ME 375: Midterm 2

November 6, 2008

Instructions

- This is a closed book examination.
- Please write your name on each page.
- You are allowed one (1), double-sided, hand-written crib sheet.
- You must show all of your work to receive credit.

Problem 1	____________________
Problem 2	____________________
Problem 3	____________________
Total	____________________

Student: ____

Professor: ____

Problem 1. (30%) You are designing a new automated parts sorter which contains a simple mass-spring-dashpot system constrained by the following performance metrics:
(i) Maximum percent overshoot: %OS = 10%
(ii) Allowable settling time: $t_s = 0.25$ s
If the effective mass of the parts sorter is $m = 0.1$ kg, determine:
(a) The values of c and k required to meet the performance metrics.
(b) The transfer function which governs the system.
(c) The steady-state response of the system when actuated by a unit impulse input force $\delta(t)$.

(a) $\% OS = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100\%$

$.1 = e^{-\pi\xi/\sqrt{1-\xi^2}} \Rightarrow \xi = .591$

$t_s = 4\tau = \frac{4}{\xi\omega_n} \Rightarrow .25 = \frac{4}{(.591)\omega_n}$

$\Rightarrow \omega_n = 27.07 \text{ rad/s}$

$\omega_n = \sqrt{\frac{k}{m}} \Rightarrow k = m\omega_n^2 = (.1 \text{ kg})(27.07 \text{ rad/s})^2$
$= 73.28 \frac{N}{m}$

$c = 2\xi\omega_n m \Rightarrow c = 2(.591)(27.07 \text{ rad/s})$
$\times(.1 \text{ kg}) = 3.2 \text{ kg/s}$

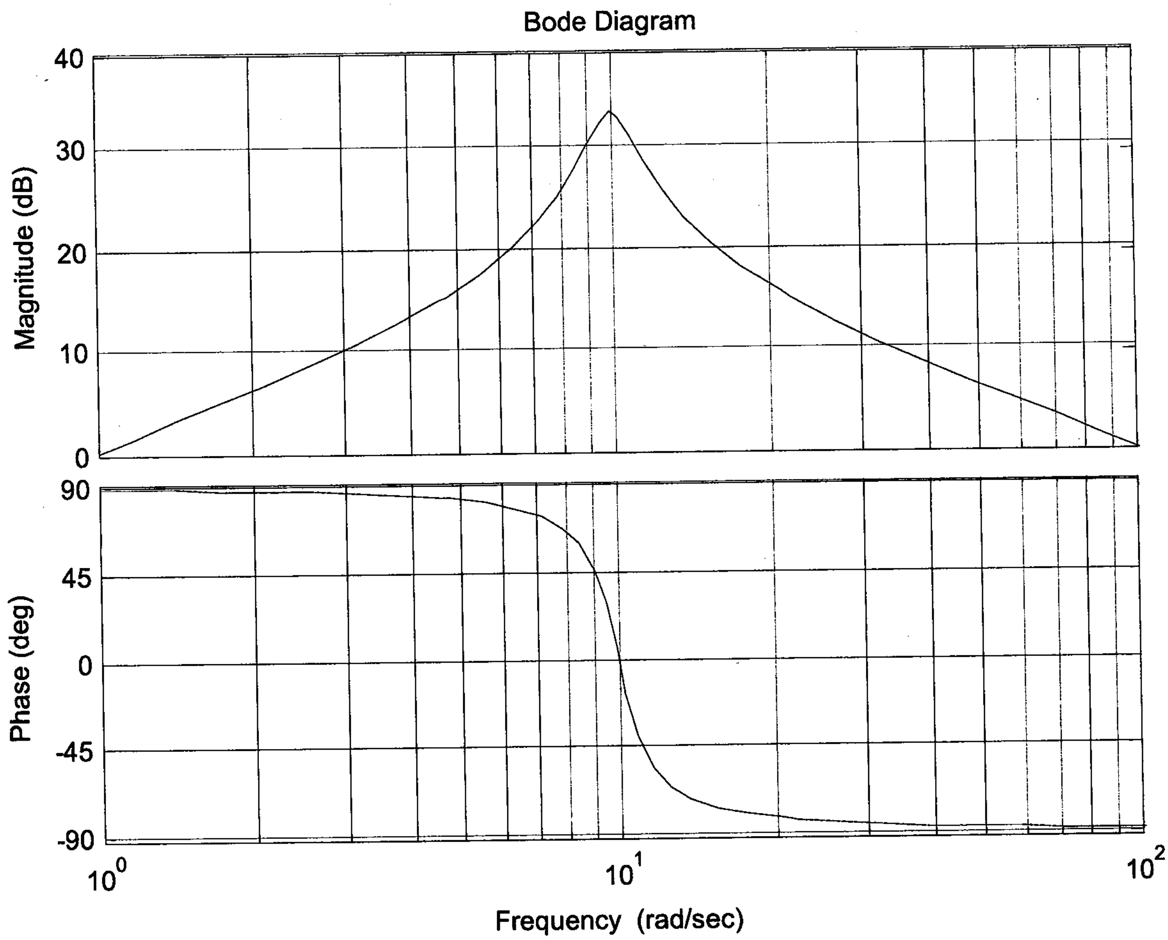
(b) $G(s) = \frac{1}{.1s^2 + 3.2s + 73.28}$

(c) $x_{ss} = 0$

This page is for extra work related to Problem 1.

Problem 2. (30%) Given the Bode diagram shown below:
(a) Find the steady-state response of the system under the force input $u(t) = \sin(3t) + 5\sin(9t + 20°)$.
(b) Find the steady-state response of the system under the force input $u(t) = 67$.
(c) Find the constants A and B which appear in the transfer function governing the system, which is given by

$$G(s) = \frac{Bs}{s^2 + As + B}.$$



Problem 2

This page is for extra work related to Problem 2.

(a) $x_{ss} = 10^{10/20}\sin(3t+85^\circ) + 5\times 10^{30/20}\sin(9t+20^\circ+45^\circ)$

$= 3.16\sin(3t+85^\circ) + 158.11\sin(9t+65^\circ)$

(b) $x_{ss} = 0 \quad (\omega = 0 \rightarrow -\infty\,dB)$

(c) $B = \omega_n^2 = 100$

$$|G(j\omega)| = \frac{B\omega}{\sqrt{(B-\omega^2)^2 + (A\omega)^2}}$$

At $\omega = \omega_n$

$$|G(j\omega_n)| = \frac{100}{A}$$

$$10^{34/20} = \frac{100}{A} \Rightarrow A \simeq 2$$

Problem 3. **(40%)** Mechanical generators can be use to scavenge the kinetic energy associated with an applied force and store the energy in a capacitive element. A primitive linear generator can be modeled as a mass-dashpot system electromechanically coupled to an RC circuit. Assuming that the equivalent force generated in the system F_{gen} is related to the armature current i_A through the relationship

$$F_{gen} = K_t i_A$$

and the generated electromotive force E_{emf} is related to the plunger velocity $\dot{x}$ through the relationship

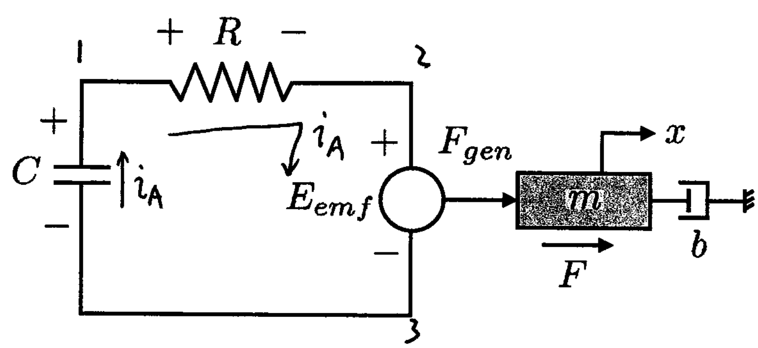
$$E_{emf} = K_b \dot{x}$$

find:

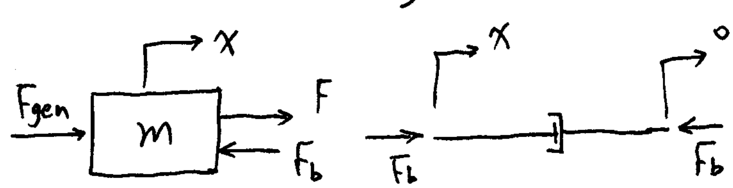
(a) The transfer function relating the voltage across the capacitor $E_c(s)$ and the applied external force $F(s)$.

(b) An input/output model relating the voltage across the capacitor $e_c(t)$ and the applied external force $F(t)$.

(c) The steady-state voltage across the capacitor if a unit step force of magnitude F_0 is applied to the system.



(a). FBD:



Mechanical Subsystem,

$$\left.\begin{aligned} F_b &= b(\dot{x} - 0) = b\dot{x} \\ m\ddot{x} &= F + F_{gen} - F_b \end{aligned}\right\} \Rightarrow \quad m\ddot{x} + b\dot{x} = F + F_{gen} \qquad (1)$$

Electrical Subsystem:

$$i_A = -C\dot{e}_{13}, \quad e_{13} = e_c(t) \qquad (2)$$

$$e_{12} = i_A R \qquad (3)$$

This page is for extra work related to Problem 3.

$$e_{23} = E_{emf} \qquad (4)$$

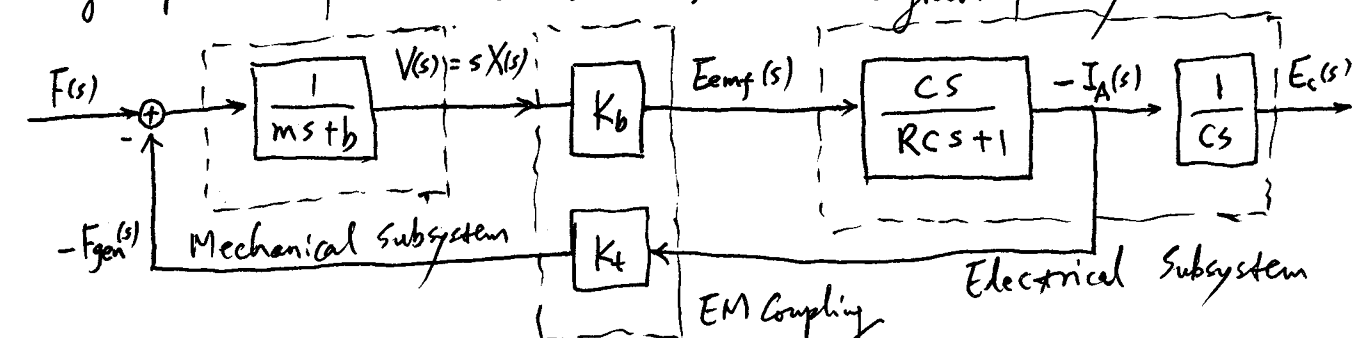
$$e_{13} = e_{12} + e_{23} \qquad (5)$$

EM Coupling:

$$F_{gen} = K_t i_A \qquad (6)$$

$$E_{emf} = K_b \dot{x} \qquad (7)$$

Taking laplace transform, we can obtain the block diagram of the system as:



$$\Rightarrow \boxed{\frac{E_c(s)}{F(s)} = \frac{K_b}{mRCs^2 + (bRC + m + K_t K_b C)s + b}} = G(s) \qquad (8)$$

(b). From (8):

$$mRC\,\ddot{e}_c + (bRC + m + K_t K_b C)\,\dot{e}_c + b\,e_c = K_b F(t) \qquad (9)$$

or

$$\ddot{e}_c + \underbrace{\left(\frac{1}{RC} + \frac{b}{m} + \frac{K_t K_b}{mR}\right)}_{2\zeta\omega_n}\dot{e}_c + \underbrace{\frac{b}{mRC}}_{\omega_n^2}\,e_c = \underbrace{\frac{K_b}{mRC}}_{K\omega_n^2}\,F(t)$$

(c). As (8) is stable, for a step force of magnitude F_0, the output $e_c(t)$ will converge to a constant value of:

$$e_c(t) \longrightarrow \boxed{e_{ss} = G(0)\,F_0 = \frac{K_b}{b}F_0} \qquad (10)$$