Name: Solution

Section Time (please circle): 9:30 3:30

ME 375

EXAM II

Tuesday, 29 October 2013

6:30 pm-7:30 pm

CL 50

Problem 1 ________________/35

Problem 2 ________________/35

Problem 3 ________________/30

Total Score: ______________/100

- Don't forget your name and your section time.
- Exam is 7 pages including this cover page: make sure you're not missing any pages.
- **If you use extra pages, indicate on the problem page that you're continuing onto an extra page.**
- Pay attention to units if they are given.
- Show your work. Correct answer with wrong explanation gets no credit.

Problem 1.

You'll be sketching a straight-line approximation Bode plot of the following transfer function:

$$G(s) = 10\frac{s^2 + 2s + 100}{s + 1000}$$

a) What are the break frequencies?

10 r/s and 1000 r/s

b) Will there be a "peak"? If so, calculate here how much of an adjustment you'll need to make in the Bode plot to account for the peak.

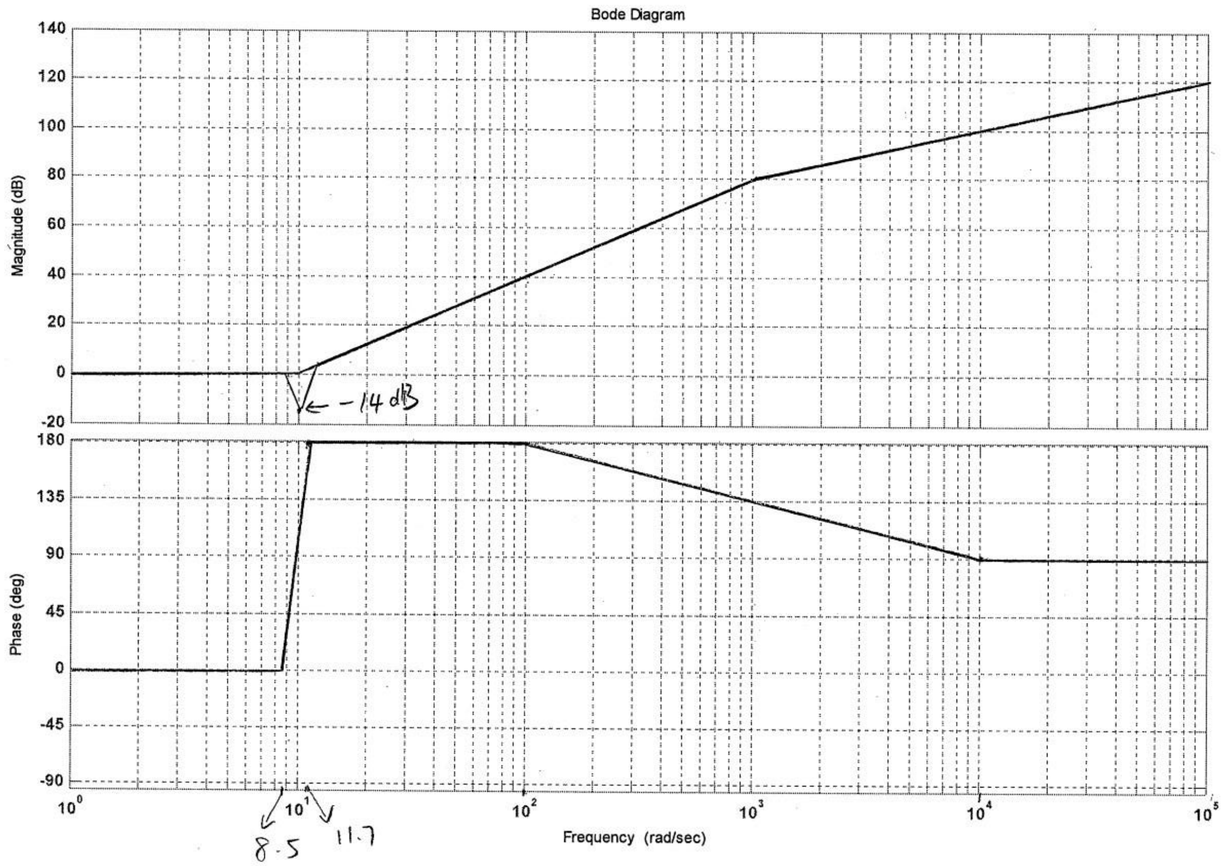
Yes. $\omega_n = 10$ r/s $\quad 2\zeta\omega_n = 2 \quad \zeta = \frac{1}{\omega_n} = 0.1$

$|Peak| = \frac{1}{2\zeta} = \frac{1}{0.2} = 5 = 20\log 5 = \underline{\underline{14\text{ dB}}}$

c) Sketch the straight line approximation Bode plot (next page). Hint: You will want to re-arrange the transfer function so that each relevant term is in its standard form and also so that it's clear what happens at what frequency. Pay attention to the static gain (i.e. the magnitude at low frequencies) because that can affect your entire magnitude plot. <u>If there is a peak in the magnitude, show it in your plot</u>.

$$G(s) = 10\,\frac{s^2+2s+100}{100} \times 100 \times \frac{1}{\frac{s}{1000}+1} \times \frac{1}{1000}$$

$$= \frac{s^2+2s+100}{100} \times \frac{1}{\frac{s}{1000}+1}$$



$$\left(\frac{1}{5}\right)^{0.1} \times 10 = 8.5 \ r/s$$

$$(5)^{0.1} \times 10 = 11.7 \ r/s$$

Problem 2.

Part 1

You are given the following transfer function: $G(s)=\dfrac{s+5}{s^2+4s+25}$

a) What is the frequency response function $G(j\omega)$?

$$G(j\omega) = \frac{j\omega+5}{(j\omega)^2+4j\omega+25} = \frac{j\omega+5}{4j\omega+25-\omega^2}$$

b) What are the magnitude and the phase of this frequency response function?

$$|G(j\omega)| = \frac{\sqrt{\omega^2+25}}{\sqrt{16\omega^2+(25-\omega^2)^2}}$$

$$\angle G(j\omega) = \tan^{-1}\frac{\omega}{5} - \operatorname{atan2}(4\omega,\ 25-\omega^2)$$

c) If this system is excited by the input $x=10\cos(2t-0.5)$, what will be the steady state output $y_{ss}(t)$?

$$y_{ss} = 10\,|G(j\underbrace{\boxed{\omega}}_{2})|\cos(2t-0.5+\angle G(j2))$$

$$|G(j\omega)| = \sqrt{4+25}\Big/\sqrt{64+(25-4)^2} = 0.24$$

$$\angle G(j2) = \tan^{-1}\frac{2}{5} - \tan^{-1}\frac{8}{21} = 0.017$$

$$y_{ss} = 2.4\cos(2t-0.48) \quad \text{or } -27.7^\circ$$

d) If this system is excited by a unit step input, what will be the steady state response?

$$y_{ss} = \frac{1}{s}\,s\,\frac{s+5}{s^2+4s+25}\bigg|_{s=0} = \frac{1}{5} = 0.2$$

Part 2.

You are given the transfer function: $G(s) = \dfrac{1}{s^2 + 4s + 100}$

If the system with the above transfer function is excited with a unit step input,

a) What will be the % overshoot?

$2\zeta\omega_n = 4 \qquad \omega_n = 10\,r/s$

$\zeta = \frac{4}{2\omega_n} = \frac{4}{20} = 0.2 \qquad \%\,OS = 100\, e^{-\pi \times 0.2/\sqrt{1-0.2^2}} = 52.7\%$

b) What will be the steady state response?

$s \times \frac{1}{s} \times \frac{1}{s^2+4s+100}\Big|_{s=0} = 0.01$

c) What will be the 2% settling time?

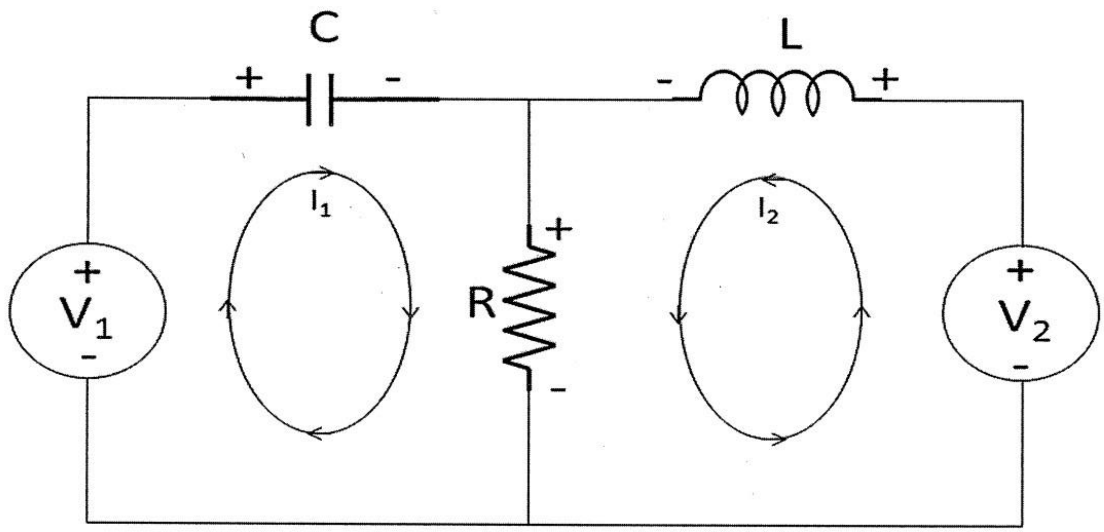
$t_s = \frac{4}{\zeta\omega_n} = 2\,sec$

d) What will be the peak time?

$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{10\sqrt{1-0.2^2}} = 0.32\,s$

Problem 3.

You will be modeling the circuit below. You are interested in <u>the current that flows through the inductor (I_2), so that's your output</u>. You obviously have 2 input sources.



We are asked to use a combination of <u>the impedance method</u> and <u>the loop approach</u>. The direction of the loop currents have been assumed for you (stick with them). According to Ohm's Law, remember that V=IZ where Z is the impedance of the element. <u>For now, keep the impedances symbolic: Z_C, Z_L, and Z_R</u>.

a) Write the Kirchoff's voltage Law <u>for the first loop</u>. Your equation should only be in terms of the symbolic impedances (Z_C, Z_L, and Z_R), the voltage source(s), and the loop current(s) I_1 and I_2. (No derivatives or integrals in the equation).

$$V_1 - I_1 Z_C - (I_1 + I_2) Z_R = 0$$

$$V_1 - I_1 (Z_C + Z_R) - I_2 Z_R = 0$$

b) Do the same for the second loop.

$$V_2 - I_2 Z_L - (I_1 + I_2) Z_R = 0$$

c) Now use your answer in part b to solve for I_1.

$$V_2 - I_2 Z_C - I_1 Z_R - I_2 Z_R = 0$$

$$I_1 = \frac{V_2}{Z_R} - \frac{(Z_R + Z_L)}{Z_R} \cdot I_2$$

d) Now substitute what you found in part c into your equation in part a to eliminate I_1. Solve for I_2. Your answer should be in the form:

$I_2 = (\quad\quad)V_1 + (\quad\quad)V_2$

$$V_1 - \left[\frac{V_2}{Z_R} - \frac{(Z_R + Z_L)}{Z_R} I_2\right](Z_C + Z_R) - I_2 Z_R = 0$$

$$V_1 - \frac{V_2 (Z_C + Z_R)}{Z_R} + \frac{(Z_R + Z_L)(Z_C + Z_R)}{Z_R} I_2 - Z_R I_2 = 0$$

$$V_1 - \frac{V_2 (Z_C + Z_R)}{Z_R} + I_2 \left[\frac{(Z_R + Z_L)(Z_C + Z_R) - Z_R^2}{Z_R}\right] = 0$$

$$I_2 = \frac{-Z_R}{(Z_R + Z_L)(Z_C + Z_R) - Z_R^2} V_1 + \frac{Z_C + Z_R}{(Z_R + Z_L)(Z_C + Z_R) - Z_R^2} V_2$$

$$I_2 = \frac{-Z_R}{Z_R Z_C + Z_L Z_C + Z_L Z_R} V_1 + \frac{Z_C + Z_R}{Z_R Z_C + Z_L Z_C + Z_L Z_R} V_2$$

e) What are the Z_C, Z_L, and Z_R in terms of C, L, R and the complex variable "s"? Only write what they are. <u>Do NOT go back to previous parts to plug them into your equations</u>.

$$Z_C = \frac{1}{Cs} \qquad Z_L = Ls \qquad Z_R = R.$$