ME 375 EXAM #2
Tuesday, October 30, 2012

Division Hall / Deng (circle one) **Name** SOLUTION

Instructions

(1) This is a closed book examination, but you are allowed two 8.5×11 crib sheets.

(2) You have one hour to work all three problems on the exam.

(3) Use the solution procedure we have discussed: what are you given, what are you asked to find, what are your assumptions, what is your solution, does your solution make sense. You must show all of your work to receive <u>any</u> credit.

(4) You must write neatly and should use a logical format to solve the problems. You are encouraged to really <u>think</u> about the problems before you start to solve them. Please write your name in the top right-hand corner of each page.

(5) A table of Laplace transform pairs and properties of Laplace transforms is located on the last page of the exam.

Problem No. 1 (30%) ____________________

Problem No. 2 (30%) ____________________

Problem No. 3 (40%) ____________________

TOTAL (*/100%)** ____________________

PROBLEM 1 (30%)

(a) For a second order system with a damping ratio of 0.2 and a natural frequency of 8 rad/sec, determine the 2% settling time and the percent overshoot for the step response.

$$\xi = 0.2$$

$$\omega_n = 8 \text{ rad/sec}$$

$$t_{s,2\%} = \frac{4}{\xi\omega_n} = \frac{4}{0.2(8)} = \boxed{2.5 \text{ sec.} = t_s}$$

$$\%OS = 100 e^{-\pi\xi / \sqrt{1-\xi^2}}$$

$$= 100 e^{-0.2\pi / \sqrt{1-(0.2)^2}}$$

$$\boxed{\%O.S. = 52.7\%}$$

PROBLEM 1 (Continued)

(b) For the following transfer function:

$$\frac{Y(s)}{F(s)} = G(s) = \frac{64}{s^2 + 3.2s + 64}$$

determine the steady state output y_{ss} to an input of:

$$f(t) = 5 + 10\cos(5t) - 20\sin(15t).$$

(Reduce as far as possible but leave as a function of t.)

$$y_{ss}(t) = 5|G(0)| + 10|G(j5)|\cos(5t + \angle G(j5)) - 20|G(j15)|\sin(15t + \angle G(j15))$$

$$G(j\omega) = \frac{64}{(j\omega)^2 + 3.2j\omega + 64} = \frac{64}{(64-\omega^2) + 3.2\omega j}$$

$$|G(0)| = \frac{64}{64} = 1$$

$$|G(5j)| = \frac{64}{\sqrt{(64-5^2)^2 + ((3.2)(5))^2}} = 1.52$$

$$|G(15j)| = \frac{64}{\sqrt{(64-15^2)^2 + (3.2(15))^2}} = 0.381$$

$$\angle G(5j) = \angle 64 - \angle[(64-5^2) + 3.2(5)j]$$

$$= -\tan^{-1}\left(\frac{3.2(5)}{64-5^2}\right) = -0.389 \text{ rad}$$

$$\angle G(15j) = -\tan^{-1}\left(\frac{3.2(15)}{64-15^2}\right) = -2.85 \text{ rad}$$

$$y_{ss}(t) = 5 + 10(1.52)\cos(5t + -0.389) - 20(0.381)\sin(15t - 2.85)$$

$$\therefore \boxed{y_{ss}(t) = 5 + 15.2\cos(5t - 0.389) - 7.62\sin(15t - 2.85)}$$

PROBLEM 2 (30%)

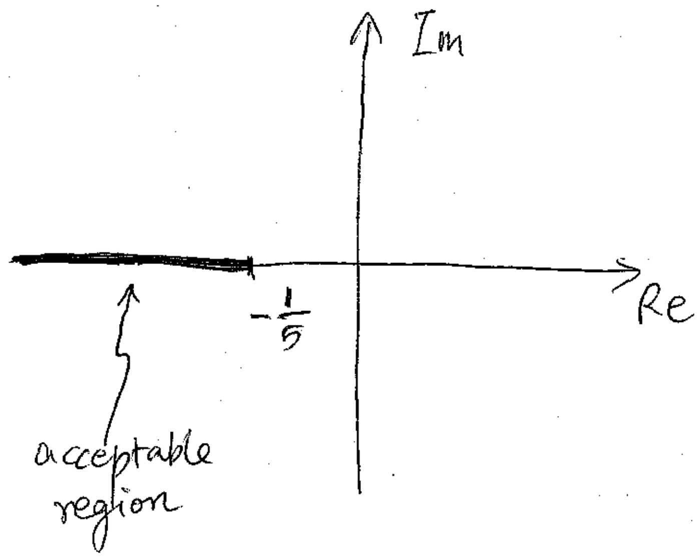
(a) A first order system is given a step input. Clearly and accurately make a drawing of the complex plane that shows the pole locations corresponding to a time constant less than 5 seconds.

$\tau < 5$

$\tau = \frac{1}{+P_K}$

$\Rightarrow P_K = 1/5$ at $\tau = 5$

(For general system with a characteristic eq of $s + P_K = 0$)



(b) A second order system is given a step input. Clearly and accurately make a drawing of the complex plane that shows the pole locations corresponding to a system %OS less than 5% and 2% settling time between 2 and 6 seconds.

$\%O.S. < 5\%$

At $\%O.S. = 5\%$

$100e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 5$

$\Rightarrow \zeta = 0.69$

$\cos\phi = \zeta$

$\Rightarrow \phi = 46.4^\circ$

$2 < t_{s,2\%} < 6$

At upper bound:

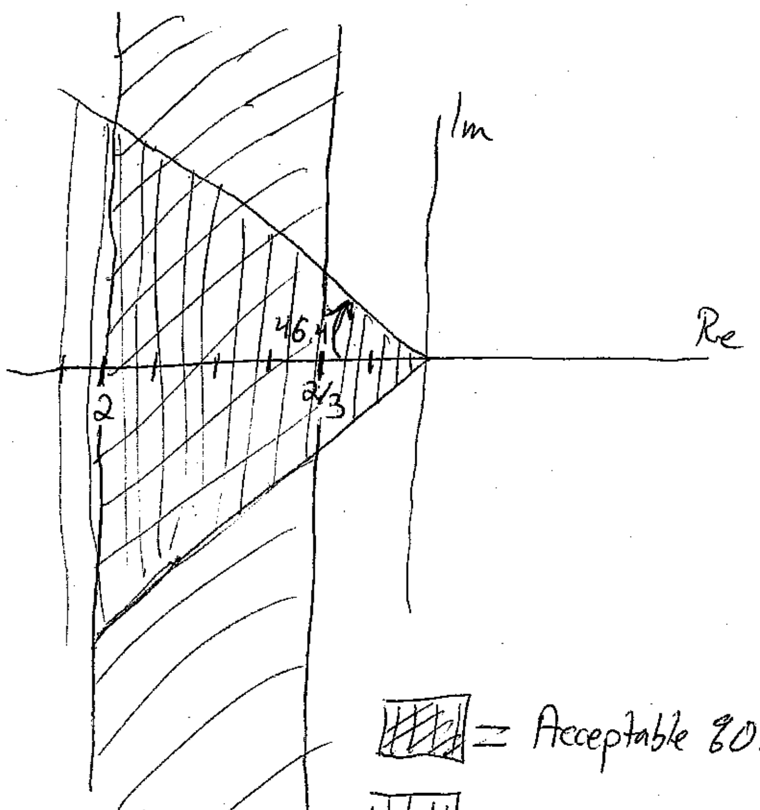
$t_s = \frac{4}{\zeta\omega_n} = 6$

$\zeta\omega_n = 2/3$

At lower bound

$t_s = \frac{4}{\zeta\omega_n} =$

$\zeta\omega_n = 2$



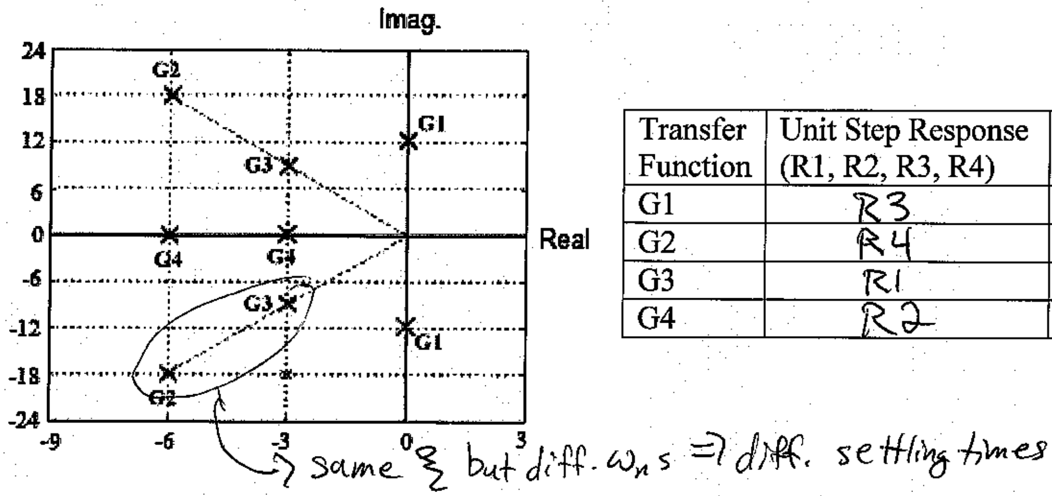
= Acceptable %O.S. & t_s

= Acceptable %O.S. Region

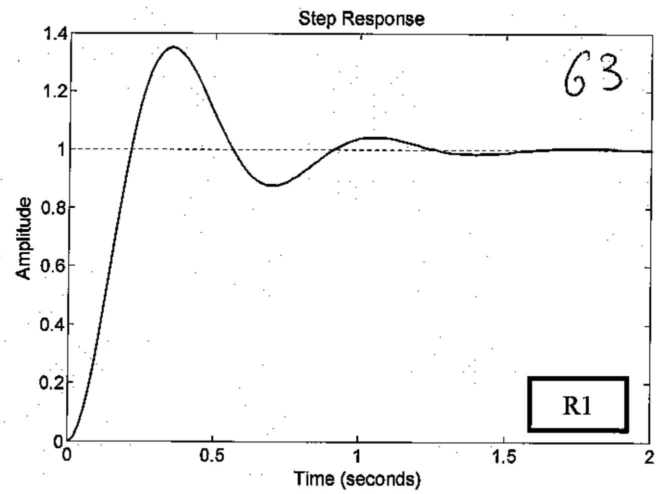
= Acceptable Settling Time Region

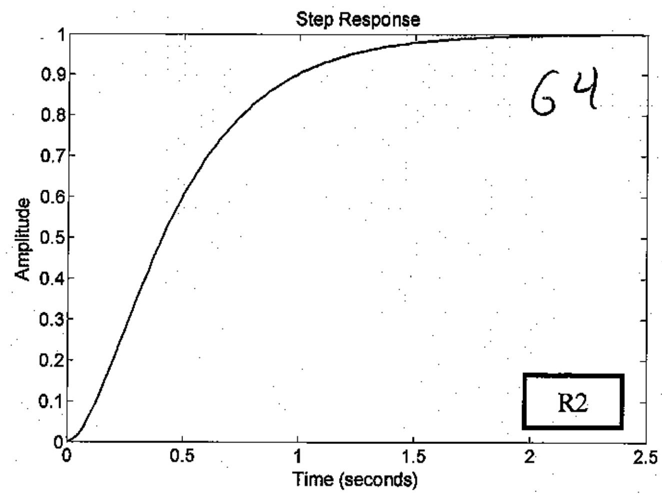
PROBLEM 2 (Continued)

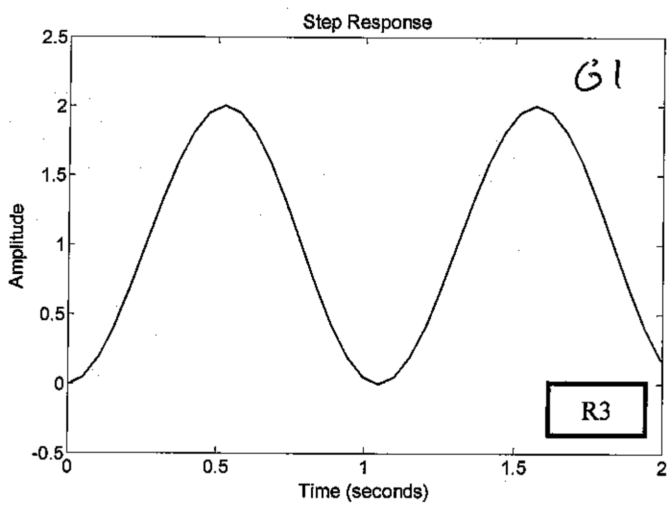
(c) The pole positions of four 2nd order systems are shown below. The unit step responses and the Bode Plots of these systems are also shown. Match up the pole position (labeled G1, G2, G3, G4) with the correct unit step response (labeled R1, R2, R3, R4) as well as the correct Bode Plots (labeled A, B, C, D).

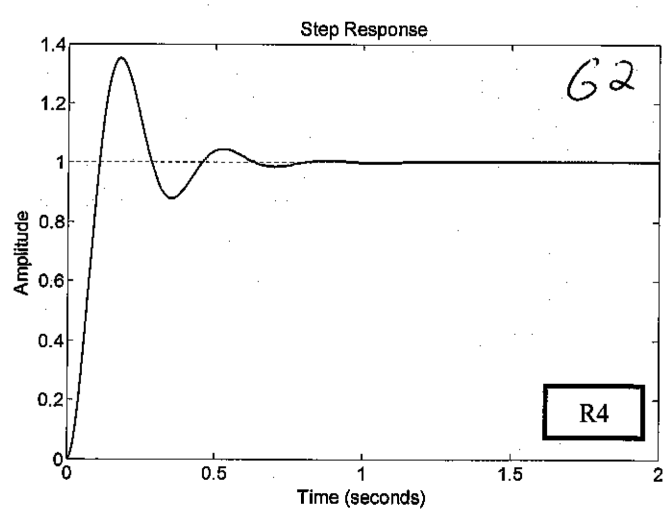


Transfer Function	Unit Step Response (R1, R2, R3, R4)	Bode Plot (A, B, C, D)
G1	R3	A
G2	R4	C
G3	R1	D
G4	R2	B



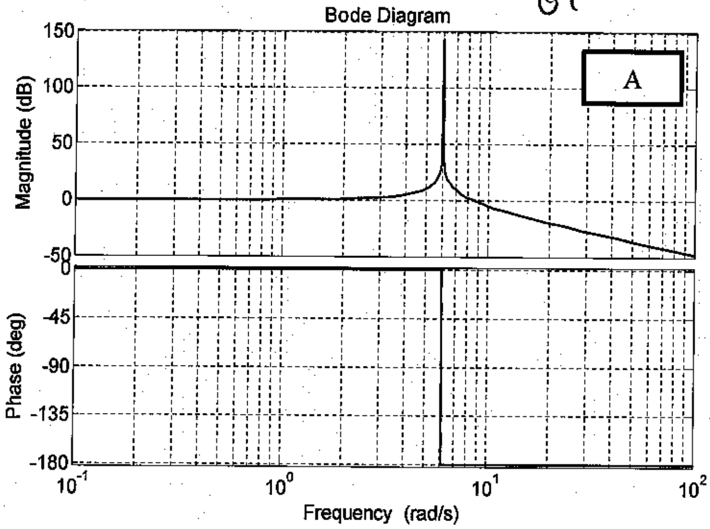




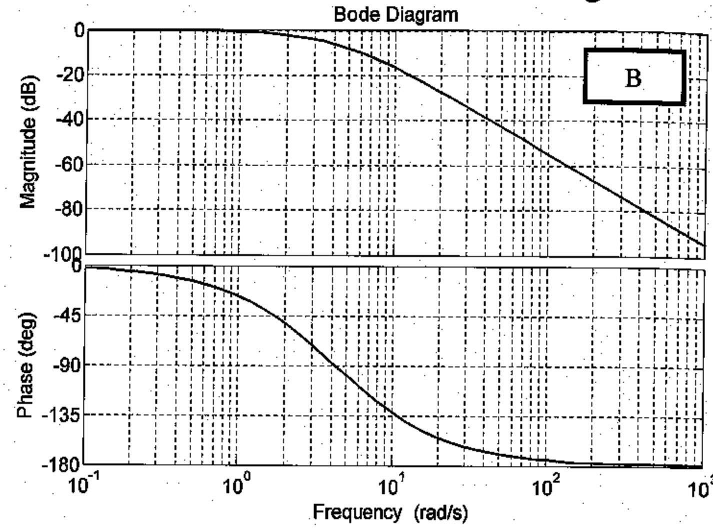


PROBLEM 2 (Continued)

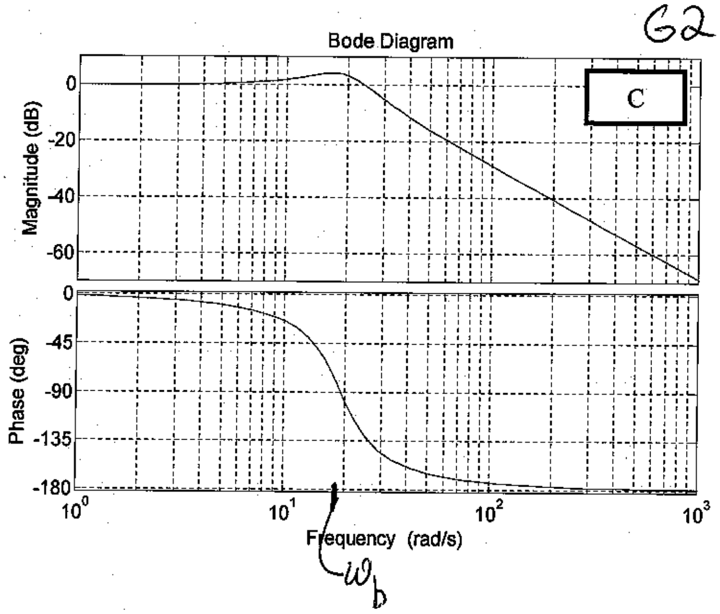
G1



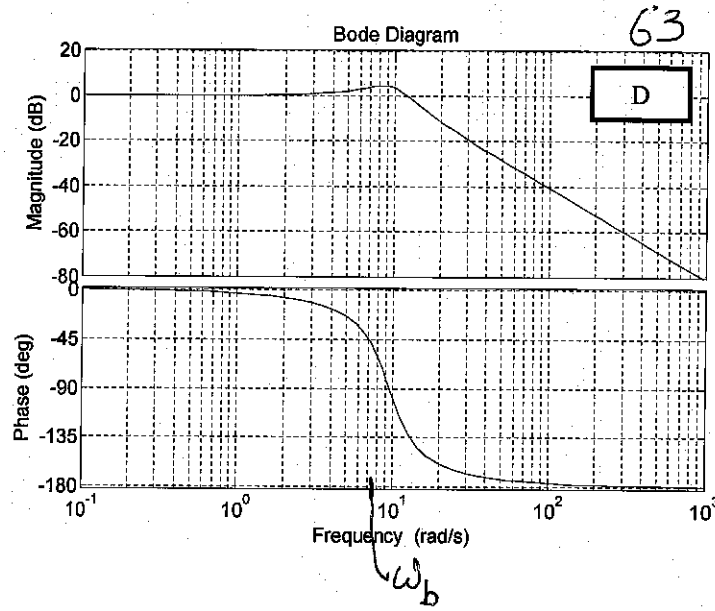
G4



G2



G3



ω_n for G_3 $<$ ω_n for G_2

So should see difference in break frequencies

PROBLEM 3 (40%)

Given the transfer function $G(s) = \dfrac{40(s+1)}{s(s^2+12s+400)}$, draw the Bode straight line approximation using the following steps:

(a) Find the poles, zeros, and static gain of the transfer function $G(s)$.

Poles: $p_1 = 0$

$$p_{2,3} = \frac{-12 \pm \sqrt{144 - 4(400)}}{2}$$

$$= -6 \pm j19.08$$

Zeros: $z_1 = -1$

$$N(s) = (s+1) = 0$$

Static Gain: $K_S = G(0) = \dfrac{40}{0}$

$$G(0) \to \infty$$

(b) Write all first order terms in the form ($\tau s + 1$) (except s or $1/s$) and all second order complex terms in the form $\dfrac{1}{\frac{s^2}{\omega_n^2}+\frac{2\zeta}{\omega_n}s+1}$ or $\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$.

$$\frac{40(s+1)}{s(s^2+12s+400)} \frac{\left(\frac{1}{400}\right)}{\left(\frac{1}{400}\right)} = \frac{0.1(s+1)}{s\left(\frac{s^2}{20^2}+\frac{12}{20^2}s+1\right)}$$

Standard Form:

$$\boxed{G(s) = \frac{0.1(s+1)}{s\left(\frac{s^2}{20^2}+\frac{0.6}{20}s+1\right)}}$$

$$\frac{12}{20^2} = \frac{2\xi}{\omega_n} = \frac{2\xi}{20}$$

$$\Rightarrow \xi = 0.3$$

PROBLEM 3 (Continued)

(c) Provide the break frequencies for all of the terms in transfer function $G(s)$.

$$G(s) = \frac{K\,N(s)}{D_1(s)\,D_2(s)}$$

For $N(s) = s+1$

$$\tau = 1 \,,\quad \omega_{b_1} = \frac{1}{\tau} = 1 \text{ rad/sec}$$

For $D_2(s) = \frac{s^2}{20^2} + \frac{0.6}{20}s + 1$

$$\omega_{b_2} = \omega_n = 20 \text{ rad/sec}$$

(d) Plot the straight line approximation for the magnitude and phase of all elemental transfer functions separately on the graphs on the next page. You do NOT need to include resonant peaks in the magnitude plot for any second order terms.

Label the elemental transfer functions on the graph along with all break frequencies and slopes.

(e) Plot the Bode plot of the total transfer function made up of the elemental transfer functions in part (d).

$\checkmark K = 0.1 \Rightarrow \|K\|_{dB} = 20\log_{10}(0.1) = -20$

$\checkmark N = s+1$

$\checkmark D_1 = s$

$$D_2 = \frac{1}{\frac{s^2}{20^2} + \frac{0.6}{20}s + 1}$$

$$\zeta = 0.3$$

$$\omega_n = 20$$

$$\omega_1 = \left(\frac{1}{5}\right)^{\zeta}\omega_n = 12.3$$

$$\omega_2 = (5)^{\zeta}\omega_n = 32.4$$

PROBLEM 3 (Continued)