot y}_{R\ddot\theta} = -F_1 - F_2 - F_g = -B(R+r)\dot\theta + B\dot x - KR\theta + Kx - F_g$

$$\rightarrow F_g = -B(R+r)\dot\theta + B\dot x - KR\theta + Kx - \bar m R\ddot\theta \quad \boxed{2}$$

$\bullet\ I\ddot\theta = \tau - F_1 r + F_g R \quad \boxed{3}$

Plug $\boxed{2}$ into $\boxed{3}$:

$$I\ddot{\theta} = \tau - B(R+r)r\dot{\theta} + Br\dot{x} - B(R+r)R\dot{\theta} + BR\dot{x} - KR^2\theta + KRx - \bar{m}R^2\ddot{\theta}$$

$$\rightarrow (I+\bar{m}R^2)\ddot{\theta} + \left[B(R+r)r + B(R+r)R\right]\dot{\theta} + KR^2\theta - \left[Br + BR\right]\dot{x} - KRx = \tau$$

$$\rightarrow \boxed{(I+\bar{m}R^2)\ddot{\theta} + B(R+r)^2\dot{\theta} + KR^2\theta - B(R+r)\dot{x} - KRx = \tau}$$

Boxed answers are EOMs

Problem 2

consider the following eqns. of motion

$$\ddot{x} + 2\dot{x} + 3x - \dot{y} - 2y = 0 \quad (1)$$

$$2\ddot{y} + \dot{y} + y - 3\dot{x} - x = \dot{u} + 2u$$

(a) determine the transfer fcn. from $u(t)$ to $x(t)$

$$(1) \rightarrow X(s)[s^2+2s+3] = Y(s)[s+2] \rightarrow Y(s) = \frac{s^2+2s+3}{s+2}X(s) \quad \boxed{1}$$

$$(2) \rightarrow Y(s)[2s^2+s+1] - X(s)[3s+1] = U(s)[s+2] \quad \boxed{2}$$

Plug $\boxed{1}$ into $\boxed{2}$:

$$\frac{(s^2+2s+3)}{(s+2)} \cdot (2s^2+s+1)X(s) - X(s)[3s+1] = U(s)[s+2]$$

$$\rightarrow X(s)\left[(s^2+2s+3)(2s^2+s+1) - (3s+1)(s+2)\right] = U(s)[s^2+4s+4]$$

$$\rightarrow X(s)\left[2s^4 + \cancel{s^3} + \cancel{s^2} + 4s^3 + \cancel{2s^2} + \cancel{2s} + 6s^2 + \cancel{3s} + 3 - 3s^2 - \cancel{6s} - s - 2\right] = U(s)\left[s^2 + 4s + 4\right]$$

$$\rightarrow X(s)\left[2s^4 + 5s^3 + 6s^2 - 2s + 1\right] = U(s)\left[s^2+4s+4\right]$$

$$\rightarrow \frac{X(s)}{U(s)} = \frac{s^2+4s+4}{2s^4+5s^3+6s^2-2s+1} \quad (a)$$

$$\rightarrow 2\ddddot{x} + 5\dddot{x} + 6\ddot{x} - 2\dot{x} + x = \ddot{u} + 4\dot{u} + 4u \quad (b)$$

$\boxed{\text{Problem 3}}$

Consider the following input/output eqn

$$\ddot{y} + 7\dot{y} + 12y = 2\dot{u} + u$$

(a) Find $Y(s)$ where initial conditions are $y_0 + \dot{y}_0 + u_0$

$$\rightarrow \left(s^2Y(s) - sy_0 - \dot{y}_0\right) + 7\left(sY(s) - y_0\right) + 12Y(s) = 2\left(sU(s) - u_0\right) + U(s)$$

$$\rightarrow Y(s)\left[s^2 + 7s + 12\right] = U(s)\left[2s+1\right] + \left(sy_0 + \dot{y}_0 + 7y_0 - 2u_0\right)$$

$$\rightarrow \boxed{Y(s) = \frac{2s+1}{s^2+7s+12}U(s) + \frac{(\dot{y}_0 + 7y_0 - 2u_0) + sy_0}{s^2+7s+12}}$$

↑ Transfer Function

(b) now, let $\dot{y}_0 = 10$, $y_0 = -1$, $u_0 = -1$ + $u(t)$ = unit impulse

$$\rightarrow Y(s) = \frac{2s+1}{s^2+7s+12} + \frac{(10-7-2) - s}{s^2+7s+12} = \frac{2s+1+1-s}{s^2+7s+12}$$

$$= \frac{s+2}{s^2+7s+12} = \frac{A}{s+3} + \frac{B}{s+4} = \frac{As + 4A + Bs + 3B}{s^2+7s+12}$$

so: $A + B = 1 \rightarrow B = 1 - A$

$4A + 3B = 2 \rightarrow 4A + 3 - 3A = A + 3 = 2 \rightarrow \boxed{A = -1 \mid B =}$

$$\rightarrow Y(s) = \frac{-1}{s+3} + \frac{2}{s+4} \rightarrow \boxed{y(t) = -e^{-3t} + 2e^{-4t}} \longrightarrow$$