Exam II Solution

Problem 1

(a)

$$P(s) = \frac{s^2 + 1000 + 250{,}000}{10{,}000\,(s^2 + .1s + 25)} \quad \leftarrow (s+500)^2$$

$(s^2 + .1s + 25)$ → complex

$$= \left(\frac{1}{10{,}000}\right)\frac{(s+500)}{1}\frac{(s+500)}{1} \cdot \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\omega_n^2 = 25 \rightarrow \omega_n = 5\ \text{rad/s}$ ← $\zeta = \frac{0.1}{2\omega_n} = \frac{0.1}{2 \cdot 5} = 0.01 = \zeta$

$K\omega_n^2 = 1 \rightarrow K = 1/\omega_n^2 = \frac{1}{25}$

$$= \left(\frac{1}{10{,}000}\right) 500\left(\tfrac{s}{500}+1\right) 500\left(\tfrac{s}{500}+1\right) \cdot \frac{(1/25)\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\rightarrow \frac{500^2}{10{,}000} \cdot \frac{1}{25} = 1$

$$P(s) = (1)\left(\tfrac{s}{500}+1\right)^2 \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\left(\tfrac{s}{500}+1\right)^2$ ← break freq at 500 rad/s

$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ ← break freq of $\omega_n = 5\ \text{rad/s}$

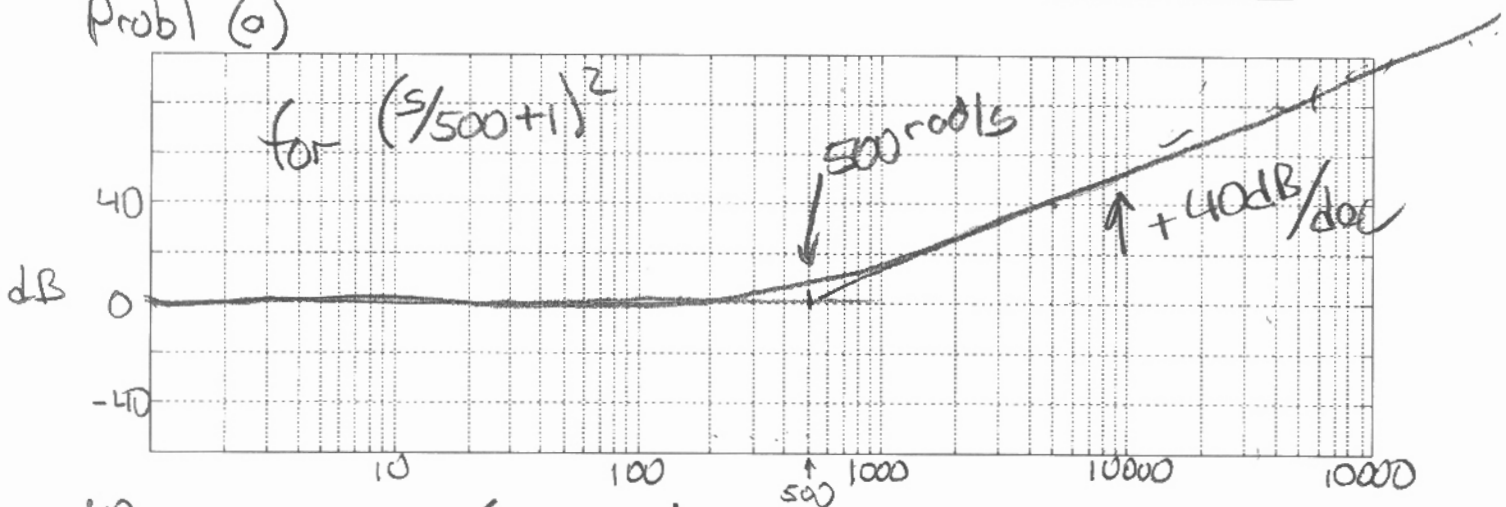
resonant freq $= \omega_r = \omega_n\sqrt{1-2\zeta^2}$

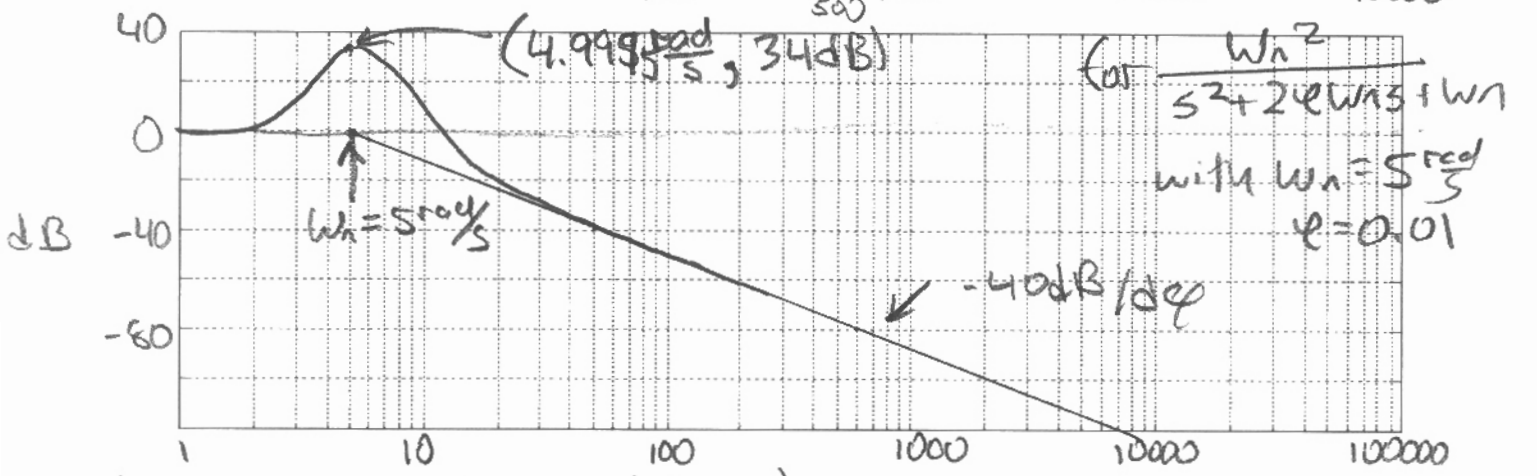
$= (5)\sqrt{1-2\cdot.01^2} = 4.9995\ \text{rad/s} = \omega_r$

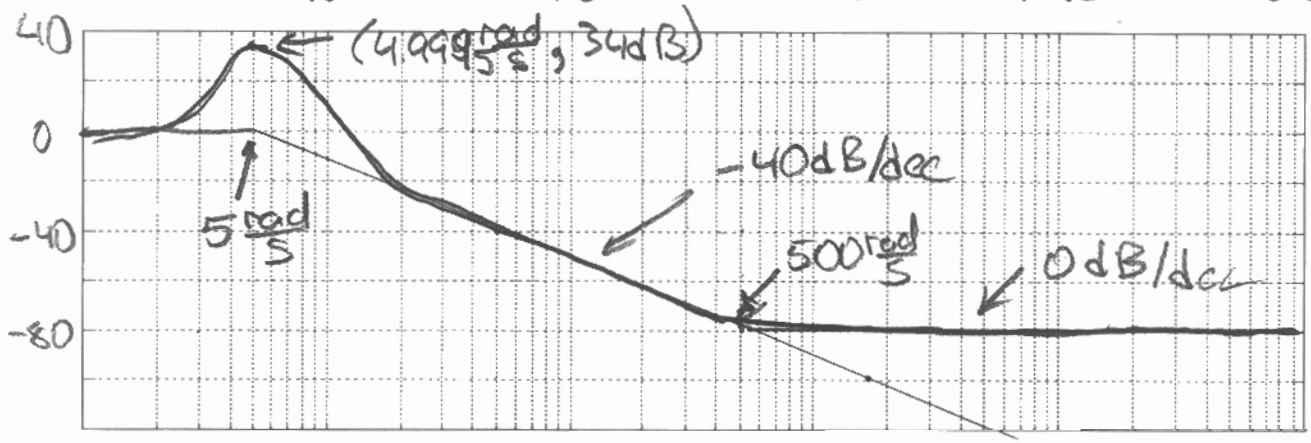
peak @ $20\log_{10}\left(\frac{K}{2\zeta\sqrt{1-\zeta^2}}\right)$

$= 20\log_{10}\left(\frac{1}{2\cdot.01\sqrt{1-.01^2}}\right) = 34\text{dB} = \text{Peak}$

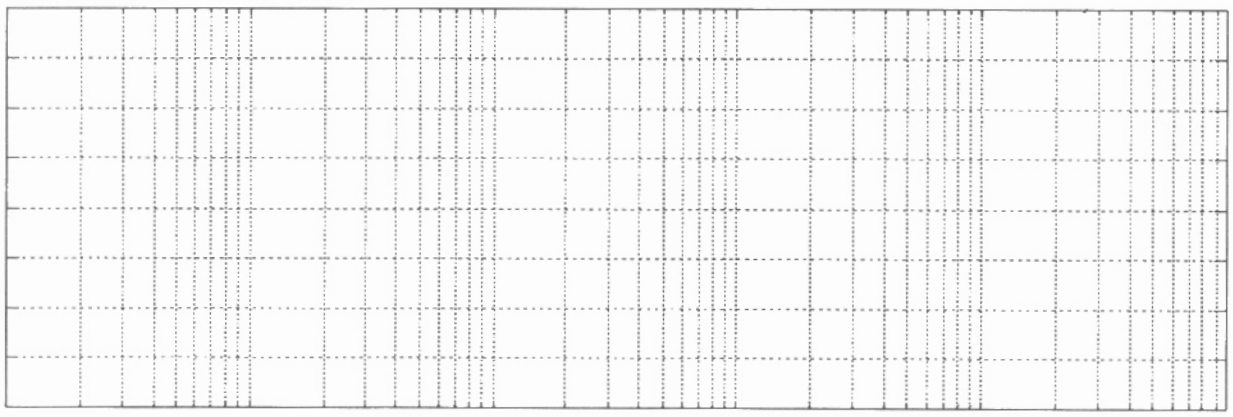
Probl (a)







Add them

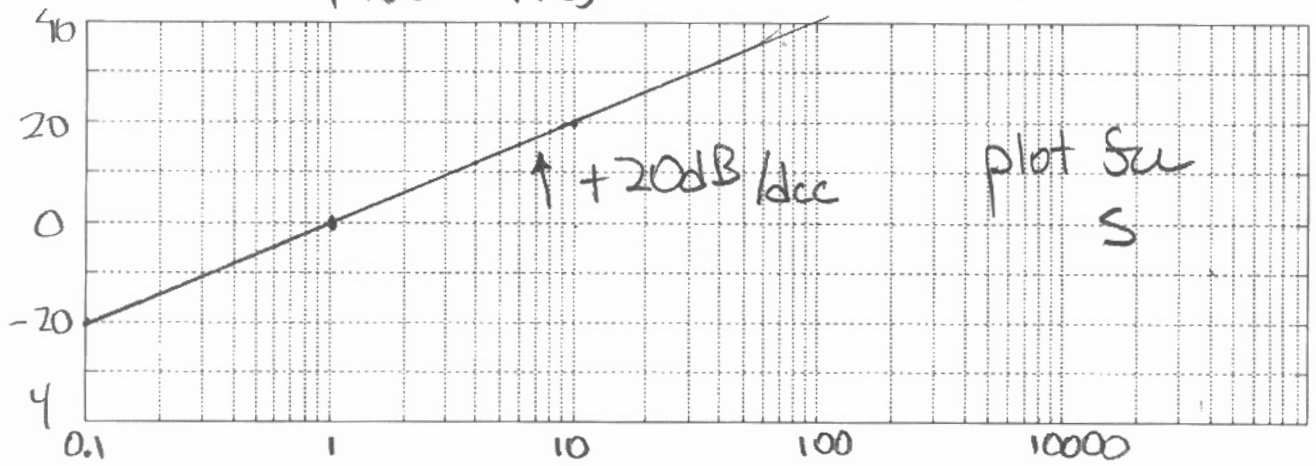


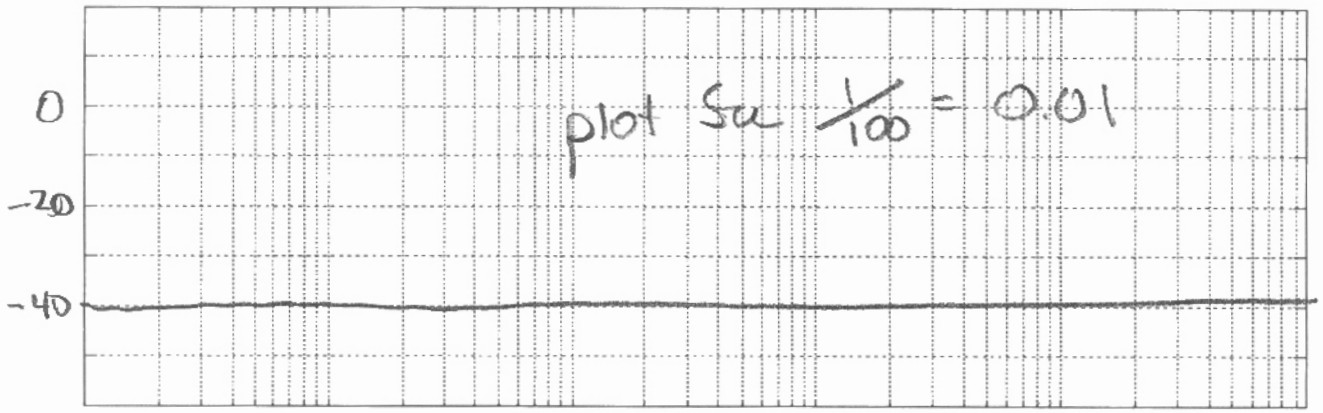
Problem 1

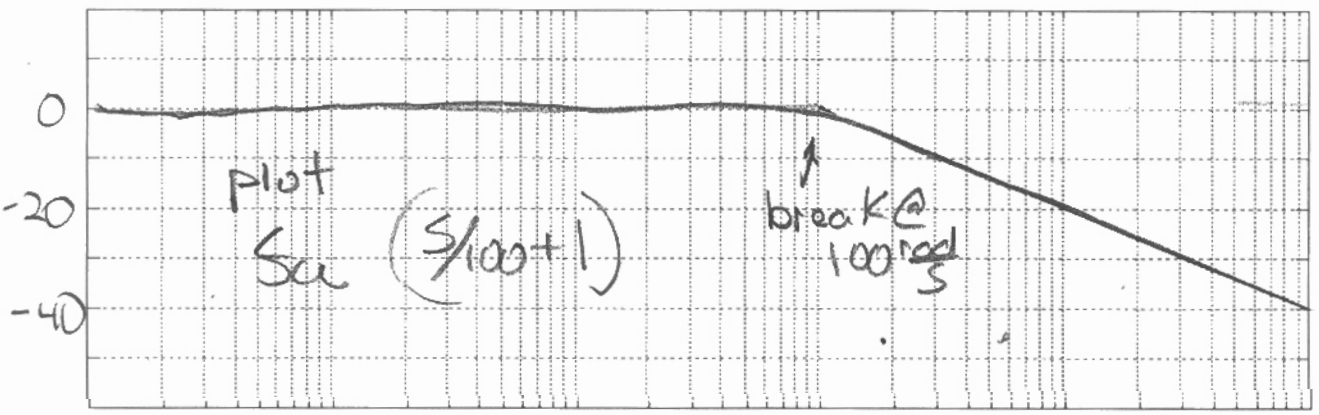
(b) $P(s) = \frac{s}{s+100}$

$$\frac{s}{100(s/100+1)} = \frac{1}{100} \cdot \frac{s}{1} \cdot \frac{1}{s/100+1} = (0.01)(s) \cdot \frac{1}{s/100+1}$$

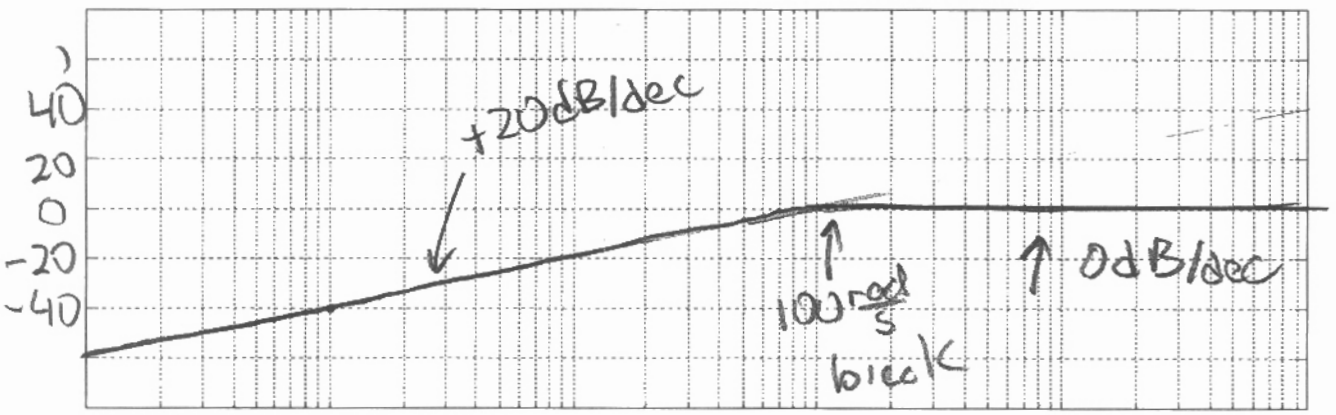
$20\log_{10}(0.01) \Rightarrow -40\text{dB}$


40
20
0
-20
0.1
1
10
100
10000
+20dB/dec
plot $\frac{5\omega}{s}$


0
-20
-40
plot $5\omega \frac{1}{100} = 0.01$


0
-20
-40
plot $5\omega \left(\frac{s}{100}+1\right)$
break @ $100 \frac{rad}{s}$

Add them


40
20
0
-20
-40
+20dB/dec
$100 \frac{rad}{s}$
break
0dB/dec

Problem 2

(a) 4 Elements

$$C_1\dot{P}_1 = W_1 - W_{12} \quad [1]$$

$$P_1 - P_2 = I\dot{W}_{12} \quad [2]$$

$$C_2 P_2 = W_{12} - W_2 \quad [3]$$

$$P_2 - P_a = RW_2 \rightarrow P_2 = RW_2 + P_a \quad [4]$$

} 20 pts.

(b) Combine.

1) [4] $\rightarrow \dot{P}_2 = R\dot{W}_2$ (a)

2) (a) into [3] $\rightarrow C_2R\dot{W}_2 = W_{12} - W_2 \rightarrow W_{12} = C_2R\dot{W}_2 + W_2$ (b)

$\rightarrow \dot{W}_{12} = C_2R\ddot{W}_2 + \dot{W}_2$ (c)

3) (c) into [2]:

$$P_1 - P_2 = IC_2R\ddot{W}_2 + I\dot{W}_2 \quad (d)$$

4) [4] into (d) $\rightarrow$ ($P_2 = RW_2 + P_a$)

$$P_1 = IC_2R\ddot{W}_2 + I\dot{W}_2 + P_2$$

$$= IC_2R\ddot{W}_2 + I\dot{W}_2 + RW_2 + P_a$$

$$\rightarrow \dot{P}_1 = IC_2R\dddot{W}_2 + I\ddot{W}_2 + R\dot{W}_2 \quad (e)$$

5) (e) + (b) into [1]

$$\rightarrow C_1IC_2R\dddot{W}_2 + C_1I\ddot{W}_2 + C_1R\dot{W}_2 = W_1 - C_2R\dot{W}_2 - W_2$$

$$\rightarrow C_1C_2RI\dddot{W}_2 + C_1I\ddot{W}_2 + R(C_1+C_2)\dot{W}_2 + W_2 = W_1$$

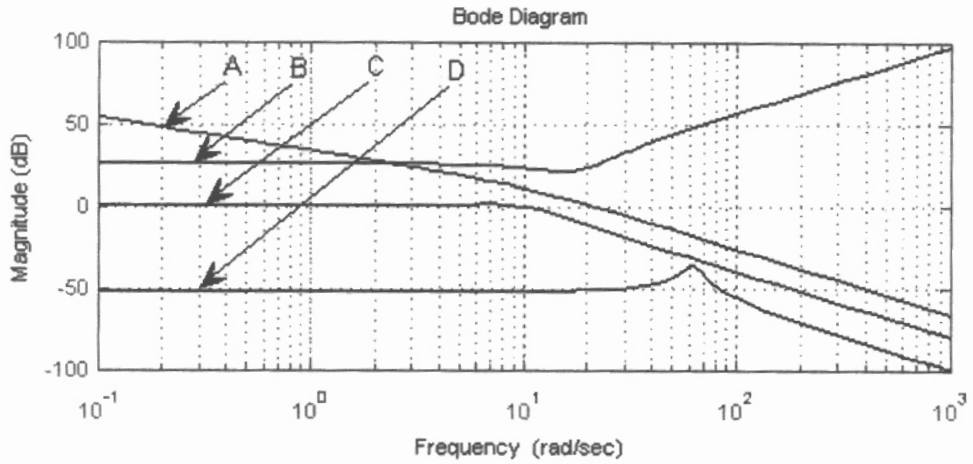
$$\rightarrow \boxed{\frac{W_2(s)}{W_1(s)} = \frac{1}{C_1C_2RIs^3 + C_1Is^2 + R(C_1+C_2)s + 1}}$$

PROBLEM 3: (30%)

PROBLEM MC-1) Which of the following Bode Magnitude plots correctly corresponds to the following transfer function?

Circle one: **A)** **B)** **C)** **D)**

$$G(s) = \frac{100}{s^2 + 10s + 100}$$



PROBLEM MC-2) Consider the root locus plot for a second order system shown below. What is the natural frequency for the pole location at A shown below.

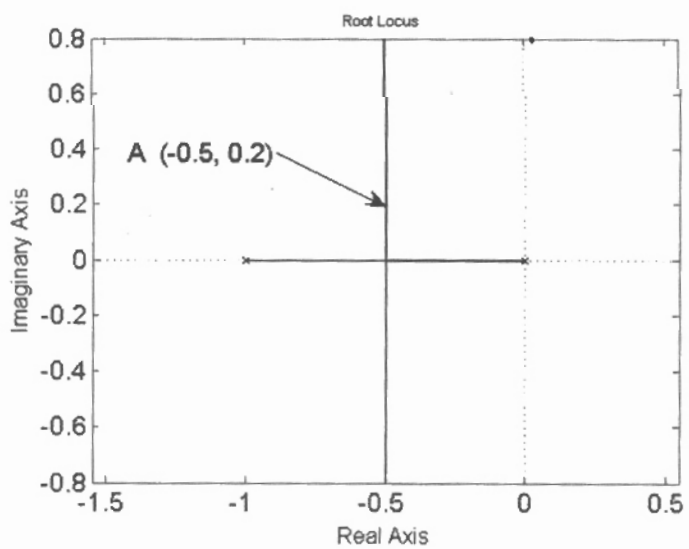
Circle one:

a) ω_n = 0.3713 **b) ω_n = 0.5385** **c) ω_n = 0.9284** **d) ω_n = 1.3713** **e) none of the others**

PROBLEM MC-3) Consider the root locus plot for a second order system shown below. What is the damping ratio for the system corresponding to the pole location at A shown below.

Circle one:

a) ζ = 0.3713 **b) ζ = 0.5385** **c) ζ = 0.9284** **d) ζ = 1.3713** **e) none of the others**



PROBLEM MC-4) The poles for a system with the characteristic equation $s^2 + s + 25$:

$\frac{-1 \pm \sqrt{1^2 - 100}}{2}$

Circle one:

a) have a complex part **b) are unstable** **c) are real**

d) more than one of the above **e) none of the above**

PROBLEM MC-5) The phenomena/components in a hydraulic system that can be model with a capacitive law of the form ($C\dot{P}$ = 'flow in' minus 'flow out') include:

Circle one:

a) tank with variable fluid height **b) fluid compressibility**

c) flow across of valve **d) more than one of the above** **e) none of the above**

PROBLEM MC-6) Possible units for a DC motor torque constant are

Circle one:

a) N/A **b) (kg m)/A** **c) V/(m/s)** **d) more than one of the above**

e) none of the above

E is the answer

PROBLEM MC-7) Match the system poles in the complex plane plot with the corresponding step response (you can just draw arrows from a particular pole to a particular step response)