ME 375 EXAM #1

Tuesday, February 27, 2007

Division Shaver 11:30 / Meckl 2:30 (circle one) **Name** Solution

Instructions

(1) This is a closed book examination, but you are allowed one 8.5×11 crib sheet.

(2) You have one hour to work all three problems on the exam.

(3) Use the solution procedure we have discussed: what are you given, what are you asked to find, what are your assumptions, what is your solution, does your solution make sense. You must show all of your work to receive any credit.

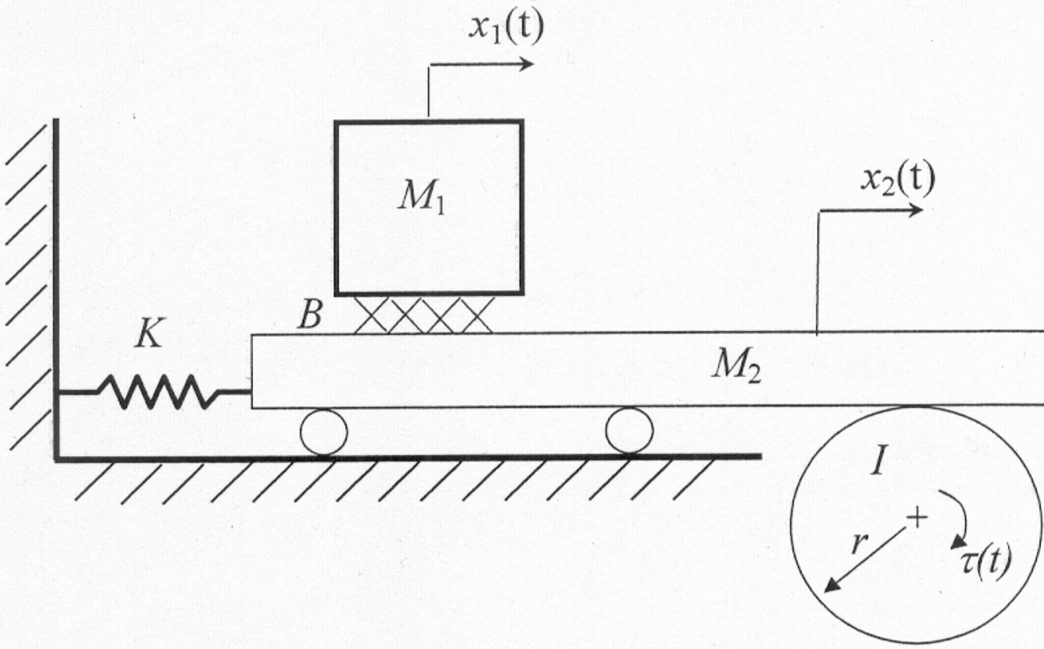
(4) You must write neatly and should use a logical format to solve the problems. You are encouraged to really "think" about the problems before you start to solve them. Please write your name in the top right-hand corner of each page.

(5) A table of Laplace transform pairs and properties of Laplace transforms is attached at the end of this exam set.

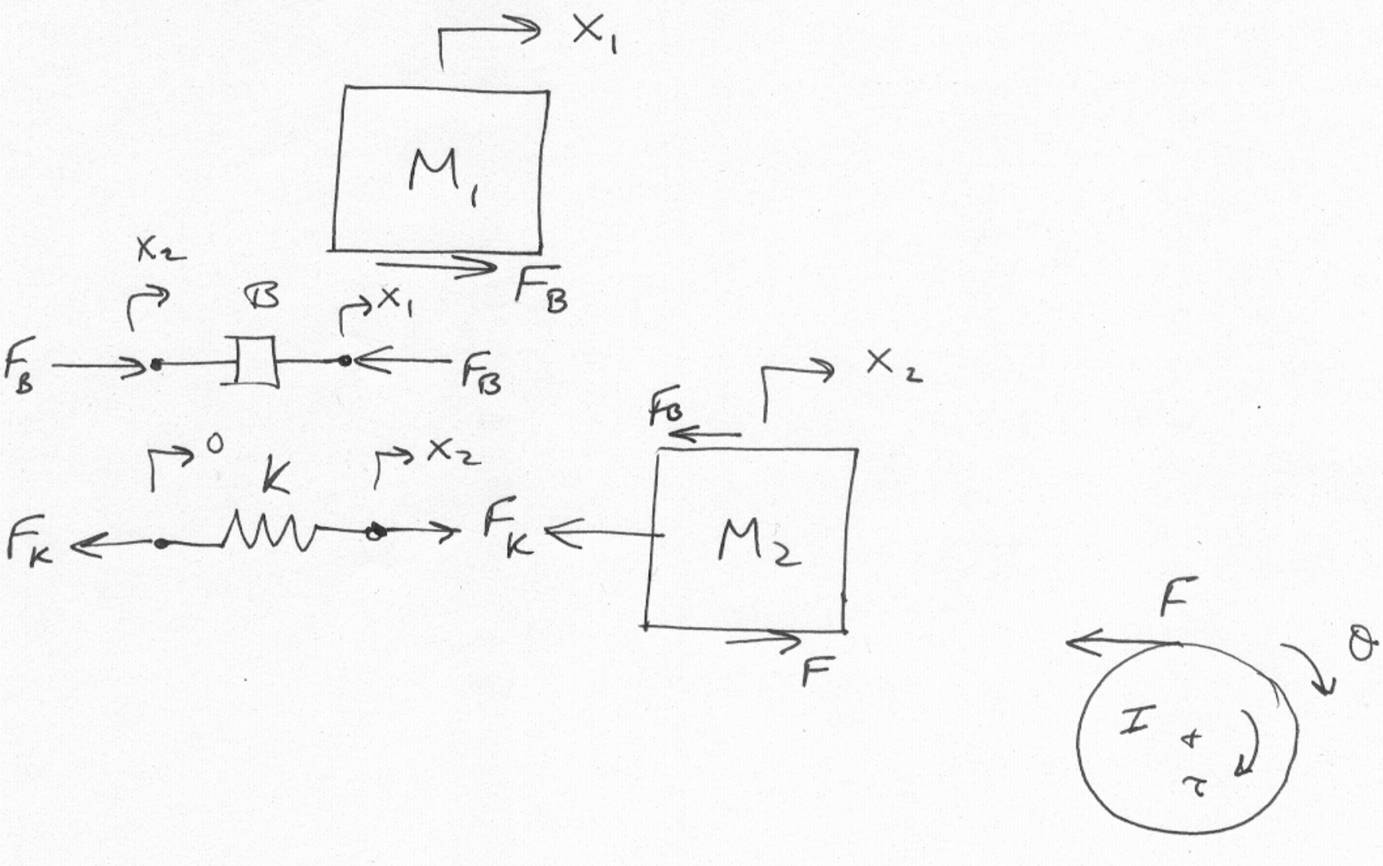
Problem No. 1 (40%)	
Problem No. 2 (30%)	
Problem No. 3 (30%)	
TOTAL (*/100%)**	

PROBLEM 1: (40%)

Consider the following system. Assume that the wheel rolls without slipping on the mass M_2.



a) Draw free body diagrams for all the elements in this system. Make sure to include all displacements and label all the forces.



b) Write down the elemental equations.

$$M_1 \ddot{x}_1 = F_B$$

$$F_B = B(\dot{x}_2 - \dot{x}_1)$$

$$M_2 \ddot{x}_2 = F - F_K - F_B$$

$$F_K = K x_2$$

$$I\ddot{\theta} = \tau - rF \quad , \quad x_2 = r\theta$$

c) Using $\tau(t)$ as the input to the system, show that the equations of motion (EOM) are given by:

$$M_1\ddot{x}_1 + B\dot{x}_1 - B\dot{x}_2 = 0$$
$$\left(M_2 + I/r^2\right)\ddot{x}_2 + B\dot{x}_2 + Kx_2 - B\dot{x}_1 = \tau / r$$

$$M_1\ddot{x}_1 = B(\dot{x}_2 - \dot{x}_1) \Rightarrow M_1\ddot{x}_1 + B\dot{x}_1 - B\dot{x}_2 = 0 \checkmark$$

$$F = \frac{1}{r}\left(\tau - I\ddot{\theta}\right) = \frac{1}{r}\left(\tau - \frac{I}{r}\ddot{x}_2\right)$$

$$M_2\ddot{x}_2 = \frac{1}{r}\left(\tau - \frac{I}{r}\ddot{x}_2\right) - Kx_2 - B(\dot{x}_2 - \dot{x}_1)$$

$$\left(M_2 + \frac{I}{r^2}\right)\ddot{x}_2 + B\dot{x}_2 + Kx_2 - B\dot{x}_1 = \frac{1}{r}\tau \checkmark$$

d) Find the transfer function from $\tau(t)$ to x_1.

$$(M_1 s^2 + Bs) X_1(s) = Bs X_2(s) \Rightarrow X_2(s) = \frac{1}{Bs}(M_1 s^2 + Bs)$$

$$\left[\left(M_2 + \frac{I}{r^2}\right)s^2 + Bs + K\right] X_2(s) - Bs X_1(s) = \frac{1}{r}\tau(s)$$

$$\left\{(M_1 s^2 + Bs)\left[\left(M_2 + \frac{I}{r^2}\right)s^2 + Bs + K\right] - B^2 s^2\right\} X_1(s) = \frac{B}{r} s \tau(s)$$

$$\left\{M_1\left(M_2 + \frac{I}{r^2}\right)s^4 + \left[BM_1 + B\left(M_2 + \frac{I}{r^2}\right)\right]s^3 + KM_1 s^2 + KBs\right\} X_1(s)$$
$$= \frac{B}{r} s \tau(s)$$

$$\boxed{\frac{X_1(s)}{\tau(s)} = \frac{\frac{B}{r}s}{M_1\left(M_2 + \frac{I}{r^2}\right)s^4 + \left[BM_1 + B\left(M_2 + \frac{I}{r^2}\right)\right]s^3 + KM_1 s^2 + KBs}}$$

e) Provide the input/output differential equation that relates the output x_1 to the input $\tau(t)$.

$$M_1\left(M_2 + \frac{I}{r^2}\right)\ddddot{x}_1 + \left[BM_1 + B\left(M_2 + \frac{I}{r^2}\right)\right]\dddot{x}_1 + KM_1 \ddot{x}_1 + KB\dot{x}_1 = \frac{B}{r}\dot{\tau}$$

or

$$\boxed{M_1\left(M_2 + \frac{I}{r^2}\right)\dddot{x}_1 + \left[BM_1 + B\left(M_2 + \frac{I}{r^2}\right)\right]\ddot{x}_1 + KM_1 \dot{x}_1 + KBx_1 = \frac{B}{r}\tau}$$

PROBLEM 2: (30%)

Given the following differential equation,

$$\ddot{y}+10\dot{y}+125y=250f(t)$$

with zero initial conditions.

(a) Find the response $y(t)$ to a unit step input in $f(t)$.

$$(s^2+10s+125)Y(s)=\frac{250}{s}$$

$$\Rightarrow Y(s)=\frac{250}{s(s^2+10s+125)}=\frac{A}{s}+\frac{Bs+C}{s^2+10s+125}$$

$$=\frac{A(s^2+10s+125)+Bs^2+Cs}{s(s^2+10s+125)}$$

s^2: $A+B=0 \Rightarrow B=-2$

s^1: $10A+C=0 \Rightarrow C=-20$

s^0: $125A=250 \Rightarrow A=2$

$$Y(s)=\frac{2}{s}-\frac{2s+20}{s^2+10s+125}$$

$$=2\left[\frac{1}{s}-\frac{s+10}{(s+5)^2+10^2}\right]=2\left[\frac{1}{s}-\frac{s+5}{(s+5)^2+10^2}-\frac{\frac{1}{2}(10)}{(s+5)^2+10^2}\right]$$

$$\boxed{y(t)=2\left[1-e^{-5t}\cos 10t-\frac{1}{2}e^{-5t}\sin 10t\right]}$$

(b) Is this system stable? Please explain.

Yes – when system is 2nd order, all coefficients having same sign is sufficient for stability

or

- poles at $s=-5\pm j10$ have negative real part

PROBLEM 3: (30%)

For the following transfer functions:

(i) determine the poles

(ii) compute time constant, natural frequency and/or damping ratio

(iii) indicate which time response (1, 2, 3, or 4) below represents the unit step response of each transfer function.

(a) $\frac{10}{s^2+11s+10} = \frac{10}{(s+10)(s+1)}$

poles: $s = -10, -1$

$\tau_1 = \frac{1}{10}s, \tau_2 = 1s$

(4) $\left.\begin{array}{l}\omega_n = \sqrt{10} = 3.16\ r/s \\ \zeta = 1.74\end{array}\right\}$ ok

(b) $\frac{3}{s+3}$

poles: $s = -3$

$\tau = \frac{1}{3}s$

(1)

(c) $\frac{25}{s^2+6s+25} = \frac{25}{(s+3)^2+4^2}$

poles: $s = -3 \pm j4$

$\omega_n = 5\ r/s$

$\zeta = 0.6$

(3)

(d) $\frac{25}{s^2+3s+25} = \frac{25}{(s+1.5)^2+4.77^2}$

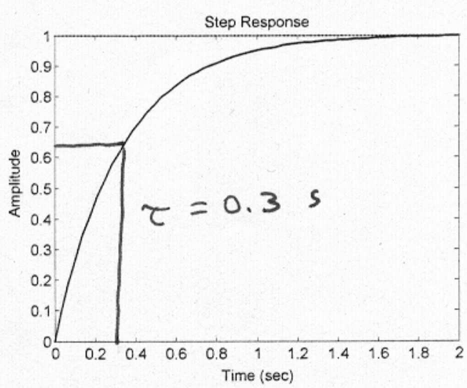
poles: $s = -1.5 \pm j4.77$

$\omega_n = 5\ r/s$

$\zeta = 0.3$

(2)

(1)



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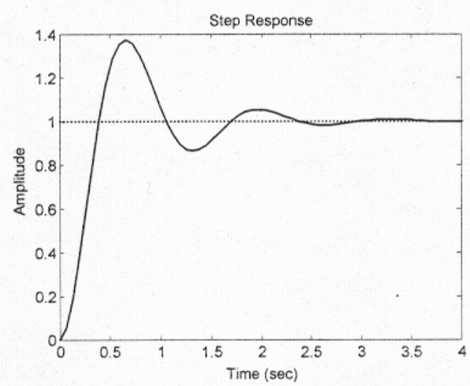

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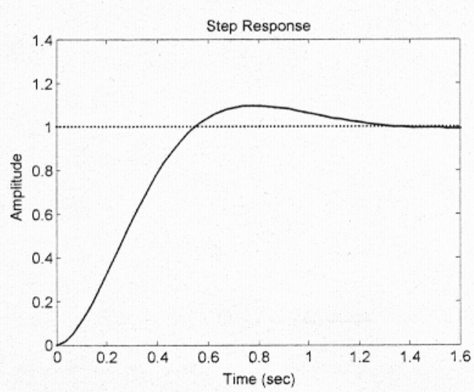

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