ME 375 EXAM #2
Tuesday, March 30, 2004

Division Meckl 10:30 / King 1:30 (circle one) **Name** Solution

Instructions

(1) This is a closed book examination, but you are allowed two 8.5×11 crib sheets.

(2) You have one hour to work all two problems on the exam.

(3) Use the solution procedure we have discussed: what are you given, what are you asked to find, what are your assumptions, what is your solution, does your solution make sense. You must show all of your work to receive <u>any</u> credit.

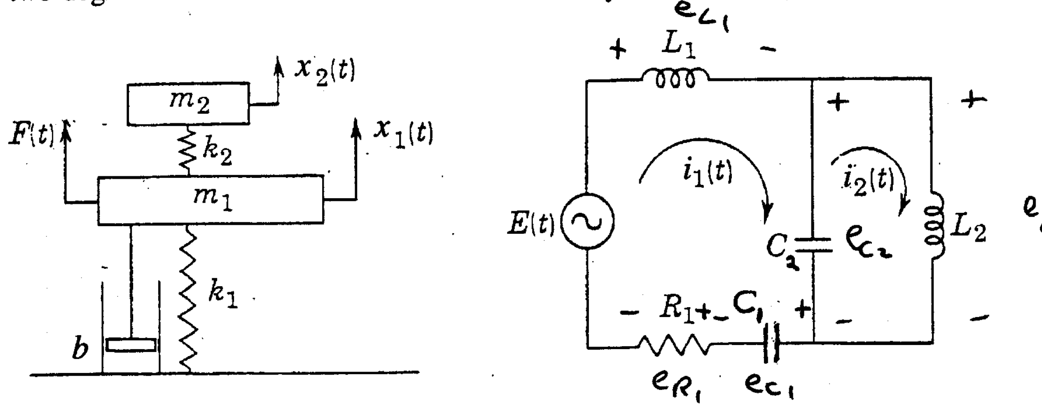
(4) You must write neatly and should use a logical format to solve the problems. You are encouraged to really "think" about the problems before you start to solve them. Please write your name in the top right-hand corner of each page.

(5) A table of Laplace transform pairs and properties of Laplace transforms is attached at the end of this exam set.

Problem No. 1 (50%)	________
Problem No. 2 (50%)	________
TOTAL (*/100%)**	________

PROBLEM NO. 1 **(50%)**

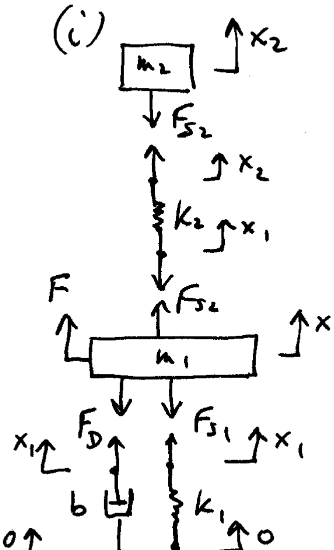
Analogous two-degree-of-freedom mechanical and electrical systems are shown in the figure below:



(a) For the mechanical system, determine the system equations in the time domain by:

   (i) Drawing the free body diagrams
   (ii) Writing the elemental equations
   (iii) Combining the elemental equations to find two governing differential equations
   (iv) Converting the ODEs from part (iii) into differential equations with *velocities* $v_1 = \dot{x}_1$ and $v_2 = \dot{x}_2$ (and derivatives) as the output variables and force *F(t)* (and possibly its derivative) as the input variable.

(i)



(ii)

$$m_2 \ddot{x}_2 = -F_{s2}$$
$$F_{s2} = k_2 (x_2 - x_1)$$
$$m_1 \ddot{x}_1 = F_{s2} - F_d - F_{s1} + F$$
$$F_d = b \dot{x}_1$$
$$F_{s1} = k_1 x_1$$

(iii)

$$m_1 \ddot{x}_1 + b \dot{x}_1 + (k_1 + k_2) x_1 - k_2 x_2 = F$$
$$m_2 \ddot{x}_2 + k_2 x_2 - k_2 x_1 = 0$$

(iv)

$$m_1 \dot{v}_1 + b v_1 + (k_1 + k_2) \int v_1 dt - k_2 \int v_2 dt = F$$
$$m_2 \dot{v}_2 + k_2 \int v_2 dt - k_2 \int v_1 dt = 0$$

or

$$m_1 \ddot{v}_1 + b \dot{v}_1 + (k_1 + k_2) v_1 - k_2 v_2 = \dot{F}$$
$$m_2 \ddot{v}_2 + k_2 v_2 - k_2 v_1 = 0$$

PROBLEM NO. 1 (Continued)

(b) For the electrical system, determine the two system equations in the time domain using the loop method. You should get one differential equation for each loop, in terms of i_1 and i_2 (and their derivatives, and input voltage $E(t)$ (and possibly its derivative).

$$E(t) - e_{L_1} - e_{C_2} - e_{C_1} - e_{R_1} = 0$$

$$e_{C_2} - e_{L_2} = 0$$

$$e_{L_1} = L_1 \frac{di_1}{dt}, \quad e_{C_2} = \frac{1}{C_2}\int (i_1 - i_2)\,dt$$

$$e_{C_1} = \frac{1}{C_1}\int i_1\,dt, \quad e_{R_1} = R_1 i_1, \quad e_{L_2} = L_2 \frac{di_2}{dt}$$

$$\boxed{\begin{cases} L_1 \frac{di_1}{dt} + R_1 i_1 + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int i_1\,dt - \frac{1}{C_2}\int i_2\,dt = E(t) \\ L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int i_2\,dt - \frac{1}{C_2}\int i_1\,dt = 0 \end{cases}}$$

or

$$\boxed{\begin{array}{l} L_1 \frac{d^2 i_1}{dt^2} + R_1 \frac{di_1}{dt} + \left(\frac{1}{C_1} + \frac{1}{C_2}\right) i_1 - \frac{1}{C_2} i_2 = \dot{E}(t) \\ L_2 \frac{d^2 i_2}{dt^2} + \frac{1}{C_2} i_2 - \frac{1}{C_2} i_1 = 0 \end{array}}$$

(c) Given that for the mechanical system the Effort variable is Force and the Flow variable is Velocity, and that for the electrical system the Effort variable is Voltage and the Flow variable is Current; identify the corresponding elements between the mechanical and the electrical systems and the corresponding impedance terms. (Recall impedance is the Effort variable divided by the Flow variable.)

Mechanical System	Mechanical impedance	Electrical System	Electrical impedance
Displacement (x)	------------------	Charge (q)	----------------------
Force (F)	------------------	Voltage (e)	----------------------
Velocity (v)	------------------	Current (i)	----------------------
Mass (m)	$m\frac{d}{dt}$ or ms	Inductance L	$L\frac{d}{dt}$ or Ls
Spring (k)	$k\int dt$ or $\frac{k}{s}$	Inverse Capacitance $1/C$	$\frac{1}{C}\int dt$ or $\frac{1}{Cs}$
Dashpot (b)	b	Resistance R	R

PROBLEM NO. 2 (50%)

Given the transfer function $G(s) = \dfrac{2000s}{\underbrace{s^2+1001s+1000}_{(s+1)(s+1000)}}$, draw the Bode straight line approximation using the following steps:

(a) Find the poles, zeros, and static gain of the transfer function $G(s)$.

poles: $s = -1$, $s = -1000$

zeros: $s = 0$

static gain: 0

(b) Write all first order terms in the form ($\tau s + 1$) (except s or $1/s$) and all second order complex terms in the form $\omega_n^2/(s^2+2\zeta\omega_n s+\omega_n^2)$.

$$G(s) = \frac{2000s}{(s+1)(s+1000)} = \frac{2s}{(s+1)(s/1000+1)}$$

$$G_0(s) = 2, \quad G_1(s) = s$$

$$G_2(s) = \frac{1}{s+1}, \quad G_3(s) = \frac{1}{s/1000+1}$$

(c) Provide the break frequencies for all of the terms in transfer function $G(s)$.

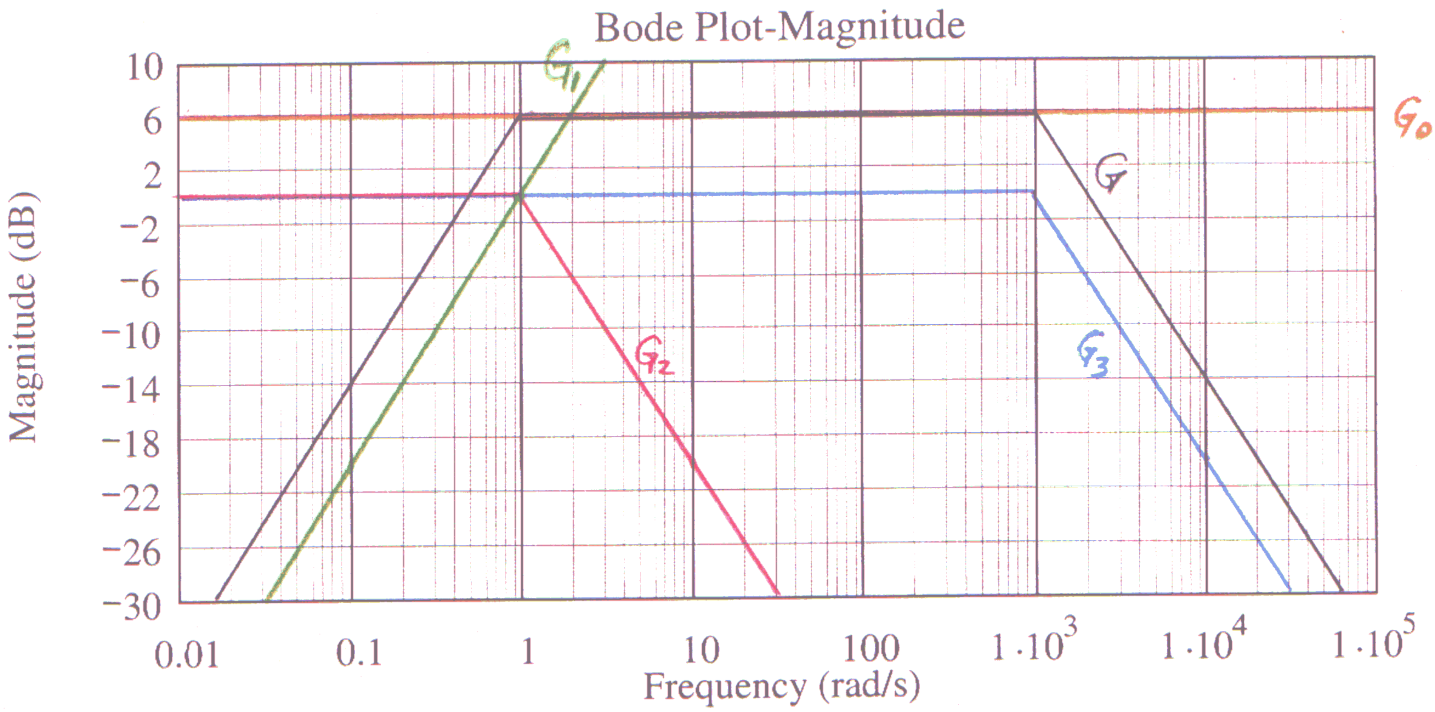
$\omega_0 \rightarrow \infty$, $\omega_1 = 0$ r/s optional

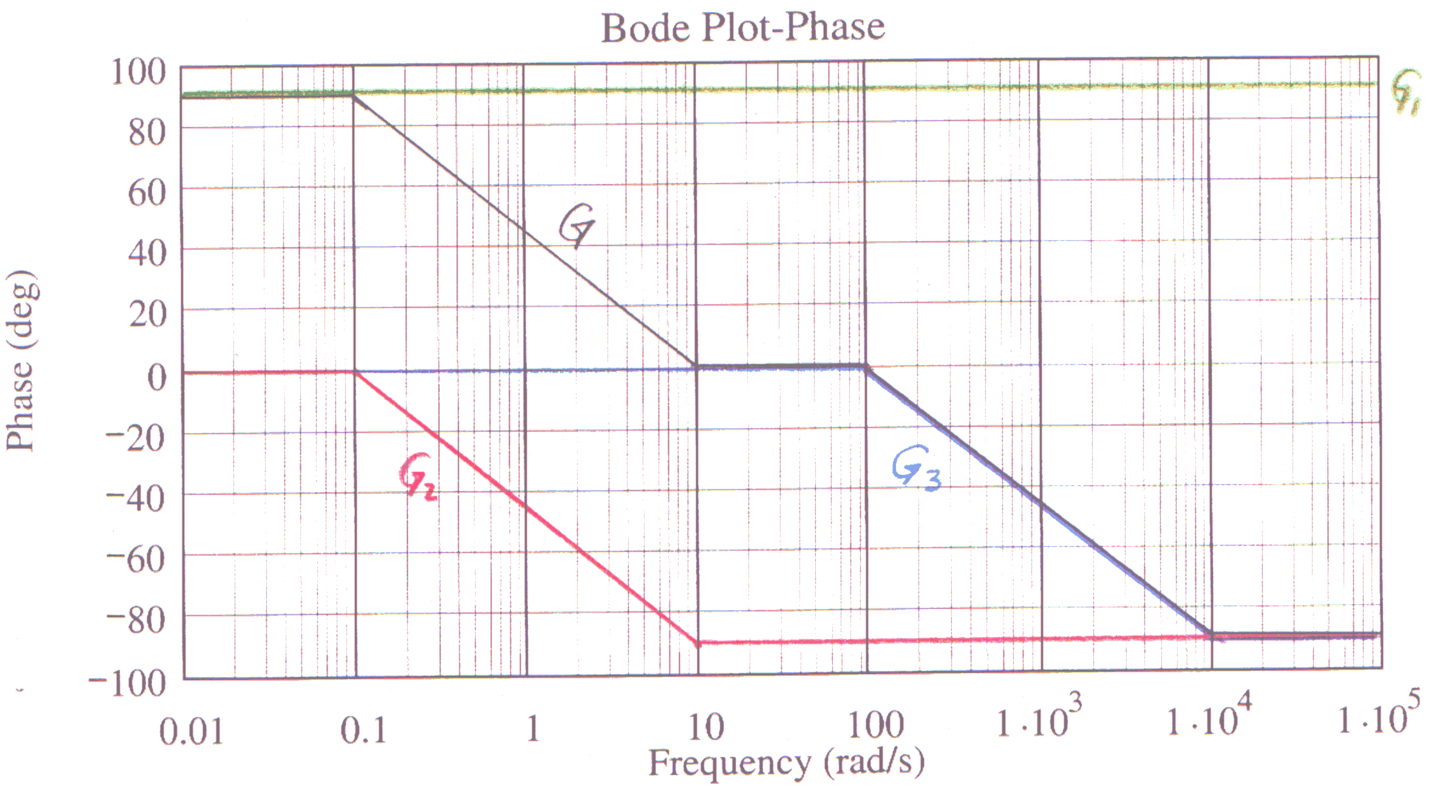
$\boxed{\omega_2 = 1 \text{ r/s}, \quad \omega_3 = 1000 \text{ r/s}}$

(d) Plot magnitude and phase of all elemental transfer functions separately on the graphs on the next page.

Label the elemental transfer functions on the graph along with all break frequencies and slopes.

(e) Plot the total transfer function made up of the elemental transfer functions in part (d).


Bode Plot-Magnitude
Magnitude (dB)
10
6
2
-2
-6
-10
-14
-18
-22
-26
-30
0.01
0.1
1
10
100
$1 \cdot 10^3$
$1 \cdot 10^4$
$1 \cdot 10^5$
Frequency (rad/s)
G_1
G_0
G
G_2
G_3


Bode Plot-Phase
Phase (deg)
100
80
60
40
20
0
-20
-40
-60
-80
-100
0.01
0.1
1
10
100
$1 \cdot 10^3$
$1 \cdot 10^4$
$1 \cdot 10^5$
Frequency (rad/s)
G_1
G
G_2
G_3

PROBLEM NO. 2 (Continued)

(f) For $G(s)$ given on the previous page, write down expressions for the magnitude and phase of the frequency response as a function of ω.

$$G(j\omega) = \frac{2j\omega}{(j\omega+1)(j\omega/1000+1)}$$

$$|G(j\omega)| = \frac{2\omega}{\sqrt{\omega^2+1}\sqrt{\left(\frac{\omega}{1000}\right)^2+1}}$$

$$\angle G(j\omega) = 90^\circ - \tan^{-1}\frac{\omega}{1} - \tan^{-1}\frac{\omega}{1000}$$

(g) Assume that the transfer function on the previous page represents $G(s) = \frac{Y(s)}{U(s)}$, where $u(t)$ is the input signal and $y(t)$ is the output signal. Write down an expression for the steady-state response $y_{ss}(t)$ when the input is $u(t) = 0.5\cos(0.1t) + 3\sin(20t)$.

$$y_{ss}(t) = 0.5|G(j0.1)|\cos(0.1t + \angle G(j0.1)) + 3|G(j20)|\sin(20t + \angle G(j20))$$

$$\boxed{y_{ss}(t) = 0.5(0.2)\cos(0.1t + \underset{1.47\text{ rad}}{\underline{84.3^\circ}}) + 3(2)\sin(20t + \underset{0.03\text{ rad}}{\underline{1.72^\circ}})}$$