ME 375 EXAM #1
Tuesday, February 24, 2004

Division Meckl 10:30 / King 1:30 (circle one) **Name** Solution

Instructions

(1) This is a closed book examination, but you are allowed one 8.5×11 crib sheet.

(2) You have one hour to work all three problems on the exam.

(3) Use the solution procedure we have discussed: what are you given, what are you asked to find, what are your assumptions, what is your solution, does your solution make sense. You must show all of your work to receive any credit.

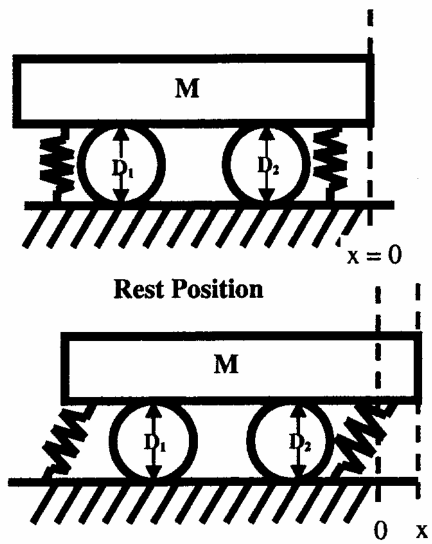
(4) You must write neatly and should use a logical format to solve the problems. You are encouraged to really "think" about the problems before you start to solve them. Please write your name in the top right-hand corner of each page.

(5) A table of Laplace transform pairs and properties of Laplace transforms is attached at the end of this exam set.

Problem No. 1 (40%)	____________
Problem No. 2 (30%)	____________
Problem No. 3 (30%)	____________
TOTAL (*/100%)**	____________

PROBLEM NO. 1 (40%)

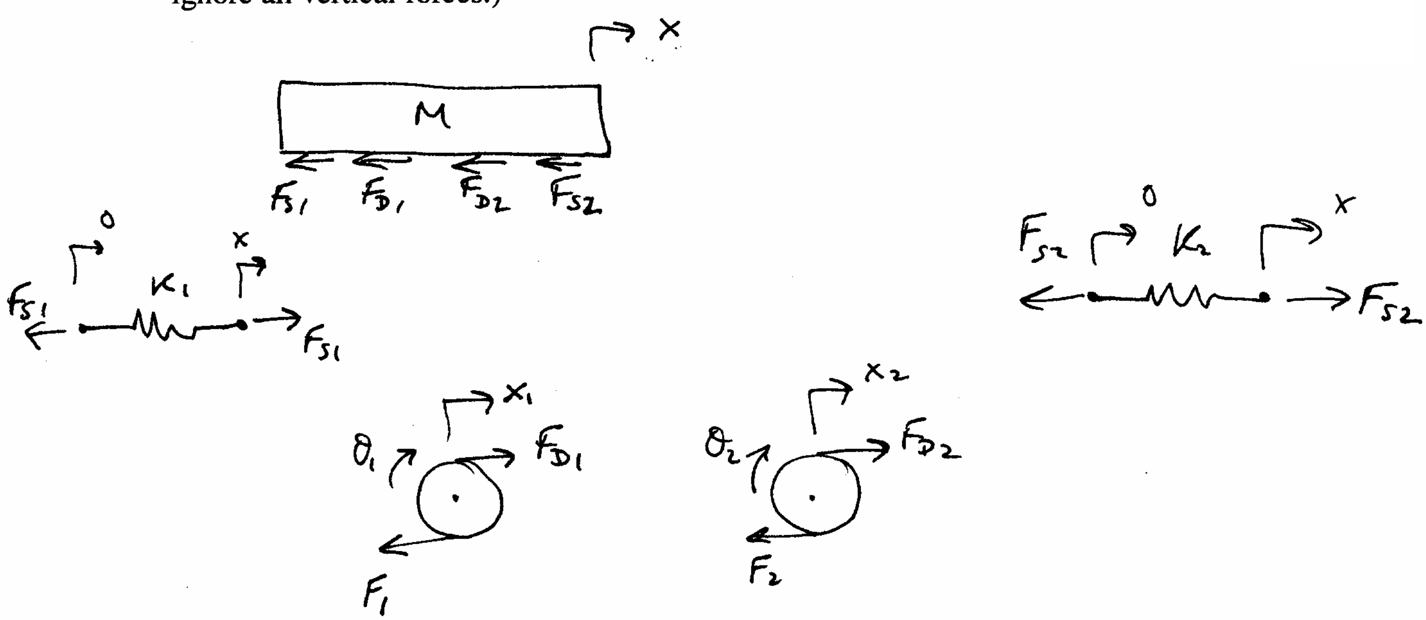
Consider the "shaker table" system shown in the figure below. The following information is known: M is the moving mass of the table. The rollers D_1 and D_2 have masses of m_1 and m_2, respectively, and similar diameters of 2r and they roll without slipping. D_1 and D_2 are cylinders and have moments of inertia $J_i = \frac{1}{2} m_i r^2$ about their respective centers of mass. The springs have equivalent horizontal spring constants of K_1 and K_2, respectively, and are unstretched in the rest position shown, when x = 0. Treat the springs as acting only horizontally.



Starting Displacement

Develop a model for this system using the following steps:

(a) Draw free body diagrams for all the components of the system shown above. (You can ignore all vertical forces.)



PROBLEM NO. 1 (Continued)

(b) Write governing equations for each of the free body diagrams in part (a). Don't forget to include kinematic relationships.

$$M\ddot{x} = -F_{S1} - F_{D1} - F_{D2} - F_{S2}$$

$$F_{S1} = K_1 x \quad , \quad F_{S2} = K_2 x$$

$$m_1\ddot{x}_1 = F_{D1} - F_1 \quad , \quad m_2\ddot{x}_2 = F_{D2} - F_2$$

$$J_1\ddot{\theta}_1 = rF_{D1} + rF_1 \quad , \quad J_2\ddot{\theta}_2 = rF_{D2} + rF_2$$

$$x_1 = r\theta_1 \quad , \quad x_2 = r\theta_2 \Rightarrow x_1 = x_2 = \frac{x}{2}$$

$$x = 2r\theta_1 = 2r\theta_2 \Rightarrow \theta_1 = \theta_2 = \frac{x}{2r}$$

(c) Develop a single differential equation of the system in terms of only the variable x.

$$F_1 = F_{D1} - \frac{m_1}{2}\ddot{x} \quad , \quad F_2 = F_{D2} - \frac{m_2}{2}\ddot{x}$$

$$\begin{cases} \frac{1}{2}m_1 \cancel{r^2}\frac{\ddot{x}}{2\cancel{r}} = \cancel{r}F_{D1} + \cancel{r}\left(F_{D1} - \frac{m_1}{2}\ddot{x}\right) \\ \frac{1}{2}m_2 \cancel{r^2}\frac{\ddot{x}}{2\cancel{r}} = \cancel{r}F_{D2} + \cancel{r}\left(F_{D2} - \frac{m_2}{2}\ddot{x}\right) \end{cases}$$

$$F_{D1} = \frac{1}{2}\frac{3}{4}m_1\ddot{x} = \frac{3}{8}m_1\ddot{x}$$

$$F_{D2} = \frac{1}{2}\frac{3}{4}m_2\ddot{x} = \frac{3}{8}m_2\ddot{x}$$

$$M\ddot{x} + \frac{3}{8}m_1\ddot{x} + \frac{3}{8}m_2\ddot{x} + (k_1+k_2)x = 0$$

$$\boxed{\left(M + \frac{3}{8}m_1 + \frac{3}{8}m_2\right)\ddot{x} + (K_1 + K_2)x = 0}$$

PROBLEM NO. 1 (Continued)

(d) Based on your differential equation in part (c), find the roots of the characteristic equation and express them in terms of only these parameters: K_1, K_2, M, m_1, and m_2.

$$\left(M + \frac{3}{8}m_1 + \frac{3}{8}m_2\right)s^2 + (K_1 + K_2) = 0$$

$$\boxed{s = \pm j\sqrt{\frac{K_1 + K_2}{M + \frac{3}{8}m_1 + \frac{3}{8}m_2}}}$$

(e) Is this system stable? Please explain.

No — two roots are on the imaginary axis (real part is NOT negative).

PROBLEM NO. 2 (30%)

Suppose we have a mechanical system with the following model,

$$\ddot{y}+6\dot{y}+13y=13u$$

where y is the output of the system and u is the input of the system. Initial conditions are $y(0)$ and $\dot{y}(0)$.

(a) Based on the given model, generate the transfer function $Y(s)/U(s)$. set $y(0)=\dot{y}(0)=0$

$$(s^2+6s+13)\,Y(s) = 13\,U(s)$$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{13}{s^2+6s+13}}$$

(b) If the input $u(t)$ is a unit step, find the output $y(t)$ when $y(0)=0$ and $\dot{y}(0)=0$.

$$Y(s) = \frac{13}{s^2+6s+13}\cdot\frac{1}{s} = \frac{13}{s(s^2+6s+13)}$$

$$= \frac{A}{s} + \frac{Bs+C}{s^2+6s+13} = \frac{A(s^2+6s+13)+Bs^2+Cs}{s(s^2+6s+13)}$$

s^2: $A+B=0 \Rightarrow B=-1$

s: $6A+C=0 \Rightarrow C=-6$

1: $13A=13 \Rightarrow A=1$

$$Y(s) = \frac{1}{s} - \frac{s+6}{(s+3)^2+2^2} = \frac{1}{s} - \frac{s+3}{(s+3)^2+2^2} - \frac{3}{(s+3)^2+2^2}$$

$$\mathcal{L}^{-1}\quad \boxed{y(t) = 1 - e^{-3t}\cos 2t - \frac{3}{2}e^{-3t}\sin 2t}$$

(c) Write down an expression for the free response with initial conditions $y(0)=1$ and $\dot{y}(0)=0$.

$$[s^2Y(s) - s\,y(0) - \dot{y}(0)] + 6\,[sY(s) - y(0)] + 13Y(s) = 0$$

$$(s^2+6s+13)Y(s) = y(0)s + \dot{y}(0) + 6y(0) = s+6$$

$$Y(s) = \frac{s+6}{(s+3)^2+2^2} = \frac{s+3}{(s+3)^2+2^2} + \frac{3}{(s+3)^2+2^2}$$

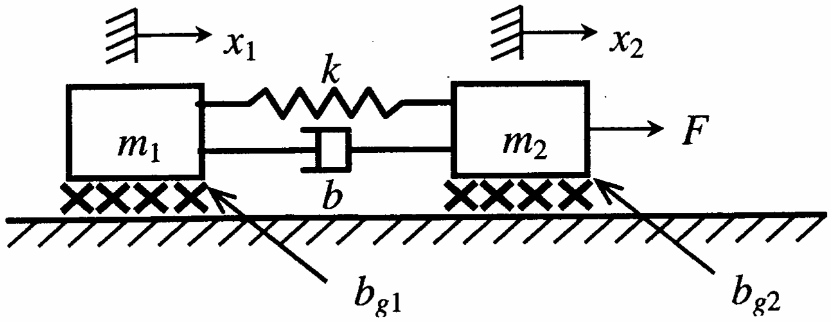
$$\mathcal{L}^{-1}\quad \boxed{y(t) = e^{-3t}\cos 2t + \frac{3}{2}e^{-3t}\sin 2t}$$

(d) Is this system stable? Please explain.

Yes, the free response decays to zero.

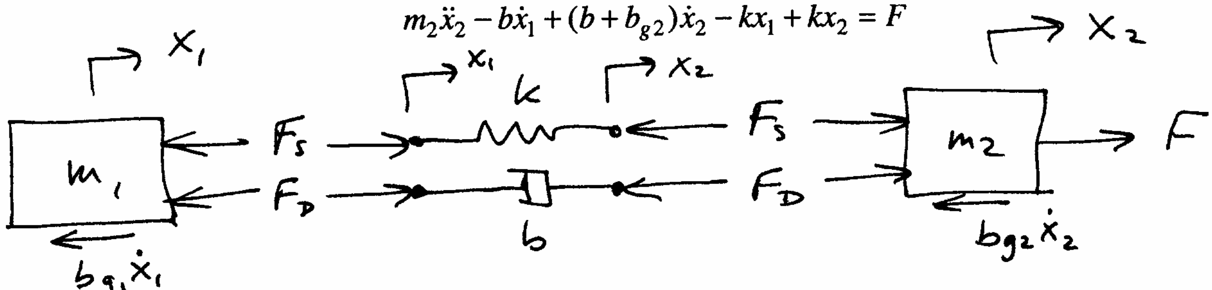
PROBLEM NO. 3 (30%)

Consider the mechanical system shown in the following figure:



(a) Show that the equations of motion for this system are given by:

$$m_1\ddot{x}_1 + (b + b_{g1})\dot{x}_1 - b\dot{x}_2 + kx_1 - kx_2 = 0$$
$$m_2\ddot{x}_2 - b\dot{x}_1 + (b + b_{g2})\dot{x}_2 - kx_1 + kx_2 = F$$



$$m_1\ddot{x}_1 = -F_s - F_D - b_{g1}\dot{x}_1 \qquad F_s = k(x_1 - x_2)$$
$$m_2\ddot{x}_2 = F + F_s + F_D - b_{g2}\dot{x}_2 \qquad F_D = b(\dot{x}_1 - \dot{x}_2)$$

$$m_1\ddot{x}_1 + b(\dot{x}_1 - \dot{x}_2) + b_{g1}\dot{x}_1 + k(x_1 - x_2) = 0$$
$$m_2\ddot{x}_2 - b(\dot{x}_1 - \dot{x}_2) + b_{g2}\dot{x}_2 - k(x_1 - x_2) = F$$

PROBLEM NO. 3 (Continued)

(b) Assume we are interested in predicting the output $x_2(t)$. Define an appropriate state vector $\mathbf{q} = [q_1 \quad q_2 \quad q_3 \quad q_4]^T$ and write the state space form of this model.

$q_1 = x_1, \quad q_2 = \dot{x}_1, \quad q_3 = x_2, \quad q_4 = \dot{x}_2$

$$\boxed{\begin{aligned}
\dot{q}_1 &= q_2 \\
\dot{q}_2 &= -\frac{k}{m_1} q_1 - \frac{b + b_{g1}}{m_1} q_2 + \frac{k}{m_1} q_3 + \frac{b}{m_1} q_4 \\
\dot{q}_3 &= q_4 \\
\dot{q}_4 &= \frac{k}{m_2} q_1 + \frac{b}{m_2} q_2 - \frac{k}{m_2} q_3 - \frac{b + b_{g2}}{m_2} q_4 + \frac{1}{m_2} F \\
y &= q_3
\end{aligned}}$$

(c) If the input force $F(t)$ is a unit step, qualitatively describe the nature of the response $x_2(t)$ at steady-state.

At steady-state, the system moves at a constant speed that is just enough for damping to balance the applied force.