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Section: 12:30 2:30

# ME 375
# FINAL EXAM

Solution.

8-10am, Smith 108
April 29, 2002

| Section A | | Section B | |
|---|---|---|---|
| Problems | ***Score*** | Problems | Score |
| 1. (10) | ______ | 1. (40) | ______ |
| 2. (10) | ______ | 2. (40) | ______ |
| 3. (10) | ______ | 3. (80) | ______ |
| 4. (10) | ______ | | |
| 5. (10) | ______ | | |
| 6. (10) | ______ | | |
| 7. (10) | ______ | | |
| 8. (20) | ______ | | |
| 9. (10) | ______ | | |
| 10. (20) | ______ | | |
| 11. (20) | ______ | | |
| Total (A) | /140 | Total (B) | /160 |
| | | Total (A+B) | /300 |

**READ THESE INSTRUCTIONS BEFORE TAKING THE TEST!**

- **This test is closed book. You are allowed two single sided hand written sheet of notes.**
- **Your solutions must be presented in a clear, concise, and well-written manner to receive any credit. Hard-to-follow solutions will receive no credit.**
- **You must CIRCLE your final answer.**
- **You may only write on the test. No additional paper can be used.**
- **DO NOT UNSTAPLE YOUR EXAM.**
- **You may have a calculator, pencil, and eraser (if you think you will need it).**
- **You must also have your student ID on your desk during the exam. If you don't have your student ID, do NOT take the exam. Notify one of the professors immediately before the test begins.**
- **You must stop immediately when the testing time has ended.**

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**Laplace Transform Pairs**

| f(t) | F(s) | Comment |
|---|---|---|
| 1. $u_I(t)=\delta(t)$ | $1$ | Unit impulse |
| 2. $u_s(t)=1$, for $t>0$ | $\frac{1}{s}$ | Unit step |
| 3. $u_r(t)=t$ | $\frac{1}{s^2}$ | Unit ramp |
| 4. $e^{-at}$ | $\frac{1}{s+a}$ | Exponential |
| 5. $\sin \omega t$ | $\frac{\omega}{s^2+\omega^2}$ | Sine |
| 6. $\cos \omega t$ | $\frac{s}{s^2+\omega^2}$ | Cosine |
| 7. $f(t)$ | $F(s)$ | Function |
| 8. $\frac{df(t)}{dt}$ | $sF(s)-f(0^-)$ | First Derivative |
| 9. $\frac{d^n f(t)}{dt^n}$ | $s^nF(s)-s^{n-1}f(0^-)-s^{n-2}\frac{df(0^-)}{dt}\cdots$ | $n^{th}$ Derivative |
| 10. $t^n$ | $\frac{n!}{s^{n+1}}$ | |
| 11. $e^{-at}f(t)$ | $F(s+a)$ | |
| 12. $te^{-at}$ | $\frac{1}{(s+a)^2}$ | |
| 13. $t^n e^{-at}$ | $\frac{n!}{(s+a)^{n+1}}$ | |
| 14. $e^{-at}\sin \omega t$ | $\frac{\omega}{(s+a)^2+\omega^2}$ | |
| 15. $e^{-at}\cos \omega t$ | $\frac{(s+a)}{(s+a)^2+\omega^2}$ | |
| 16. Initial Value Theorem: | $f(0^+)=\lim_{s\to\infty} sF(s)$ | |
| 17. Final Value Theorem: | $\lim_{t\to\infty} f(t)=\lim_{s\to 0} sF(s)$ | |

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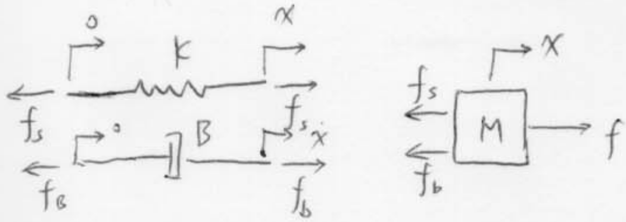
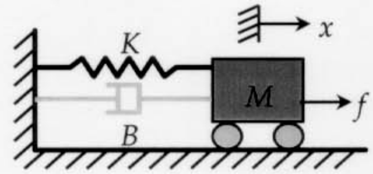
**Section A**

A1.(10 points) Consider the following system. Draw the free-body diagram of the system and obtain the transfer function of the system from the input $f(t)$ to the output $x(t)$




$$f_s = K(x - 0)$$
$$f_b = B(\dot{x} - 0)$$
$$M\ddot{x} = f - f_s - f_b$$

$$\Rightarrow \quad M\ddot{x} + B\dot{x} + Kx = f$$

$$\therefore \boxed{G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}}$$

A2.(10 points) The EOM for the system in problem A1 is given by:

$$M\,\ddot{x} + B\,\dot{x} + K\,x = f(t)$$

Let the state variables be $q_1 = x$ and $q_2 = \dot{x}$. Obtain the state-space representation of the system.

$$\dot{q}_1 = q_2$$
$$\dot{q}_2 = \ddot{x} = \frac{1}{M}\{f - Bq_2 - Kq_1\}$$
$$y = x = q_1$$

$$\Rightarrow \quad \dot{q} = \underbrace{\begin{bmatrix} 0 & 1 \\ -K/M & -B/M \end{bmatrix}}_{A} q + \underbrace{\begin{bmatrix} 0 \\ 1/M \end{bmatrix}}_{B} f$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} q$$

A3.(10 points) Consider a system described by

$$\ddot{y} + \dot{y} + y = \dot{u} - 2u$$

Obtain the transfer function and the stability of the system.

$$\boxed{G(s) = \frac{s - 2}{s^2 + s + 1}}$$

Poles: $s^2 + s + 1 = 0 \Rightarrow p_{1,2} = \frac{-1 \pm \sqrt{3}\,j}{2} \Rightarrow Re\{p_{1,2}\} < 0 \Rightarrow$

Stable

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A4.(10 points) Consider a system described by

$$\ddot{y}+2\dot{y}+y=\dot{u}+2u$$

Obtain the forced response of the system for a sinusoidal input $u(t)=2\sin(5t)$ in s-domain.

$$G(s)=\frac{s+2}{s^2+2s+1}$$

$$\therefore\ Y(s)=G(s)U(s)=\frac{s+2}{s^2+2s+1}\cdot\frac{2\times 5}{s^2+5^2}=\boxed{\frac{10(s+2)}{(s^2+2s+1)(s^2+5^2)}}$$

A5.(10 points) Consider a system described by

$$\dot{y}+2y=u$$

Obtain the steady-state response of the system for a sinusoidal input $u(t)=5\cos(t)$ in time domain.

$$G(s)=\frac{1}{s+2} \Rightarrow G(j\omega)=\frac{1}{j\omega+2} \Rightarrow |G(j\omega)|=\frac{1}{\sqrt{4+\omega^2}}$$

$$\angle G(j\omega)=-\angle(2+j\omega)=-\tan^{-1}\left(\frac{\omega}{2}\right)$$

$$\therefore\ y_{ss}(t)=5\left|G(j1)\right|\cos\left(t+\angle G(j1)\right)$$

$$\boxed{\begin{aligned}&=\frac{5}{\sqrt{5}}\cos\left(t-\tan^{-1}\left(\tfrac{1}{2}\right)\right)=\sqrt{5}\cos(t-0.46)\\&=\sqrt{5}\cos(t-26.6^\circ)\end{aligned}}$$

A6.(10 points) Consider a system described by

$$\ddot{y}+2\dot{y}-3y=\dot{u}+2u$$

Obtain the free response of the system for the initial conditions $y(0)=1$ and $\dot{y}(0)=0$ in time domain.

$$\underbrace{s^2Y(s)-sy(0)-\dot{y}(0)}_{\mathcal{L}\{\ddot{y}\}}+2\underbrace{\{sY(s)-y(0)\}}_{\mathcal{L}\{\dot{y}\}}-3Y(s)=(s+2)U(s)$$

$\Rightarrow$ Free response (i.e. $U(s)=0$)

$$Y(s)=\frac{(s+2)y(0)+\dot{y}(0)}{s^2+2s-3}=\frac{s+2}{(s+3)(s-1)}$$

$$=\frac{A_1}{s+3}+\frac{A_2}{s-1}$$

where

$$A_1=(s+3)Y(s)\Big|_{s=-3}=\frac{-3+2}{-3-1}=\frac{1}{4}$$

$$A_2=(s-1)Y(s)\Big|_{s=1}=\frac{1+2}{1+3}=\frac{3}{4}$$

$$\boxed{\therefore\ y(t)=\frac{1}{4}e^{-3t}+\frac{3}{4}e^{t}}$$

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A7.(10 points) Consider a system described by the following transfer function

$$G(s)=\frac{10}{0.1s^2+s+2}$$

Obtain the 2% band settling time of the system and the steady-state response of the system for a unit step input.

$$G(s)=\frac{100}{s^2+10s+20} \Rightarrow \begin{cases} 10 = 2\zeta\omega_n \\ 20 = \omega_n^2 \\ 100 = K\omega_n^2 \end{cases} \Rightarrow \begin{cases} \zeta\omega_n = 5 \\ K = 5 \end{cases}$$

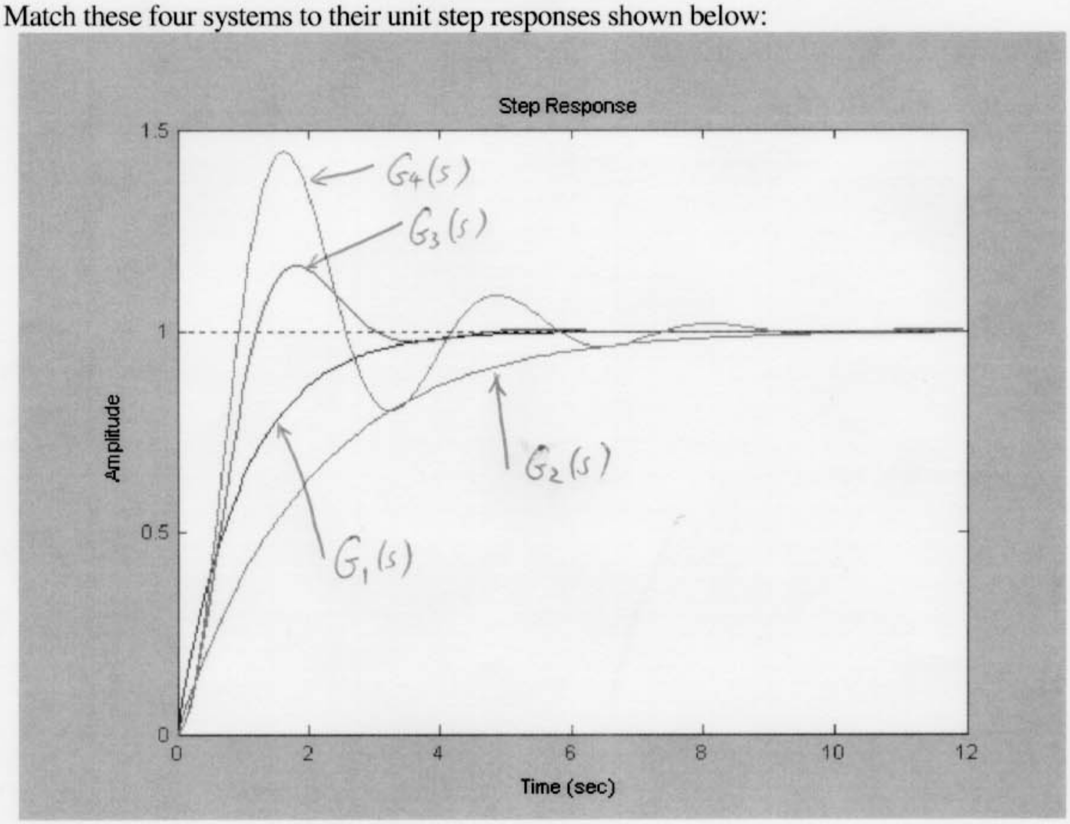
$$\therefore \boxed{\begin{aligned} t_s &= \frac{4}{\sigma} = \frac{4}{\zeta\omega_n} = \frac{4}{5} \\ y_{ss} &= K = 5 \end{aligned}}$$

A8.(20 points) Consider the four systems described by the following transfer functions

(1). $G_1(s)=\dfrac{1}{s+1}$ (2). $G_2(s)=\dfrac{1}{2s+1}$

(3) $G_3(s)=\dfrac{4}{s^2+2s+4}$ (4) $G_4(s)=\dfrac{4}{s^2+s+4}$

Match these four systems to their unit step responses shown below:



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A9.(10 points) Using straight-line approximations, sketch the Bode plots of the following system:

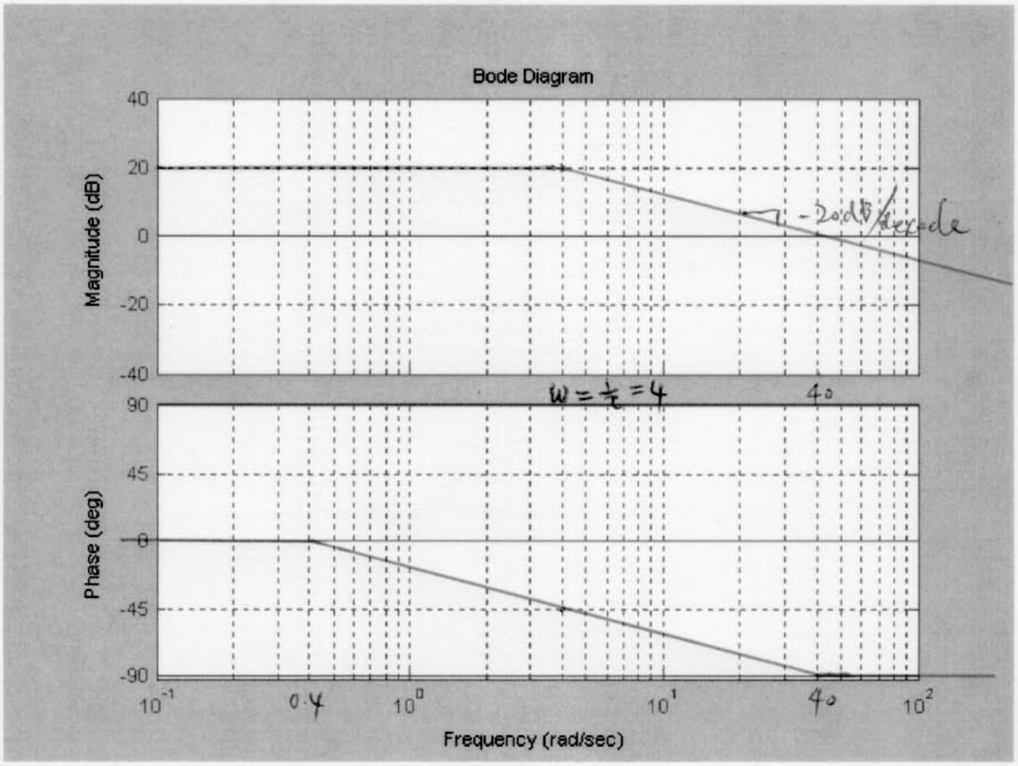
$$\dot{y} + 4y = 40u$$

$G(s) = \frac{40}{s+4}$

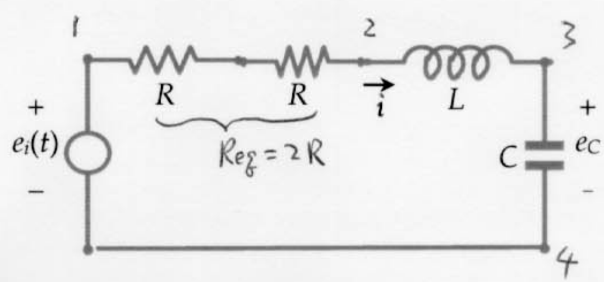
$= \frac{10}{\frac{1}{4}s+1}$

$\|G(s)\| = \|10\|$

$+ \left\|\frac{1}{\frac{1}{4}s+1}\right\|$




A10. (20 points) Obtain the transfer function of the following system where the input is $e_i$ and the output is $e_c$:




Elemental Eqs:

$e_{12} = i R_{eq}$

$e_{23} = L\frac{di}{dt}$

$= C\frac{de_{34}}{dt}$

Interconnecting Eqs:

$e_i = e_{12} + e_{23} + e_{34}$

$\Rightarrow$

UVs: $e_{12}^{\times}, i^{\times}, e_{23}^{\times}, e_{34} = e_c$

Eliminate $e_{12}, i, e_{23}$:

$e_i = R_{eq}C\frac{de_c}{dt} + LC\frac{d^2e_c}{dt} + e_c$

$\therefore \boxed{G(s) = \frac{E_c(s)}{E_i(s)} = \frac{1}{LCs^2 + R_{eq}Cs + 1}}$

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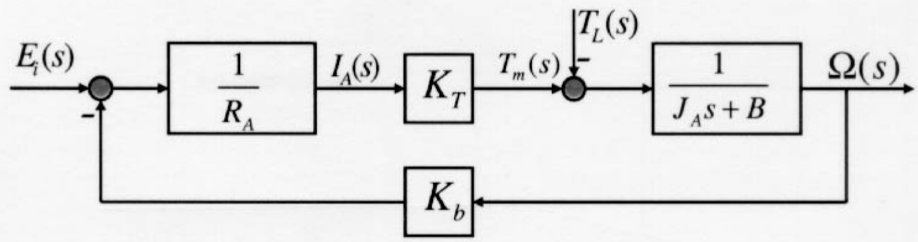
A11. (20 points) Consider the following block diagram of a DC motor with negligible electrical dynamics. Obtain the transfer functions from the inputs $E_i(s)$ and $T_L(s)$ to the output speed $\Omega(s)$ :




$$\Omega(s) = \frac{1}{J_A s+B}\left\{T_m - T_L\right\} = \frac{1}{J_A s+B}\left\{\frac{K_T}{R_A}\left(E_i(s) - K_b\,\Omega(s)\right)-T_L\right\}$$

$$\Rightarrow \left\{1 + \frac{K_T K_b}{R_A(J_A s+B)}\right\}\Omega(s) = \frac{K_T}{(J_A s+B)R_A}E_i(s) - \frac{1}{J_A s+B}T_L$$

$$\Omega(s) = \underbrace{\frac{K_T}{R_A J_A s+(R_A B+K_T K_b)}}_{G_{\Omega E}} E_i(s) - \underbrace{\frac{R_A}{R_A J_A s+(R_A B+K_T K_b)}}_{G_{\Omega T}} T_L(s)$$

$$\therefore \boxed{\begin{aligned} G_{\Omega E}(s) &= \frac{K_T}{R_A J_A s+(R_A B+K_T K_b)} \\ G_{\Omega T} &= -\frac{R_A}{R_A J_A s+(R_A B+K_T K_b)}\end{aligned}}$$

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**Section B**

**Problem B.1** (40 points)

Consider the design of blood sampler mentioned in the Lectures. A blood sampler can be modeled by a mass-spring-damper system shown below

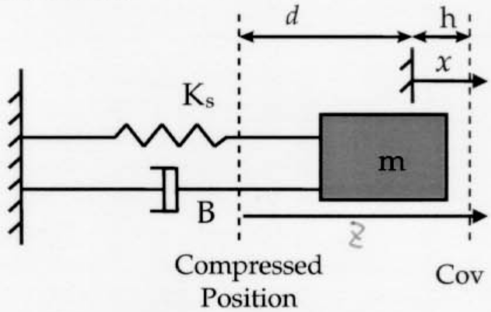


Fig3.1. Schematic of Blood Sampler

where the needle has a mass of $m$=0.01kg, and $K_s$=2500 and $d$=10mm. The system is initially at rest as shown, which is defined to be the reference position for the needle displacement $x$. When taking blood, one first pushes the needle to the compressed position and locks the needle (represented by the distance $d$ in the graph). The needle is then released so that it can penetrate the skin located at the cover position as shown. You are required to choose the damping coefficient $B$ and the cover position (i.e., the value of $h$ in the graph) such that the system satisfies the following requirements:

(1). The 2% band settling time of the system should be 40ms to have a fast enough response.

(2). The needle should pass the cover position so that it can penetrate the skin.

(3). The needle should not go over the cover position for more than 1mm; this will prevent the need penetrating the skin too much.

To solve this design problem, you need to

A. (5 points) Derive the equations of motion (mathematical model) in terms of $x$.

B. (5 points) Let $z = x - x(0)$ be the displacement relative to the lock position as shown in the figure, where $x(0) = -d$. Rewrite the equation of motion in terms of $z$.

C. (10 points) Obtain $Z(s)$, the Laplace transform of the system response $z$ when the needle is released from the compressed position. Put $Z(s)$ into the following standard form:

$$Z(s) = \frac{b_0}{s^2 + a_1 s + a_0} \cdot \frac{d}{s} \tag{E3.1}$$

where the coefficients $b_0$, $a_1$, and $a_0$ are to be determined from your answer.

D. (10points) Equation (E3.1) indicates that the response $z(t)$ can be thought as the response of a 2$^{nd}$ order system to a step input having a magnitude of $d$ (i.e., $u(t) = d, t \geq 0$). As such, one can obtain $z(t)$ quickly based on the unit step response of the 2$^{nd}$ order system taught in the lectures. For this purpose, put the transfer function of the equivalent 2$^{nd}$ order system into the standard form of

$$G(s) = \frac{K\omega_n^{\ 2}}{s^2 + 2\zeta\omega_n s + \omega_n^{\ 2}}$$

i.e., obtain $\zeta$, $\omega_n$, $K$ based on the coefficients $b_0$, $a_1$, and $a_0$ obtained in (E3.1)

E. (10points) Determine the value of $B$ and the range of values for $h$ that will satisfy the above design requirements.

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**Problem B.1** (cont.)

A. $$\boxed{m\ddot{x} + B\dot{x} + k_s x = 0} \qquad (B.1.1)$$

B.

$$x = z + x(0) = z - d$$

$$\Rightarrow m(\ddot{z}) + B\dot{z} + k_s(z-d) = 0 \Rightarrow \boxed{m\ddot{z} + B\dot{z} + k_s z = k_s d} \qquad (B.1.2)$$

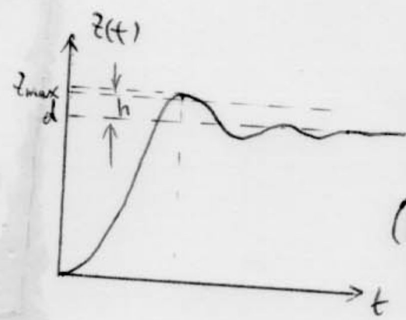
C. As $z(0) = x(0) - x(0) = 0$, the laplace transform of (B.1.2) is:

$$m s^2 Z(s) + B s Z(s) + k_s Z(s) = \frac{k_s d}{s}$$

$$\therefore \boxed{Z(s) = \frac{k_s}{m s^2 + B s + k_s} \cdot \frac{d}{s} = \frac{\frac{k_s}{m} = b_0}{s^2 + \underbrace{\frac{B}{m}}_{a_1} s + \underbrace{\frac{k_s}{m}}_{a_0}} \cdot \frac{d}{s}} \qquad (B.1.3)$$

D.

$$\boxed{\begin{aligned} \omega_n &= \sqrt{a_0} = \sqrt{\frac{k_s}{m}} \\ \zeta &= \frac{a_1}{2\omega_n} = \frac{\frac{B}{m}}{2\sqrt{\frac{k_s}{m}}} = \frac{B}{2\sqrt{m k_s}} \\ K &= \frac{b_0}{\omega_n^2} = 1 \end{aligned}} \quad \therefore$$




(B1.4)

E. (1) $t_s = 0.04$ sec $\Rightarrow \dfrac{4}{\zeta\omega_n} = .04 \Rightarrow \dfrac{4}{\frac{1}{2}\frac{B}{m}} = 0.04$

$$\therefore \boxed{B = \frac{8m}{0.04} = 2} \qquad (B1.5)$$

For $B = 2$, $\zeta = \dfrac{2}{2\sqrt{0.01 \times 2500}} = 0.2$

$$\therefore z_{max} = d(1 + OS\%) = d\left(1 + e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}\right) = 0.0153 \text{ m} = 15.3 \text{ mm}$$

To satisfy (2):

$$h \le z_{max} - d = 5.3 \text{ mm}$$

To satisfy (3):

$$0.001 + h > z_{max} - d \Rightarrow h > 4.3 \text{ mm} \qquad \therefore \boxed{4.3 \le h \le 5.3 \text{ mm}}$$

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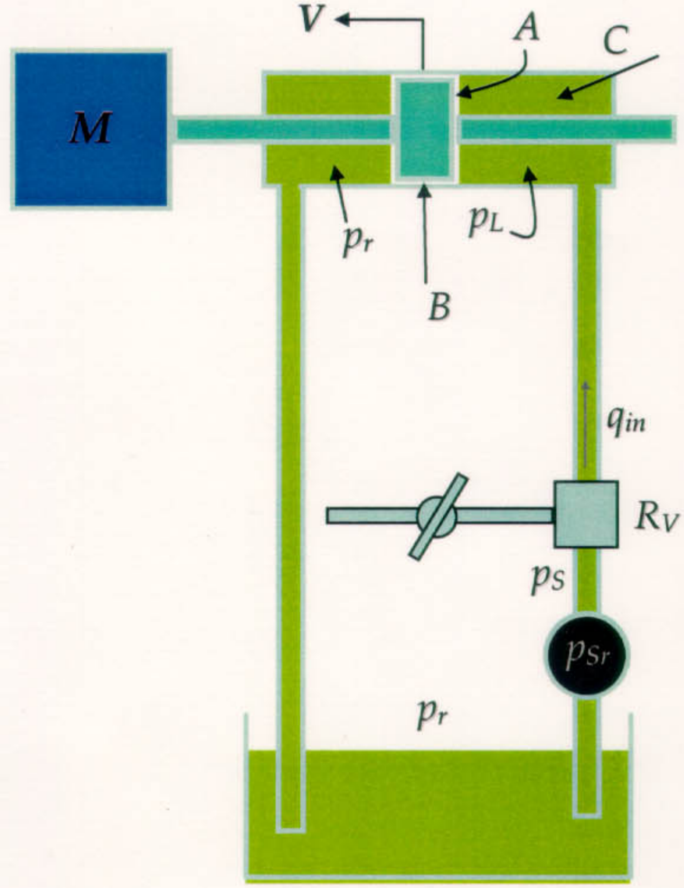
**Problem B.2** (40 points)

Consider the simplified problem of modeling and controlling an inertia load driven by a hydraulic cylinder as shown in Fig. B.2, in which the input of the system is the input flow rate $q_{in}$ and the output is the velocity of the inertia load, $V$. In the figure, $A$ is the cylinder bore area, $C$ is the cylinder chamber capacitance due to the compressibility of the fluid inside the chamber, and $B$ is the vscous friction coefficient between piston head and cylinder wall.



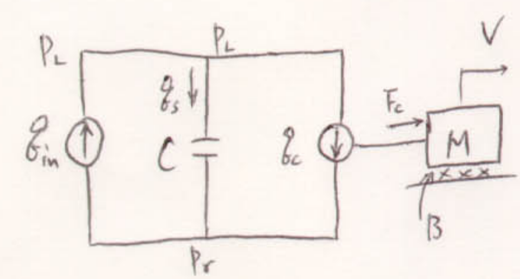

B2.1. (15 points) *Draw the equivalent electrical circuit of the system and write down all elemental and connecting equations that needed to model the system.*

B2.2. (5 points) *Draw the block diagram of the system that clearly shows the functionality of the hydraulic system, the hydraulic/mechanical coupling, and the mechanical system.*

B2.3. (10 points) *Derive the input/output model and the transfer function between* $q_{in}$ *and V.*

B2.4. (10 points) *For the hydraulic arm at Herrick Lab, typical values for the system are* $M = 50kg$, $A = 0.001m^2$, $C = 2.5\times10^{-12}$,. *and* $B = 400$. *Obtain the OS% and 2% settling time of the system to a unit step input*

B2.1.




Hydraulic Subsystem:

$$q_s = C\frac{dP_{Lr}}{dt} \quad \text{or} \quad Q_s(s) = CsP_{Lr}(s)$$

$$q_{in} = q_s + q_c \quad Q_{in}(s) = Q_s(s) + Q_c(s)$$

Coupling:

$$q_c = AV$$

$$F_c = AP_{Lr}$$

Mechanical:

$$M\dot{V} = F_c - BV \quad \text{or} \quad V(s) = \frac{1}{Ms+B}F_c(s)$$

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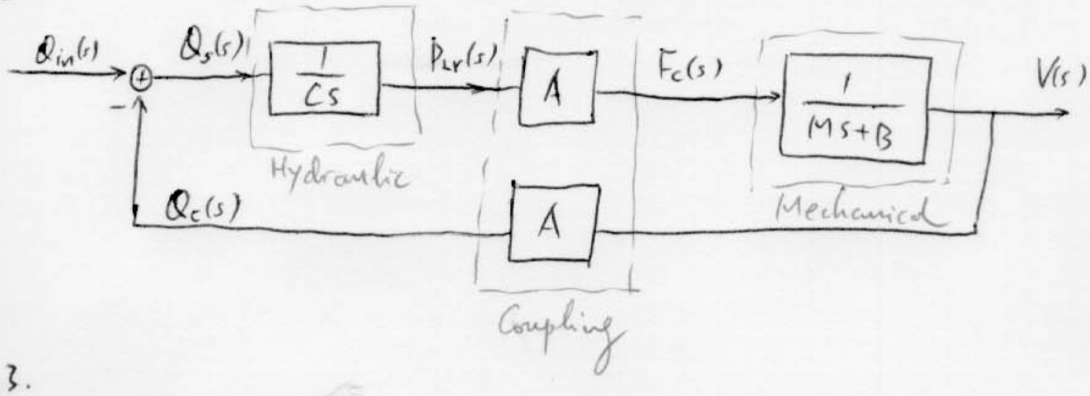
**Problem B2** (cont.)

B2.2




B2.3.

$$V(s) = \frac{1}{Ms+B} \cdot A \cdot \frac{1}{Cs} \left\{ Q_{in}(s) - A\,V(s) \right\}$$

$$\Rightarrow \left\{ 1 + \frac{A^2}{Cs(Ms+B)} \right\} V(s) = \frac{A}{(Ms+B)Cs}\, Q_{in}(s)$$

$$\therefore \boxed{G(s) = \frac{V(s)}{Q_{in}(s)} = \frac{A}{MCs^2 + BCs + A^2} = \frac{A/MC}{s^2 + \frac{B}{M}s + \frac{A^2}{MC}}}$$

I/O Model:

$$\boxed{\ddot{v} + \frac{B}{M}\dot{v} + \frac{A^2}{MC}v = \frac{A}{MC}q_{in}}$$

B2.4.

$$\omega_n = \sqrt{\frac{A^2}{MC}} = \sqrt{\frac{10^{-6}}{50 \times 2.5 \times 10^{-12}}} = 89.4 \text{ rad/sec}$$

$$\zeta = \frac{B/M}{2\omega_n} = 0.045$$

$$\therefore \boxed{\begin{aligned} t_s &= \frac{4}{\zeta\omega_n} = 1 \text{ sec} \\ OS\% &= 100 \times e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}\,\% = 86.8\% \end{aligned}}$$

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**Problem B3** (80 points)

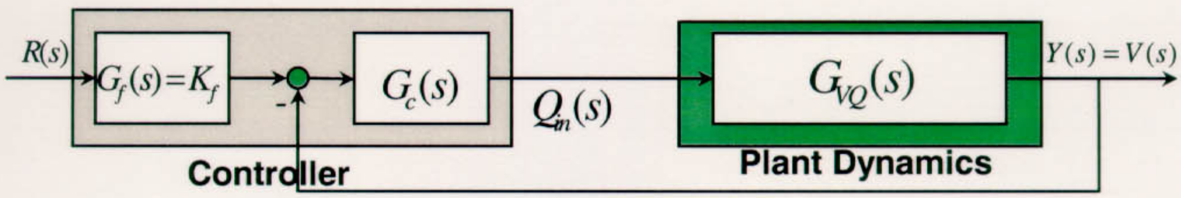
Consider the hydraulic system in Problem B2. If you are doing things correctly, the transfer function of the system with the values of $M = 150kg$, $A = 0.001m^2$, $C = 4\times10^{-12}$ and $B = 1000$ (the loaded situation) would be

$$G_{VQ}(s) = \frac{1.67\times10^6}{s^2 + 6.7s + 1670}$$

As in the Problem B2, such a system has large OS% and settling time. To solve this problem, you are asked to design a controller having the structure shown in Fig.B3.1 to meet the following transient performance requirements:

**[P1].** % overshoot should be less than 10%

**[P2].** 2% settling time should be less than 0. 1 sec




FigB3.1

To solve this problem, you are required to follow the following procedure:

B3.1. (10 points) Obtain the performance region for the closed-loop poles to meet the above transient performance requirements. Draw the performance region on the complex plane shown in Fig.B3.2

B3.2. (10 points) Obtain the closed-loop transfer function from the reference input *R(s)* to *Y(s)*

B3.3. (20 points) Sketch the root locus of the closed-loop system on Fig. B3.2 when a proportional feedback controller $G_c(s) = K_p$ is to be used. Use the sketched root locus to show that any proportional feedback controller will not be able to meet the above transient performance requirements.

B3.4. (20 points) Sketch the root locus of the closed-loop system on Fig. B3.3 when a proportional plus derivative (PD) feedback controller $G_c(s) = K_p + K_d s = K_d(s+40)$, $K_p = 40K_d$ is to be used. To receive full credit, you need to obtain the break-away/in points, the departure angles, and the imaginary axis crossing if necessary and label them clearly in Fig.B3.3. Use the sketched root locus to show that with a proper controller gain $K_d$, the above PD feedback controller will be able to meet the above transient performance requirements.

B3.5. (20 points) Determine controller gains $K_d$ and $K_f$ in B3.3 so that the output of the closed-loop system is able to follow any constant reference input $r(t) = c$ perfectly at the steady-state with ***fastest*** settling time.

B3.1. $OS\% \le 10\% \Rightarrow \boxed{\varphi \le \bar{\varphi} = \tan^{-1}\left(-\frac{\pi}{\ln(0.1)}\right) = 0.938 = 53.8^\circ}$

$t_s \le 0.1 \Rightarrow \frac{4}{\sigma} \le 0.1 \Rightarrow \boxed{\sigma \ge 40}$

B3.2. $Y(s) = G_{VQ}(s)G_c(s)\left(K_f R(s) - Y(s)\right) \Rightarrow Y(s) = \boxed{\underbrace{\frac{G_{VQ}(s)G_c(s)K_f}{1 + G_{VQ}(s)G_c(s)}}_{G_{YR}(s)}} R(s)$

CLCE: $\boxed{1 + G_{VQ}(s)G_c(s) = 0}$

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B3.3

When $G_c(s) = K_p$, the CLCE is:

$$1 + K_p \frac{\overbrace{1.67\times10^6}^{=N(s)}}{\underbrace{s^2+6.7s+1670}_{D(s)}} = 0$$

$\Rightarrow$ No open-loop zeros.

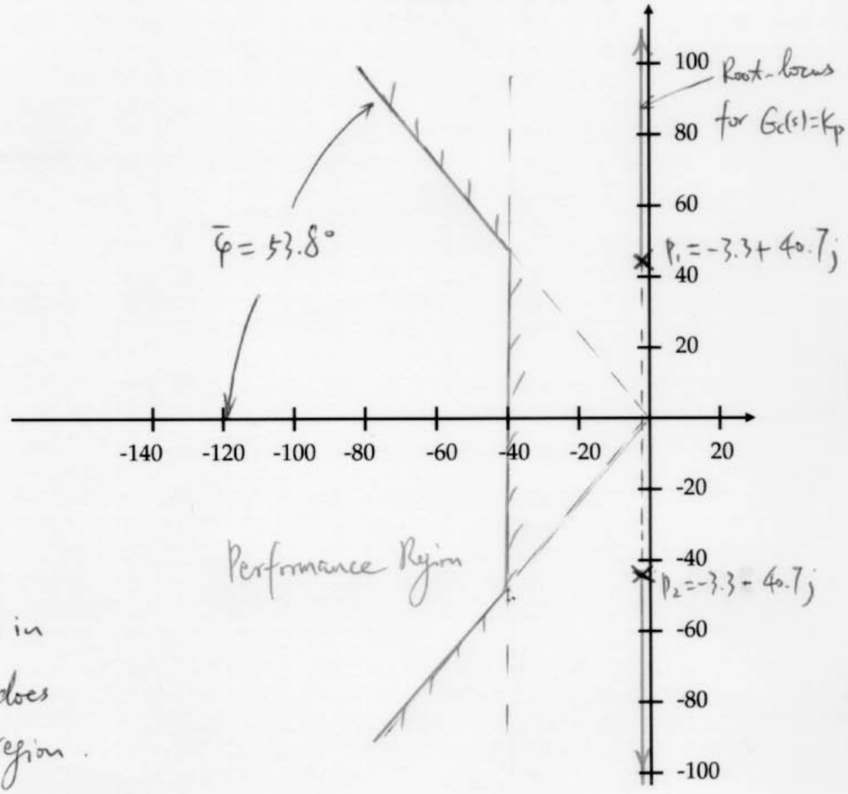
Two O.L. poles: $p_{1,2} = -3.3 \pm 40.7j$

Asymptotes:

$\sigma_0 = -3.3$

$\theta_k = (2k+1)\,90^\circ = \begin{cases} 90^\circ, & k=0 \\ 270^\circ, & k=1 \end{cases}$

The root-locus for $G_c(s)=K_p$ is shown in Fig B3.2. As seen, the root-locus does not intersect the performance region. As such, for any feedback gain $K_p$, the transient performance requirements will not be met.




FigB3.2

B3.4. When $G_c(s) = K_d(s+40)$, the CLCE is:

Step 1: $$1 + K_d \frac{\overbrace{(s+40)\times1.67\times10^6}^{N(s)}}{\underbrace{s^2+6.7s+1670}_{D(s)}} = 0$$

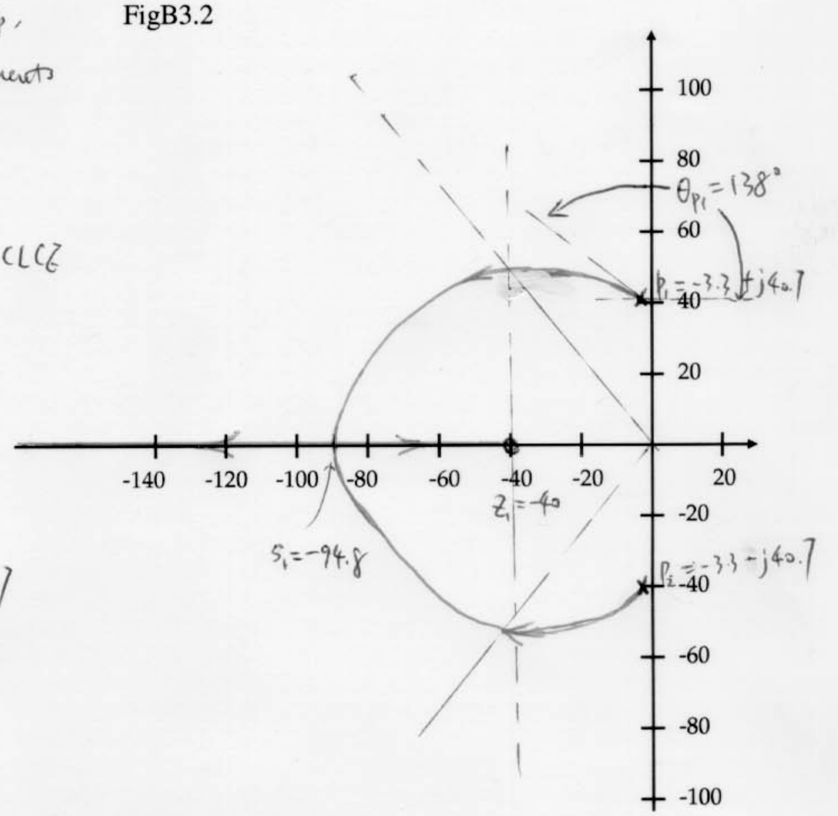
Step 2 $\Rightarrow$ One O.L. zero: $z_1 = -40$

Two O.L. poles: $p_{1,2} = -3.3 \pm j40.7$

Step 3: Real-axis segment

$(-\infty,\ z_1)$

Step 4:




FigB3.3

Name: ______________________________

**Problem B3** (cont.)

Step 5 :
$$\frac{d}{ds}\left(\frac{N(s)}{D(s)}\right) = \frac{N'(s)D(s) - N(s)D'(s)}{D^2(s)}$$

$$= \frac{1.67\times10^6(s^2+6.7s+1670) - 1.67\times10^6(s+40)(2s+6.7)}{(s^2+6.7s+1670)^2} = 0$$

$$\Rightarrow \quad -s^2 - 80s + 1402 = 0$$

$$\Rightarrow \quad \boxed{s_1 = -94.8,} \quad s_2 = 14.8$$

Step 6. Departure angle from $p_1$ :

$$\angle(s-z_1) - \angle(s-p_1) - \angle(s-p_2) = 180^\circ$$

$$\angle(p_1-z_1) - \theta_{p_1} - \angle(p_1-p_2) = 180^\circ$$

$$\angle(+36.7+j40.7) - \theta_{p_1} - 90^\circ = 180^\circ \Rightarrow \boxed{\theta_{p_1} = -270^\circ + 48^\circ = -222^\circ = 138^\circ}$$

With the above information, the root-locus can be sketched as in Fig B3.3. It is seen that for large enough gain $K_d$, the two cl poles will be within the performance region, and thus transient performance can be met.

B3.5. To follow any constant reference input perfectly at the steady-state, it is required that

$$\boxed{G_{YR}(0) = 1}$$

As

$$G_{YR}(s) = \frac{\frac{1.67\times10^6}{s^2+6.7s+1670}\cdot K_d(s+40)K_f}{1 + K_d(s+40)\frac{1.67\times10^6}{s^2+6.7s+1670}} = \frac{1.67\times10^6 K_d(s+40)K_f}{s^2+(6.7+1.67\times10^6K_d)s + 6.68\times10^7K_d + 1670} \quad (B3.5.1)\ (B3.5.2)$$

(B3.5.1) implies that

$$\frac{1.67\times10^6 K_d K_f \times 40}{1670 + 6.68\times10^7 K_d} = 1 \Rightarrow K_f = \frac{1670 + 6.68\times10^7 K_d}{6.68\times10^7 \times K_d} \quad (B3.5.3)$$

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**Problem B3** (cont.)

As seen from Fig B3.3, the closed-loop system has the fastest $\overset{\text{settling}}{\text{time}}$ when the two CL poles are at

$$p_{CL1,2} = -94.8$$

$\Rightarrow$ desired CLCE:

$$(s - p_{CL1})(s - p_{CL2}) = (s+94.8)(s+94.8) = s^2 + 189.6s + 8987 \quad (B3.5.4)$$

Compare (B3.5.4) with (B3.5.2):

$$\begin{cases} 6.7 + 1.67\times10^6 K_d = 189.6 \\ 6.68\times10^7 K_d + 1670 = 8987 \end{cases} \Rightarrow \boxed{K_d \approx 1.096\times10^{-4}}$$

From (B3.5.3):

$$\boxed{K_f = 1.227}$$