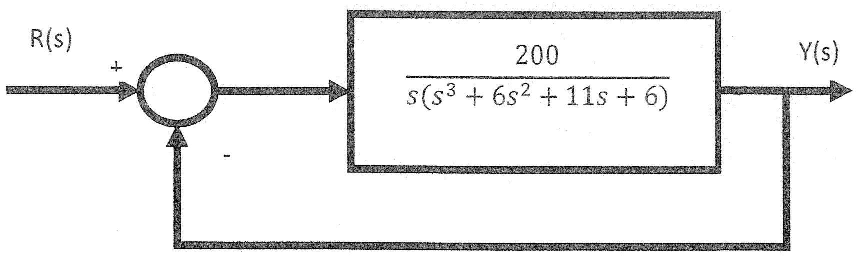
Name______________________________

Problem 1: (33%)

Determine the stability of the following unity feedback system using the Routh-Hurwitz Method.



(a) Determine the closed-loop transfer function T(s) = Y(s)/R(s) and identify the characteristic equation.

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{\frac{200}{s^4+6s^3+11s^2+6s}}{1+\frac{200}{s^4+6s^3+11s^2+6s}} = \frac{200}{s^4+6s^3+11s^2+6s+200}$$

characteristic equation:

$s^4+6s^3+11s^2+6s+200=0$

(b) State why or why not this characteristic equation satisfies or does not satisfy the Hurwitz necessary conditions for stability.

CE does satisafy the Hurwitz neccessary conditions

1) all terms of CE are present

2) all terms of CE have the same sign

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(Problem 1; continued)

(c) Construct a Routh Array and determine whether or not this system is stable. State the reasons for your conclusion regarding system stability.

$s^4 + 6s^3 + 11s^2 + 6s + 200 = 0$

s^4	1	11	200
s^3	$\cancel{6}$ 1	$\cancel{6}$ 1	0
$+ s^2$	$\cancel{10}$ 1	$\cancel{200}$ 20	0
$- s^1$	-19	0	
$+ s^0$	20		

$b_1 = \dfrac{1 \cdot 1 - 1 \cdot 11}{-1} = 10$

$b_2 = \dfrac{1 \cdot 0 - 1 \cdot 200}{-1}$

$c_1 = \dfrac{1 \cdot 20 - 1 \cdot 1}{-1}$

$1 \cdot 0$

$d_1 = \dfrac{1 \cdot 0 - (-19) \cdot (20)}{-(-19)}$

2 sign changes ⟹ 2 poles in RHP

unstable

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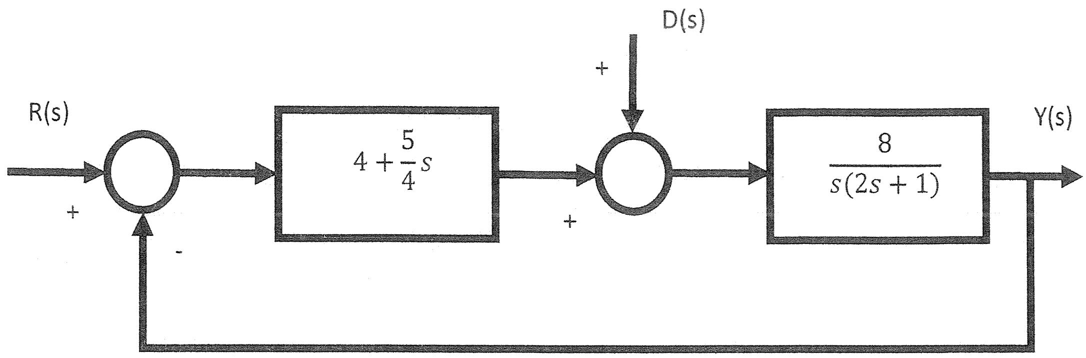
Problem 2:(33%)

For the system shown in the following figure:



Find the system steady-state error when:

(a) r(t) = unit step and d(t) = 0,

Input is a reference and unity feed back system ;

need only check for system type number of $G(s) = G_c \cdot G_p$

$G(s)$ is Type 1

$e_{ss}|_{\text{unit step (ref)}} = 0$

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(Problem 2 continued):

(b) d(t) = unit step and r(t) = 0.

$$\frac{Y(s)}{D(s)} = \frac{-8}{s(2s+1)+8(1.25s+4)}$$

$$E(s) = \cancel{R(s)}^{0} - Y(s) = -Y(s)$$

$$E(s) = \frac{8}{s(2s+1)+8(1.25s+4)} \cdot \frac{1}{s}$$

$$e_{ss}\Big|_{\substack{\text{unit step}\\ \text{dist}}} = \lim_{s\to 0} sE(s) = \frac{1}{4} = 0.25$$

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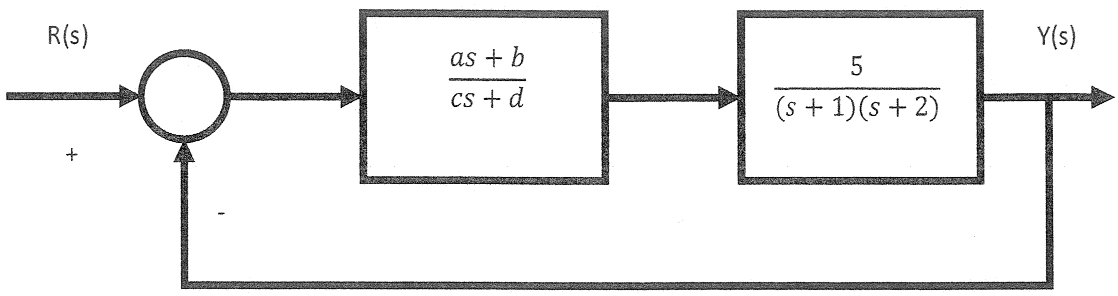
Problem 3:(34%)

For the system shown in the figure, design a compensator using **Direct Pole Placement.**



Where, $G_p(s) = \frac{5}{(s+1)(s+2)}$ is the plant; and the form of the controller is $G_c(s) = \frac{as+b}{cs+d}$. The desired operating point is: $s = -3 \pm 4j$, with an additional "fast" pole at $s = -10$.

(a) Determine the characteristic equation of the closed-loop system.

$(cs+d)(s+1)(s+2)+5(as+b) = cs^3+(3c+d)s^2+(5a+2c+3d)s+(5b+2d)=0$

(b) Determine the desired operating point characteristic equation.

$(s+3+4j)(s+3-4j)(s+10) = (s^2+16s+25)(s+10)$

$= s^3+16s^2+85s+250=0$

(Problem 3 continued):

(c) Equate like powers of s from the equations in parts (a) and (b), and solve for the controller coefficients a, b, c, & d.

$$cs^3 + (3c + d)s^2 + (5a + 2c + 3d)s + (5b + 2d) = s^3 + 16s^2 + 85s + 250$$

s^3: $c = 1$

s^2: $3c + d = 16 \Rightarrow d = 16 - 3(1) = 13$

s^1: $5a + 2c + 3d = 85 \Rightarrow a = [85 - 2(1) - 3(13)]/5 = \frac{44}{5}$

s^0: $5b + 2d = 250 \Rightarrow b = [250 - 2(13)]/5 = 224/5$

$$G_c = \frac{44s + 224}{5(s + 13)}$$

(d) Again **set up** this system of equations from parts (a) and (b) to be solved in the form of a matrix equation **Ax = y** (like you did for you myRIO project and lab) **but do not solve.**

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 1 \\ 5 & 0 & 2 & 3 \\ 0 & 5 & 0 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 16 \\ 85 \\ 250 \end{bmatrix}$$