ME 375 EXAM #2
Wednesday, Nov. 18, 2009

Division Meckl 10:30 / Deng 2:30 (circle one) **Name** Solution

Instructions

(1) This is a closed book examination, but you are allowed two 8.5×11 crib sheets.

(2) You have one hour to work all three problems on the exam.

(3) Use the solution procedure we have discussed: what are you given, what are you asked to find, what are your assumptions, what is your solution, does your solution make sense. You must show all of your work to receive <u>any</u> credit.

(4) You must write neatly and should use a logical format to solve the problems. You are encouraged to really "think" about the problems before you start to solve them. Please write your name in the top right-hand corner of each page.

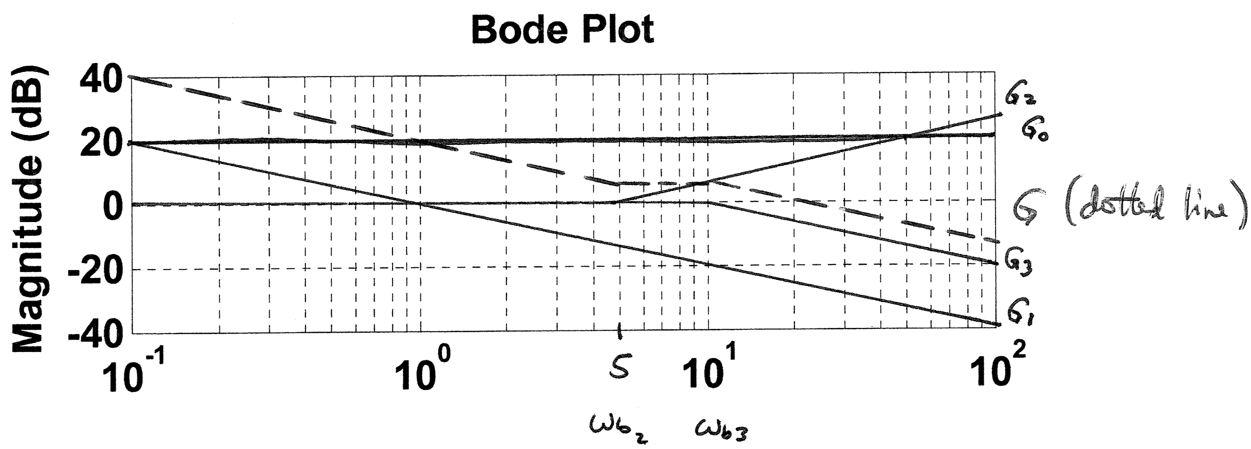
(5) A table of Laplace transform pairs and properties of Laplace transforms is attached at the end of this exam set.

Problem No. 1 (40%)	____________
Problem No. 2 (30%)	____________
Problem No. 3 (30%)	____________
TOTAL (*/100%)**	____________

PROBLEM 1: (40%)

(a) Draw the Bode magnitude plot of the following transfer function using straight-line approximate asymptotes; clearly show the steps.

$$G(s) = \frac{10(\frac{s}{5}+1)}{s(\frac{s}{10}+1)} = \frac{Y(s)}{F(s)}$$



$G_0(s) = 10$

$G_1(s) = \frac{1}{s}$

$G_2(s) = \frac{s}{5} + 1 \Rightarrow \omega_{b2} = 5\ r/s$

$G_3(s) = \frac{1}{\frac{s}{10}+1} \Rightarrow \omega_{b3} = 10\ r/s$

(b) For the transfer function in part (a), what is the steady state response of the system subject to input $f(t) = \sin(2t) + 5\sin(8t)$?

$$y(t) = |G(j2)| \sin(2t + \angle G(j2)) + 5|G(j8)| \sin(8t + \angle G(j8))$$

$$G(j\omega) = \frac{10\left(\frac{j\omega}{5}+1\right)}{j\omega\left(\frac{j\omega}{10}+1\right)} = \frac{20(j\omega+5)}{j\omega(j\omega+10)}$$

$$|G(j\omega)| = \frac{20\sqrt{\omega^2+5^2}}{\omega\sqrt{\omega^2+10^2}}$$

$$\angle G(j\omega) = \tan^{-1}\frac{\omega}{5} - 90° - \tan^{-1}\frac{\omega}{10}$$

$$|G(j2)| = 5.28, \quad \angle G(j2) = -79.5°$$

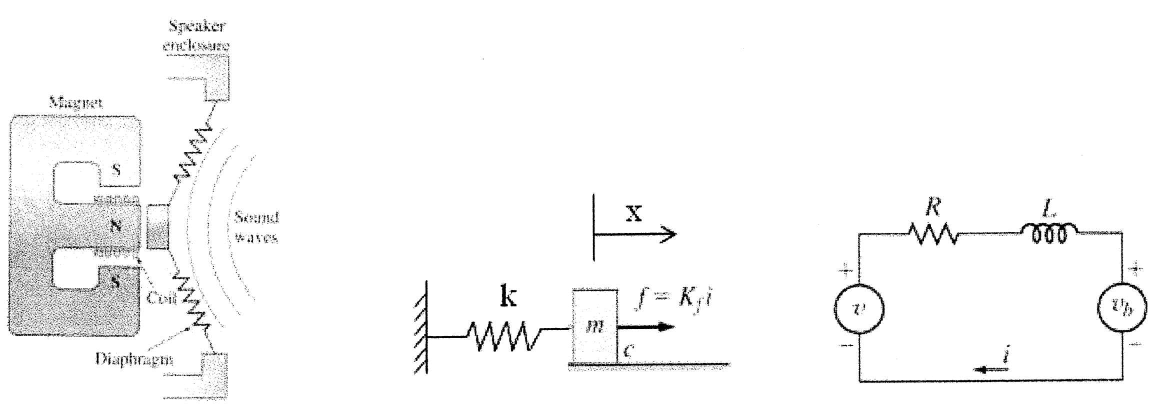
$$|G(j8)| = 1.84, \quad \angle G(j8) = -70.7°$$

$$\boxed{y(t) = 5.28\sin(2t - 79.5°) + 9.20\sin(8t - 70.7°)}$$

PROBLEM 2: (30%)

Figure 3 shows a speaker and its electromechanical model. The operation of the speaker is illustrated in Figure 3(a). A stereo amplifier produces a current in a coil that is attached to a diaphragm in the cone. This causes the coil and diaphragm to move relative to the permanent magnet. The motion of the diaphragm produces air pressure waves, which is sound. Figure 3(b) shows a simplified model of the mechanical subsystem. The mass m represents the combined mass of the diaphragm and the coil. The spring constant k and damping constant c depend on the material properties of the diaphragm. The force f is the magnetic force, which is related to the coil current i by $f = K_f i$, where K_f is the force constant. Figure 3(c) shows the electrical subsystem. The coil's inductance and resistance are L and R. The coil experiences a back emf voltage because it is a current conductor moving in a magnetic field. This back emf is given by $v_b = K_b \dot{x}$. The voltage v is the input signal from the amplifier.

You are asked to develop a model of the speaker and obtain the transfer function relating the diaphragm displacement x to the applied voltage v.



(a) (b) (c)

Figure 3. Speaker (a) and its mechanical (b) and electrical (c) subsystem models.

(a) Write down the governing mechanical equations.

$$m\ddot{x} = f - kx - c\dot{x}$$

(b) Write down the governing electrical equations.

$$v = Ri + L\frac{di}{dt} + v_b$$

(c) Write down the interconnection equations between electrical and mechanical subsystems.

$$f = K_f i$$

$$v_b = K_b \dot{x}$$

(d) Generate the transfer function between diaphragm displacement x and applied voltage v.

$$m\ddot{x} + c\dot{x} + kx = f = K_f i \qquad (1)$$

$$L\frac{di}{dt} + Ri + K_b\dot{x} = v \qquad (2)$$

eliminate i: from (1): $i = \frac{1}{K_f}(m\ddot{x} + c\dot{x} + kx)$ (3)

now plug (3) into (2):

$$\frac{L}{K_f}(m\dddot{x} + c\ddot{x} + k\dot{x}) + \frac{R}{K_f}(m\ddot{x} + c\dot{x} + kx) + K_b\dot{x} = v$$

$$Lm\dddot{x} + (Lc + Rm)\ddot{x} + (Lk + Rc + K_f K_b)\dot{x} + Rkx = K_f v$$

$$\boxed{\frac{X(s)}{V(s)} = \frac{K_f}{Lms^3 + (Lc+Rm)s^2 + (Lk+Rc+K_fK_b)s + Rk}}$$

PROBLEM 3: (30%)

A door closing mechanism, which is used to automatically close an open door, can be represented as a simple second-order system as follows:

$$J\ddot{\theta} + B\dot{\theta} + K\theta = T$$

where J is the door inertia about the hinges, B is the damping coefficient in the closing damper, and K is the spring stiffness in the closing spring. θ is the angle of door opening, and T is the applied torque when the door is pushed open.

(a) Write down the transfer function that relates the torque T to the angle θ.

$$\frac{\theta(s)}{T(s)} = \frac{1}{Js^2 + Bs + K}$$

(b) Write the transfer function in part (a) as a general second-order transfer function in terms of ω_n, ζ, and K_s. Then generate expressions for ω_n, ζ, and K_s as functions of J, B, and K.

$$\frac{\theta(s)}{T(s)} = \frac{K_s \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{\theta(s)}{T(s)} = \frac{\frac{1}{J}}{s^2 + \frac{B}{J}s + \frac{K}{J}}$$

$$\omega_n^2 = \frac{K}{J} \Rightarrow \boxed{\omega_n = \sqrt{\frac{K}{J}}}$$

$$K_s \omega_n^2 = \frac{1}{J} \Rightarrow \boxed{K_s = \frac{1}{K}}$$

$$2\zeta\omega_n = \frac{B}{J} \Rightarrow \boxed{\zeta = \frac{\frac{B}{J}}{2\sqrt{\frac{K}{J}}} = \frac{B}{2\sqrt{JK}}}$$

(c) Given the following desired step response characteristics:

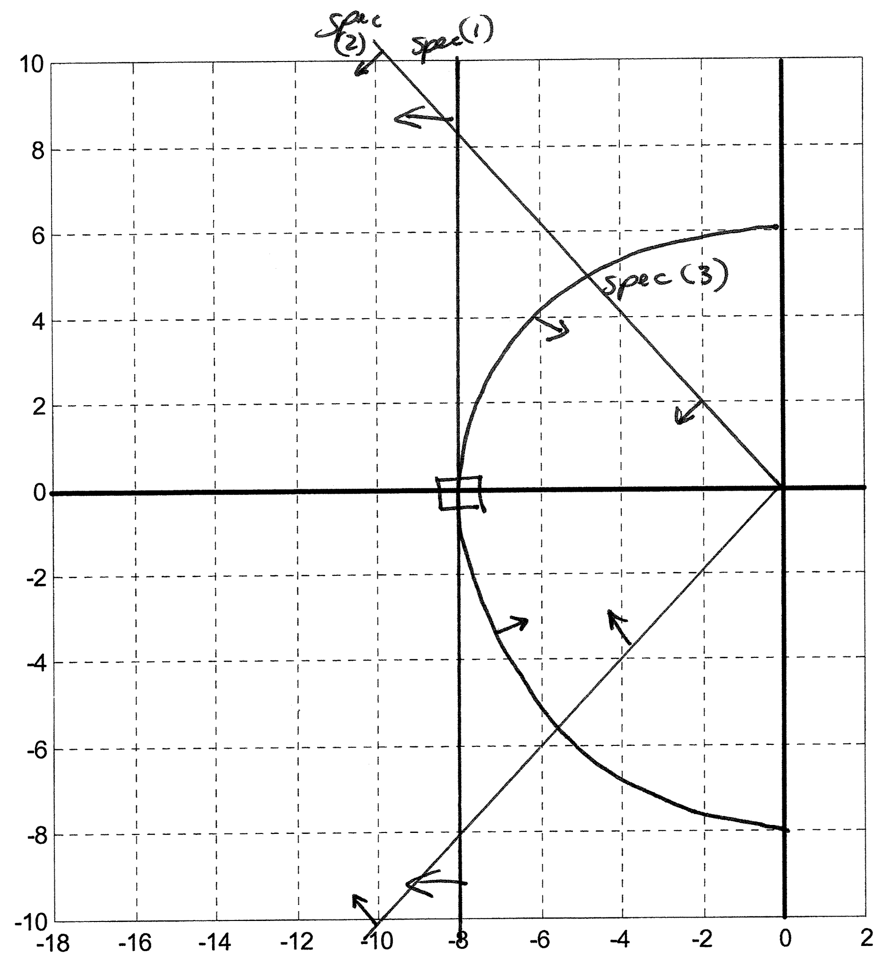
(1) 2% settling time ≤ 0.5 sec $\Rightarrow \frac{4}{\zeta\omega_n} \le 0.5 \Rightarrow \zeta\omega_n \ge 8$

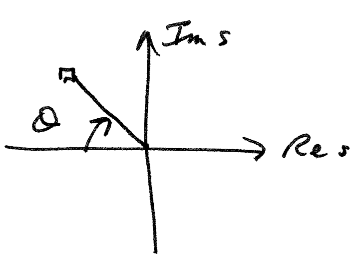
(2) %OS ≤ 5% $\Rightarrow \zeta \ge 0.69$

(3) $\omega_n \le 8$ rad/sec

Sketch the individual regions in the s-plane where each of these specifications is satisfied. Then indicate with small squares the location of the roots that satisfy all three of these specifications.

Only roots that satisfy all specs are at $s = -8$





note: $\cos\theta = \zeta$, for $\zeta = 0.69$: $\theta = 46.3^\circ$

$$\%OS = 100\, e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 5$$

$$e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 0.05$$

$$-\pi\zeta/\sqrt{1-\zeta^2} = \ln 0.05 = -3.0$$

$$\pi^2\zeta^2 = 9(1-\zeta^2)$$

$$(\pi^2+9)\zeta^2 = 9 \Rightarrow \zeta^2 = 0.477$$

$$\zeta = 0.69$$

(d) Using the desired root locations chosen in part (c), determine the values of *J*, *B*, and *K* that will achieve these desired roots. Assume that an input step torque of 20 Newton-meters leads to a steady-state door opening angle of 2 radians.

(Note: if you didn't get part (c), simply pick an arbitrary set of roots and show that you can complete the rest of this problem.)

$$K_s = \frac{2}{20} = \frac{1}{10} = 0.1 = \frac{1}{K} \Rightarrow \boxed{K = 10 \text{ Nm/rad}}$$

$$(s+8)^2 = s^2 + \frac{B}{J}s + \frac{K}{J}$$

$$(s^2 + 16s + 64) = s^2 + \frac{B}{J}s + \frac{K}{J}$$

$$\frac{K}{J} = 64 \Rightarrow \frac{10}{J} = 64 \Rightarrow \boxed{J = 0.16 \text{ kg m}^2}$$

$$\frac{B}{J} = 16 \Rightarrow \boxed{B = 16J = 2.5 \text{ Nm (rad/sec)}}$$