ME 375 EXAM #2
Thursday, April 12, 2007

Division Shaver 11:30 / Meckl 2:30 (circle one) **Name** Solution

Instructions

(1) This is a closed book examination, but you are allowed two 8.5×11 crib sheets.

(2) You have one hour to work all three problems on the exam.

(3) Use the solution procedure we have discussed: what are you given, what are you asked to find, what are your assumptions, what is your solution, does your solution make sense. You must show all of your work to receive any credit.

(4) You must write neatly and should use a logical format to solve the problems. You are encouraged to really "think" about the problems before you start to solve them. Please write your name in the top right-hand corner of each page.

(5) A table of Laplace transform pairs and properties of Laplace transforms is attached at the end of this exam set.

Problem No. 1 (40%) ________________

Problem No. 2 (30%) ________________

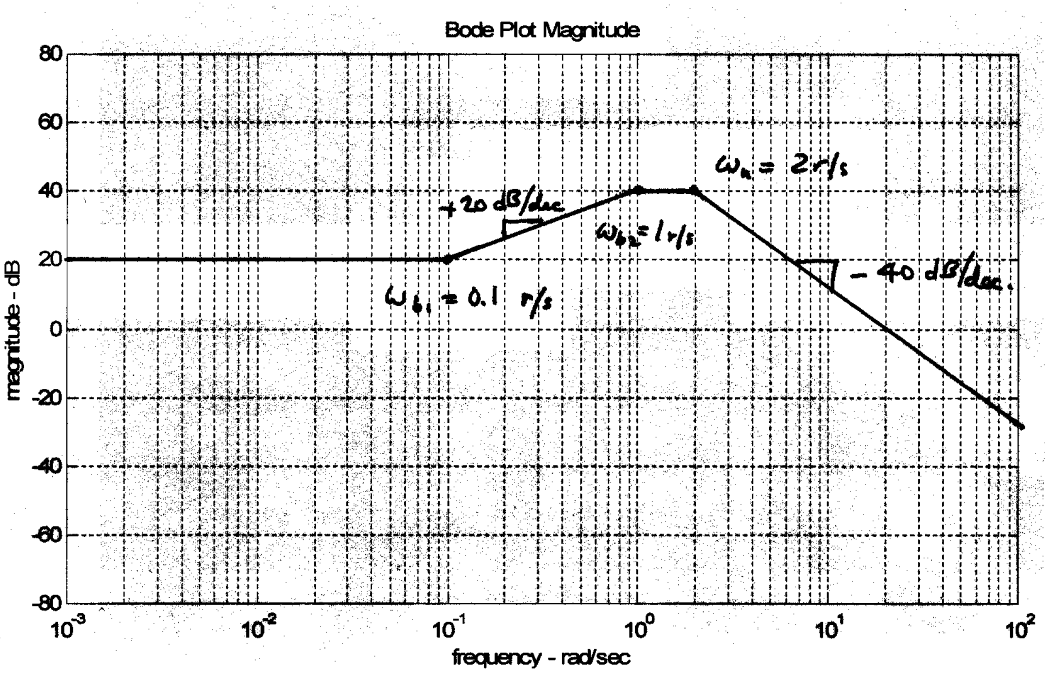
Problem No. 3 (30%) ________________

TOTAL (*/100%)** ________________

PROBLEM 1: (40%)

(a) Draw the straight-line approximation bode magnitude plot for the following transfer function. Be sure to clearly label all break frequencies and slopes.

$$G(s) = \frac{Y(s)}{U(s)} = \frac{400(s+0.1)}{(s+1)(s^2+0.4s+4)}$$



$$G(s) = \frac{40\,(s/0.1 + 1)}{(s+1)(s^2 + 0.4s + 4)} = 10\,\underbrace{(s/0.1+1)}_{\omega_{b_1} = 0.1}\,\underbrace{\frac{1}{s+1}}_{\omega_{b_2} = 1}\,\underbrace{\frac{4}{s^2+0.4s+4}}_{\omega_n = 2}$$

$\zeta = 0.1$

(b) Determine expressions for the frequency response magnitude and phase of this transfer function.

$$G(j\omega) = \frac{400\,(j\omega + 0.1)}{(j\omega + 1)\big((4-\omega^2) + 0.4j\omega\big)}$$

$$|G(j\omega)| = \frac{400\sqrt{\omega^2 + 0.1^2}}{\sqrt{\omega^2 + 1^2}\,\sqrt{(4-\omega^2)^2 + (0.4\omega)^2}}$$

$$\angle G(j\omega) = \tan^{-1}\frac{\omega}{0.1} - \tan^{-1}\frac{\omega}{1} - \tan^{-1}\frac{0.4\omega}{4-\omega^2}$$

(c) If the input $u(t)$ is given by $u(t) = 8\sin(t)$, determine an expression for the output $y(t)$ at steady-state.

$$|G(j1)| = 93.9 = 39.45 \text{ dB}$$

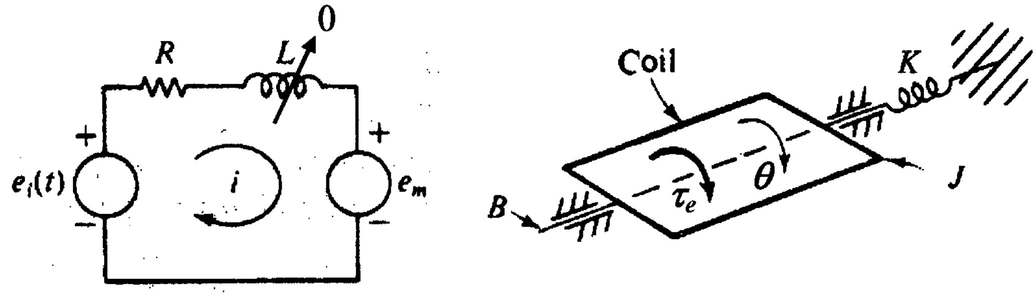
$$\angle G(j1) = 31.7^\circ = 0.553 \text{ rad.}$$

$$y(t) = 8 \cdot 93.9 \sin(t + 31.7^\circ)$$

$$\boxed{y(t) = 751.2 \sin(t + 31.7^\circ)}$$

PROBLEM 2: (30%)

A galvanometer works very much like a dc motor. It produces an angular deflection that depends on the current passing through a coil. Like a dc motor, its torque is proportional to current, i.e., $\tau_e = \alpha i$, and the back emf is related to the angular velocity, i.e., $e_m = \alpha \dot{\theta}$, where α represents the torque and/or back emf constant. The only real difference between a galvanometer and a dc motor is that the motor shaft is attached to a spring.



Assume that the inductance L is zero.

(a) Write down all the equations that relate the variables.

$$e_i(t) = Ri + e_m$$

$$J\ddot{\theta} = \tau_e - B\dot{\theta} - K\theta \Rightarrow J\ddot{\theta} + B\dot{\theta} + K\theta = \tau_e$$

$$e_m = \alpha \dot{\theta}$$

$$\tau_e = \alpha i$$

Take Laplace transforms:

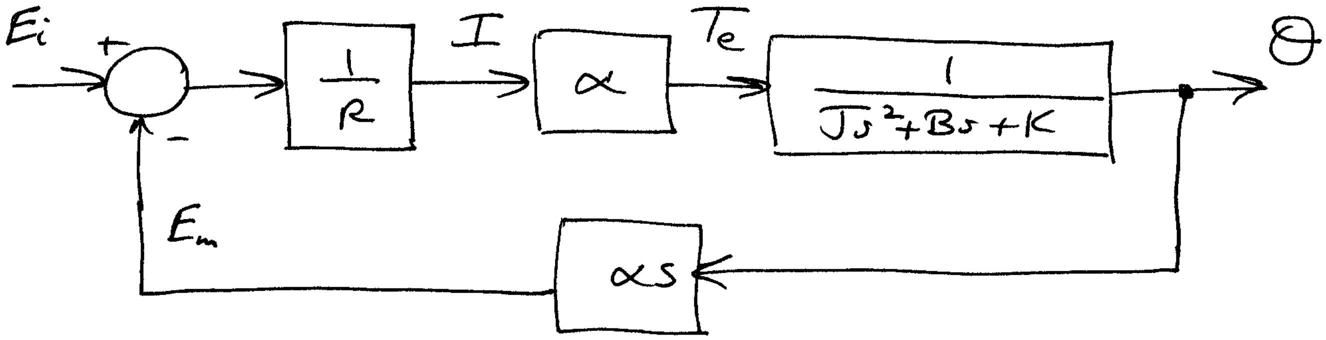
$$E_i(s) = RI(s) + E_m(s) \Rightarrow I(s) = \frac{1}{R}\left[E_i(s) - E_m(s)\right]$$

$$(Js^2 + Bs + K)\theta(s) = T_e(s) \Rightarrow \theta(s) = \frac{T_e(s)}{Js^2 + Bs + K}$$

$$E_m(s) = \alpha s\, \theta(s)$$

$$T_e(s) = \alpha I(s)$$

(b) Develop a block diagram that clearly shows the electrical and mechanical subsystems, as well as the electromechanical coupling terms, with voltage e_i as input and angular position θ as output.



(c) Using this block diagram, determine the transfer function between the voltage input e_i and the angular position output θ.

Block diagram is standard feedback loop with

$$G(s) = \frac{\alpha}{R}\frac{1}{Js^2+Bs+K}, \quad H(s) = \alpha s$$

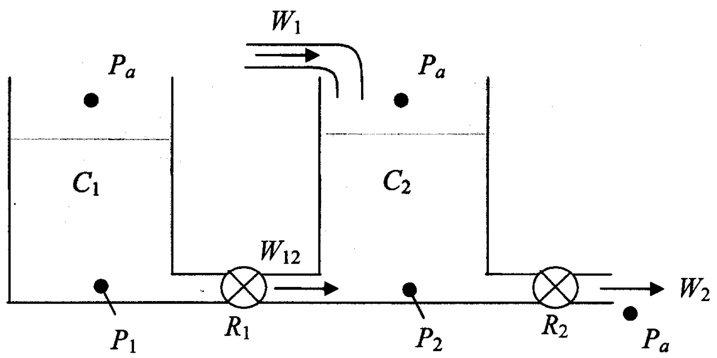
$$\frac{\theta(s)}{E_i(s)} = \frac{\frac{\alpha}{R}\frac{1}{Js^2+Bs+K}}{1+\frac{\alpha}{R}\frac{\alpha s}{Js^2+Bs+K}}$$

$$= \frac{\alpha}{R(Js^2+Bs+K)+\alpha^2 s}$$

$$\boxed{\frac{\theta(s)}{E_i(s)} = \frac{\alpha}{RJs^2+(RB+\alpha^2)s+RK}}$$

PROBLEM 3: (30%)

Given: the following hydraulic system:



Find: the transfer function between the input W_1 and the output W_2, by:

(a) Writing down all the relevant equations (20%)

(b) Combining and re-arranging to arrive at the transfer function (10%)

(a) $P_{1a} = \frac{1}{C_1}\int(-W_{12})\,dt$

$P_{12} = R_1 W_{12} = P_{1a} - P_{2a}$

$P_{2a} = \frac{1}{C_2}\int(W_1 + W_{12} - W_2)\,dt = R_2 W_2$

(b) $-\frac{1}{C_1}\int W_{12}\,dt - R_2 W_2 = R_1 W_{12}$

$R_1 C_1 \dot{W}_{12} + W_{12} = -R_2 C_1 \dot{W}_2 \Rightarrow (R_1 C_1 s + 1) W_{12}(s) = -R_2 C_1 s W_2(s)$

$\Rightarrow W_{12}(s) = -\frac{R_2 C_1 s W_2(s)}{R_1 C_1 s + 1}$

$R_2 C_2 \dot{W}_2 = W_1 + W_{12} - W_2$

$(R_2 C_2 s + 1) W_2(s) = W_1(s) + W_{12}(s) = W_1(s) - \frac{R_2 C_1 s W_2(s)}{R_1 C_1 s + 1}$

$(R_1 C_1 s + 1)(R_2 C_2 s + 1) W_2(s) + R_2 C_1 s W_2(s) = (R_1 C_1 s + 1) W_1(s)$

$$\boxed{\frac{W_2(s)}{W_1(s)} = \frac{R_1 C_1 s + 1}{(R_1 C_1 s + 1)(R_2 C_2 s + 1) + R_2 C_1 s}}$$