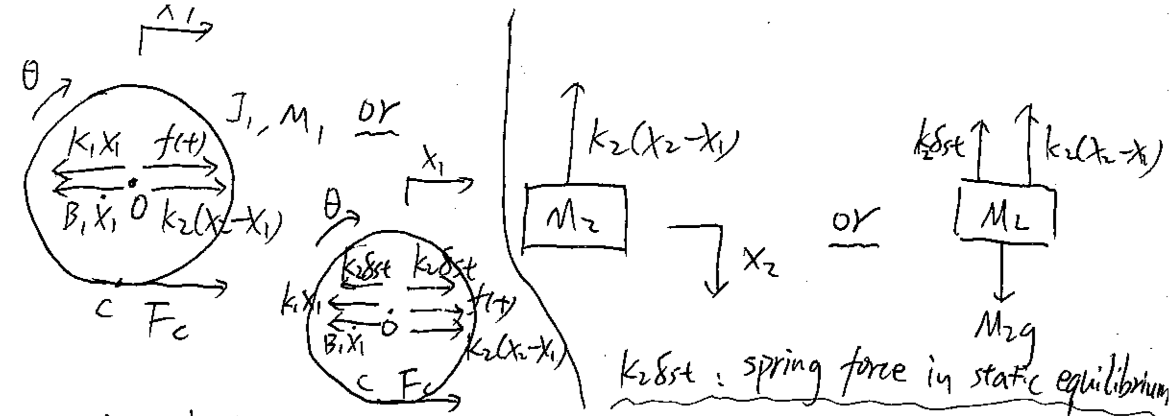
Problem 1 (a)



$k_2\delta st$: spring force in static equilibrium

(b) For the disk: $J_c \ddot{\theta} = \Sigma \tau_c$ (with respect to c)

$$\Rightarrow \underbrace{\left(\tfrac{1}{2}M_1R^2 + M_1R^2\right)}_{\frac{3}{2}M_1R^2} \ddot{\theta} = [f(t) + k_2(x_2 - x_1) - k_1 x_1 - B_1 \dot{x}_1] R \quad (1)$$

or
$$\begin{cases} J_o \ddot{\theta} = \Sigma \tau_o & \text{(with respect to o)} \\ M_1 \ddot{x}_1 = \Sigma F_o \end{cases}$$

$$\Rightarrow \begin{cases} \tfrac{1}{2}M_1R^2\ddot{\theta} = -F_c R & (2) \\ M_1 \ddot{x}_1 = f(t) + k_2(x_2 - x_1) - k_1 x_1 - B_1 \dot{x}_1 + F_c & (3) \end{cases}$$

For the block: $M_2 \ddot{x}_2 = -k_2(x_2 - x_1)$ (4)

coupling equation: $x_1 = \theta R$ (5)

(c) If the disk rotational equation~~s~~ is built with respect to C:

From (5): $\theta = \frac{x_1}{R}$, substitute it into (1):

$$\tfrac{3}{2}M_1 \ddot{x}_1 = f(t) + k_2(x_2 - x_1) - k_1 x_1 - B_1 \dot{x}_1$$

combine it with (4), we get

$$\boxed{\begin{cases} \tfrac{3}{2}M_1\ddot{x}_1 + B_1 \dot{x}_1 + (k_1 + k_2)x_1 - k_2 x_2 = f(t) \\ M_2 \ddot{x}_2 - k_2 x_1 + k_2 x_2 = 0 \end{cases}}$$

If the disk rotational equation is built with respect to o with additional translational equation:

From (5): $\theta = \frac{x_1}{R}$, substitute it into (2) $\Longrightarrow F_c = \frac{-1}{2}M_1\ddot{x}_1$

substitute into (3): $M_1\ddot{x}_1 = f(t) + k_2(x_2 - x_1) - k_1 x_1 - B_1\dot{x}_1 - \frac{1}{2}M_1\ddot{x}_1$

combine it with (4), we get

Problem 2 (a) $\mathcal{L}$: $\begin{cases} 3sX_1(s)+X_1(s)-sX_2(s)-X_2(s)=0 & ① \\ sX_1(s)+2X_1(s)+2sX_2(s)+X_2(s)=F(s) & ② \end{cases}$

From ①: $X_1(s)=\dfrac{(s+1)X_2(s)}{3s+1}$

substitute into ②:

$$\frac{(s+2)(s+1)X_2(s)}{3s+1}+(2s+1)X_2(s)=F(s)$$

$$\Rightarrow \quad (s^2+3s+2)X_2(s)+\cancel{(2s+1)X_2(s)}\,(6s^2+5s+1)X_2(s)=(3s+1)F(s)$$

$$\Rightarrow \quad (7s^2+8s+3)X_2(s)=(3s+1)F(s)$$

$$\Rightarrow \quad G(s)=\frac{X_2(s)}{F(s)}=\frac{3s+1}{7s^2+8s+3}$$

(b) $N(s)=3s+1=0 \Rightarrow$ zeros: $z=-\frac{1}{3}$

$D(s)=7s^2+8s+3=0 \Rightarrow$ poles: $p_{1,2}=-\frac{4}{7}\pm\frac{\sqrt{5}}{7}j$

(c) $K_s=G(0)=\frac{1}{3}$

(d) $(7s^2+8s+3)X_2(s)=(3s+1)F(s)$

$$\Rightarrow \quad 7\ddot{x}_2+8\dot{x}_2+3x_2=3\dot{f}(t)+f(t)$$

Problem 3 : (a) $\mathcal{L}$: $s^2Y(s) + 4sY(s) + 13Y(s) = \delta(s)$

$$G(s) = \frac{Y(s)}{\delta(s)} = \frac{1}{s^2+4s+13}$$

(b) $Y_{Forced} = G(s)\cdot\delta(s) = \frac{1}{s^2+4s+13} \times 1 = \frac{3}{(s+2)^2+3^2} \times \frac{1}{3}$

$\overset{\mathcal{L}^{-1}}{\Rightarrow} y_{Forced}(t) = \frac{1}{3}e^{-2t}\sin 3t$

(c) $\mathcal{L}$: $s^2Y(s) - s\overset{1}{f(0)} - \overset{0}{\dot{f}(0)} + 4sY(s) - 4\overset{1}{f(0)} + 13Y(s) = 0$

$\Rightarrow (s^2+4s+13)Y(s) = s+4$

$\Rightarrow \quad Y_{free}(s) = \frac{s+4}{s^2+4s+13} = \frac{s+2}{(s+2)^2+3^2} + \frac{3}{(s+2)^2+3^2}\times\frac{2}{3}$

$\overset{\mathcal{L}^{-1}}{\Longrightarrow} \quad y_{free}(t) = e^{-2t}\cos 3t + \frac{2}{3}e^{-2t}\sin 3t$

(d) $D(s) = s^2+4s+13 = 0$

poles $\frac{-4\pm\sqrt{-36}}{2} = -2\pm 3j$

real parts < 0 for both of the poles, meaning tha
they lie in the LHP

$\therefore$ stable