ME 375 FINAL EXAM
Wednesday, May 6, 2009

Division Meckl 10:30 / Adams 1:30 (circle one) **Name** Solution (Orig.)

Instructions

(1) This is a closed book examination, but you are allowed three single-sided 8.5"×11" crib sheets. A calculator is NOT allowed.

(2) You have two hours to work all five problems on the exam.

(3) Use the solution procedure we have discussed: what are you given, what are you asked to find, what are your assumptions, what is your solution, does your solution make sense. You must show all of your work to receive any credit.

(4) Circle or box-in your answers.

(5) You must write neatly and should use a logical format to solve the problems. You are encouraged to really "think" about the problems before you start to solve them. Please write your name in the top right-hand corner of each page.

(6) A table of Laplace transform pairs and properties of Laplace transforms is attached at the end of this exam set.

Problem No. 1 (20)	______________
Problem No. 2 (15)	______________
Problem No. 3 (15)	______________
Problem No. 4 (30)	______________
Problem No. 5 (20)	______________
TOTAL (*/100)**	______________

PROBLEM 1: (20%)

For the following transfer functions:

(i) determine the poles

(ii) compute time constant, natural frequency and/or damping ratio

(iii) indicate which time response (1, 2, 3, or 4) below represents the unit step response of each transfer function.

(a) $\dfrac{25}{s^2+6s+25}$

$s = -3 \pm j4$ (3)

$\omega_n = 5$ r/s

$2\zeta\omega_n = 6 \Rightarrow \zeta = 0.6$

(b) $\dfrac{3}{s+3}$

$s = -3$ (1)

$\tau = 1/3$ sec.

(c) $\dfrac{10}{s^2+11s+10}$

$s = -1, -10$ (4)

$\tau = 1$ sec

$\tau = 1/10$ sec

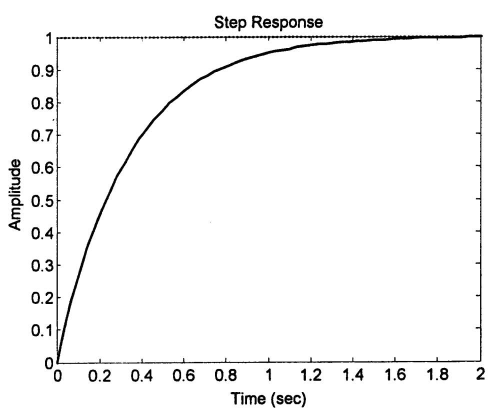
(d) $\dfrac{25}{s^2+3s+25}$

$s = -\frac{3}{2} \pm j\frac{1}{2}\sqrt{91}$ (2)

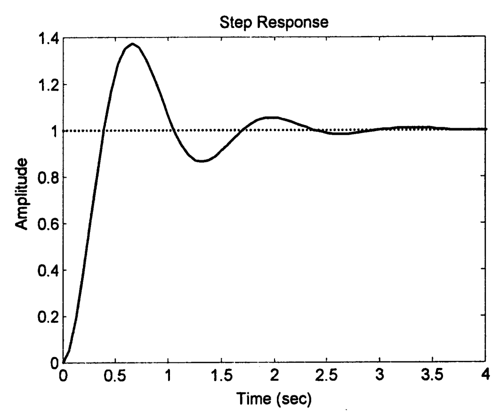
$\omega_n = 5$ r/s

$2\zeta\omega_n = 3 \Rightarrow \zeta = 0.3$

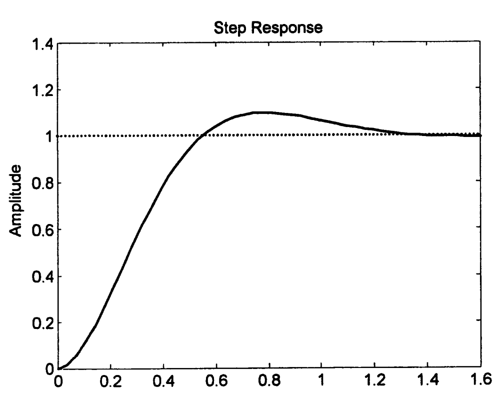
(1)



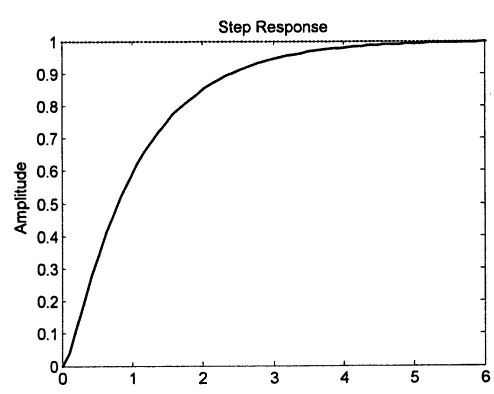
(2)



(3)

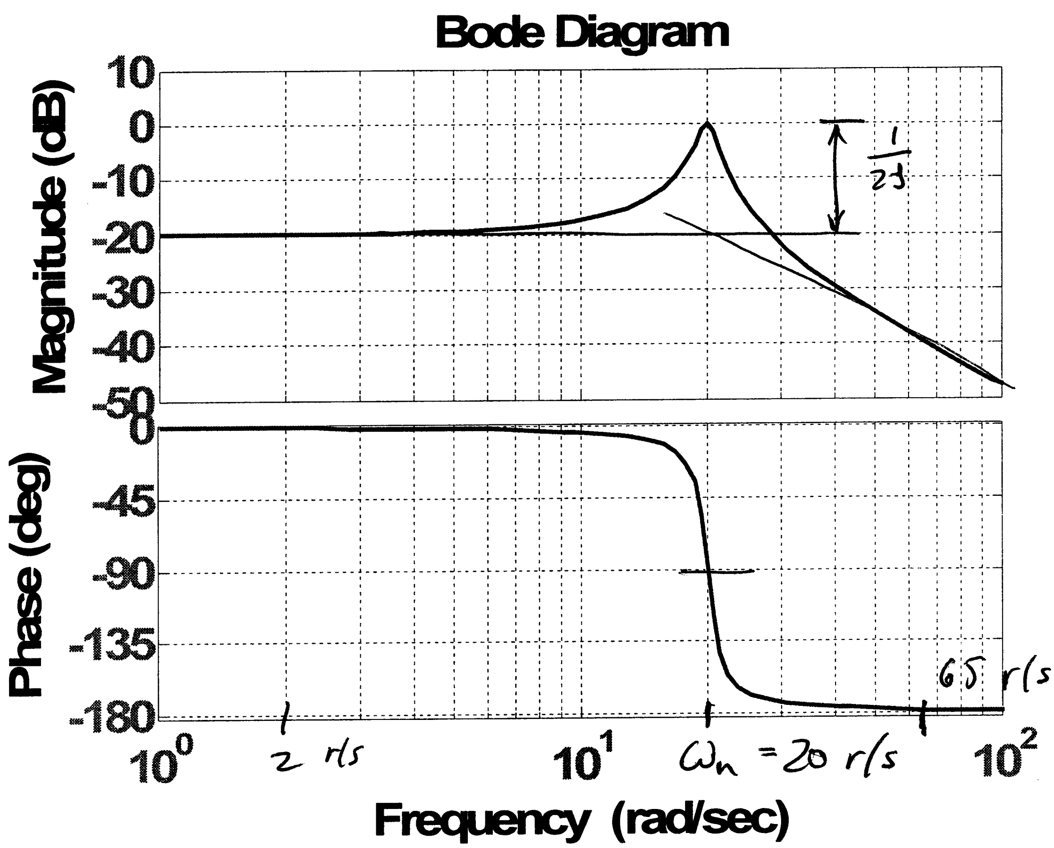


(4)



PROBLEM 2: (15%)

The frequency response for a flexible mechanical system is given in the Bode plots below:



(a) Determine the natural frequency, damping ratio, and static gain for this system.

(b) Generate an expression for the transfer function $G(s)$ for this system.

(c) Find the magnitude and phase of $G(j\omega)$ at $\omega = 2$ and 65 rad/sec.

(d) Find the steady-state output $y_{ss}(t)$ of the system to an input of $u(t) = 3 + \sin(2t) + 2\sin(65t)$.

(a) $\omega_n = 20$ r/s , $\frac{1}{2\zeta} = \left(\frac{1}{10}\right)^{-1} \Rightarrow \zeta = 0.05$, $K_s = \frac{1}{10} = 0.1$

(b) $G(s) = \frac{K_s \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{0.1(20)^2}{s^2 + 2s + 400} = \frac{40}{s^2 + 2s + 400}$

(c) $|G(j2)| = 0.1$, $\angle G(j2) = 0°$

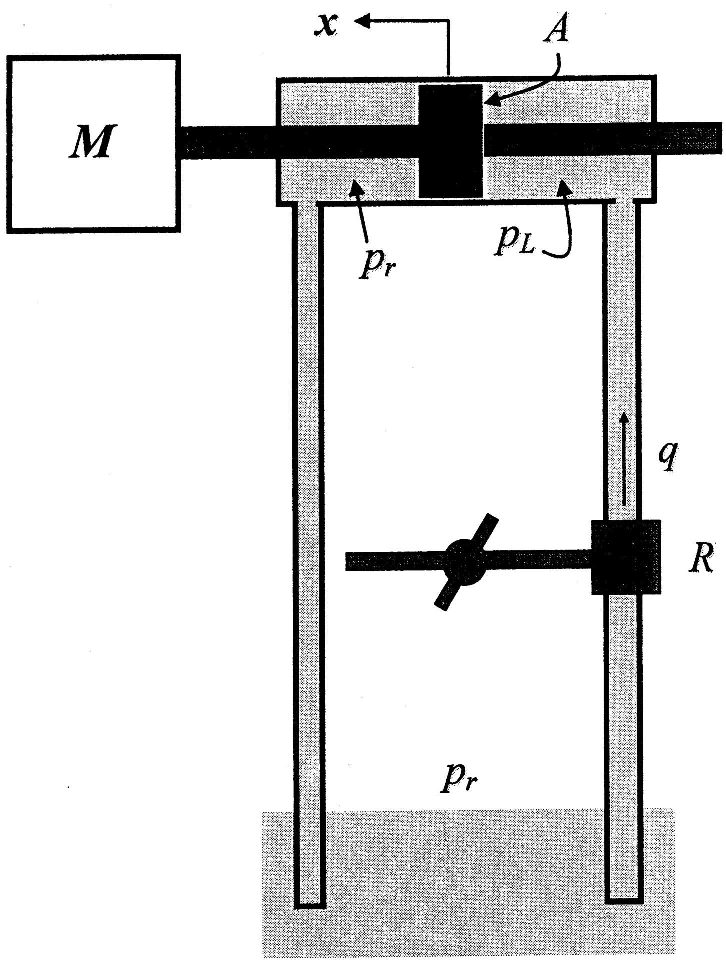
$|G(j65)| = 0.01$, $\angle G(j65) = -180°$

PROBLEM 2: (cont)

(d) $y_{ss}(t) = 3\left(\frac{1}{10}\right) + \frac{1}{10}\sin\left(2t - 0°\right) + 2\frac{1}{100}\sin\left(65t - 180°\right)$

PROBLEM 3: (15%)

A shock absorber (with attached mass *M*) can be represented as in the schematic diagram below:

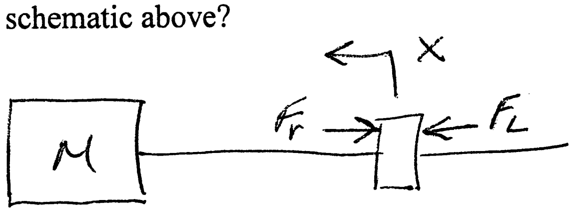


Assume there is no leakage of fluid between the piston and cylinder, and also no friction damping between piston and cylinder. Also assume that the hydraulic fluid is incompressible.

(a) Derive a differential equation that describes the free response of the piston displacement *x*.

(b) What is the equivalent damping constant *b* for this shock absorber in terms of the parameters given in the schematic above?

(a)



$$M\ddot{x} = F_L - F_r = (p_L - p_r)A$$

$$\left.\begin{aligned} p_r - p_L &= Rq \\ q &= A\dot{x} \end{aligned}\right\} \quad p_r - p_L = RA\dot{x}$$

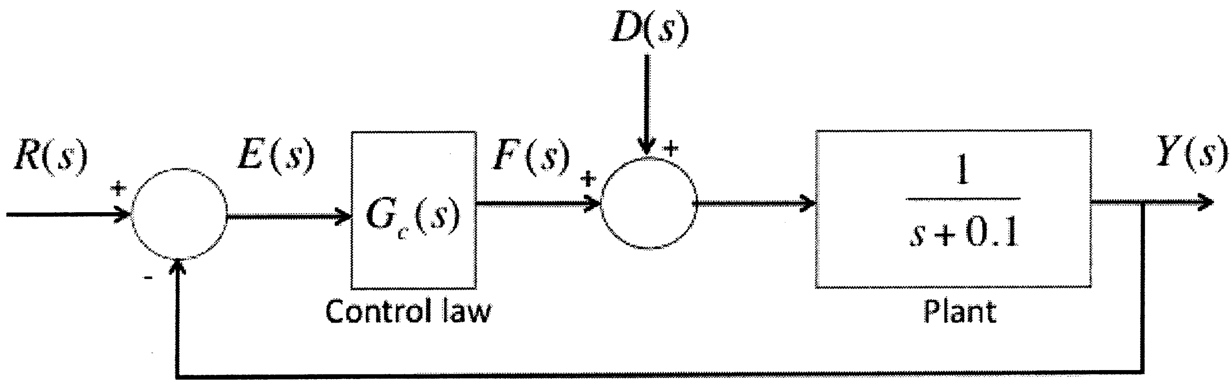
PROBLEM 3: (cont)

$$M\ddot{x} = -RA^2\dot{x} \quad \Rightarrow \quad \boxed{M\ddot{x} + RA^2\dot{x} = 0}$$

(b) $\boxed{b = RA^2}$

PROBLEM 4: (30%)

Consider the feedback control system represented below in block diagram form. The reference input R(s), system output Y(s), and disturbance D(s) are denoted in the figure along with the error E(s) and control effort F(s). You will design the control law $G_c(s)$ to achieve certain performance criteria.



Answer the following questions (assume D(s)=0 in all parts except part (i)):

(a) Show that the transfer function relating the reference R(s) to the output Y(s) is given by:

$$\frac{Y(s)}{R(s)} = \frac{G_c(s)}{s+0.1+G_c(s)}$$

(b) Assuming a proportional control law, $G_c(s) = K_p$, choose K_p to limit the steady state error between the reference and the output to 5% for a unit step reference.

(c) For the proportional control law, $G_c(s) = K_p$, choose K_p so that 63% of the steady state response for a unit step input is reached ~~within~~ in exactly 1 second.

(d) Explain why you cannot achieve both criteria listed in (b) and (c) using a proportional control law. What control law would you use (and why) to achieve both of these criteria?

(e) For an integral control law, $G_c(s) = K_I/s$, find the steady state error between the reference and the output for a unit step reference.

(f) Choose K_I in the integral control law from part (e) to limit the peak overshoot to 10%.

(g) Assuming a proportional-integral control law, $G_c(s) = K_P + K_I/s$, choose K_I and K_P to simultaneously limit the peak overshoot to 10% and settling time (for response to be within 2% of the final value) to 1 second.

(h) Show that the transfer function relating the disturbance D(s) to the error E(s) is given by,

$$\frac{E(s)}{D(s)} = \frac{-1}{s+0.1+G_c(s)}$$

(i) For a unit step disturbance, calculate the <u>total</u> steady state error in the output Y(s) assuming you use the proportional control design from part (b).

PROBLEM 4: (cont)

(a) $$\frac{Y(s)}{R(s)} = \frac{\frac{G_c(s)}{s+0.1}}{1+\frac{G_c(s)}{s+0.1}} = \frac{G_c(s)}{s+0.1+G_c(s)}$$

(b) $$\frac{Y(s)}{R(s)} = \frac{K_p}{s+0.1+K_p}$$

at steady-state : $\frac{Y(0)}{R(0)} = \frac{K_p}{0.1+K_p}$

for 5% error : $0.95 \le \frac{K_p}{0.1+K_p} \le 1.05$

$0.95(0.1+K_p) \le K_p \Rightarrow 0.095 \le 0.05K_p \Rightarrow \boxed{K_p \ge 1.9}$

(c) $$\frac{Y(s)}{R(s)} = \frac{K_p}{0.1+K_p}\,\frac{1}{\frac{s}{0.1+K_p}+1} \Rightarrow \tau = \frac{1}{0.1+K_p}$$

for $\tau = 1 \Rightarrow 0.1+K_p = 1 \Rightarrow \boxed{K_p = 0.9}$

(d) A single value of K_p won't satisfy both criteria. Use PI control.

(e) $$\frac{Y(s)}{R(s)} = \frac{K_I/s}{s+0.1+K_I/s} = \frac{K_I}{s(s+0.1)+K_I}$$

at steady-state : $\frac{Y(0)}{R(0)} = 1 \Rightarrow \boxed{e_{ss} = 0}$

(f) char. polynomial: $s^2 + \underbrace{0.1}_{2\zeta\omega_n}s + \underbrace{K_I}_{\omega_n^2} = 0$

$2\zeta\omega_n = 0.1 \Rightarrow 2\zeta\sqrt{K_I} = 0.1 \Rightarrow \boxed{K_I = \left(\frac{1}{20\zeta}\right)^2}$

where $\zeta = \frac{\ln 0.1}{\sqrt{\pi^2+(\ln 0.1)^2}}$

(g) $G_c(s) = K_p + K_I/s$

$$\frac{Y(s)}{R(s)} = \frac{K_p + K_I/s}{s + 0.1 + K_p + K_I/s} = \frac{K_p s + K_I}{s(s+0.1+K_p) + K_I}$$

char. eq: $s^2 + \underbrace{(0.1+K_p)}_{2\zeta\omega_n} s + \underbrace{K_I}_{\omega_n^2} = 0$

for settling time spec: $\frac{4}{\zeta\omega_n} \le 1 \Rightarrow \zeta\omega_n \ge 4$

$$0.1 + K_p = 2(4) = 8 \Rightarrow \boxed{K_p = 7.9}$$

$\zeta\omega_n = 4 \Rightarrow \omega_n = \frac{4}{\zeta}$ where ζ is given in part (f)

$$\Rightarrow \boxed{K_I = \left(\frac{4}{\zeta}\right)^2}$$

(h) let $R(s) = 0$:

$Y(s) = \frac{1}{s+0.1}(D(s) + F(s))$; $F(s) = G_c(s)E(s)$

$$E(s) = \cancel{R(s)}^{\,0} - Y(s) = -Y(s)$$

$$Y(s) = \frac{1}{s+0.1}(D(s) + G_c(s)E(s))$$

$$Y(s) = \frac{1}{s+0.1}[D(s) - G_c(s)Y(s)]$$

$$\left[1 + \frac{G_c(s)}{s+0.1}\right] Y(s) = \frac{1}{s+0.1} D(s)$$

$$\frac{Y(s)}{D(s)} = \frac{\frac{1}{s+0.1}}{1 + \frac{G_c(s)}{s+0.1}} = \frac{1}{s+0.1+G_c(s)} \Rightarrow \boxed{\frac{E(s)}{D(s)} = \frac{-1}{s+0.1+G_c(s)}}$$

(i) $E = R - Y = R\left(1 - \frac{Y}{R}\right) = \left(1 - \frac{K_p}{s+0.1+K_p}\right)R$ for $D = 0$

$= \left(\frac{s+0.1}{s+0.1+K_p}\right)R$

$E = \frac{-1}{s+0.1+K_p} D$ for $R = 0$ $\Rightarrow \boxed{e_{ss} = \frac{0.1}{0.1+K_p} - \frac{1}{0.1+K_p}}$ $\boxed{e_{ss} = -\frac{1.1}{2}}$

PROBLEM 5: (20%)

Consider the feedback control system from PROBLEM 4 to answer the following questions about root locus.

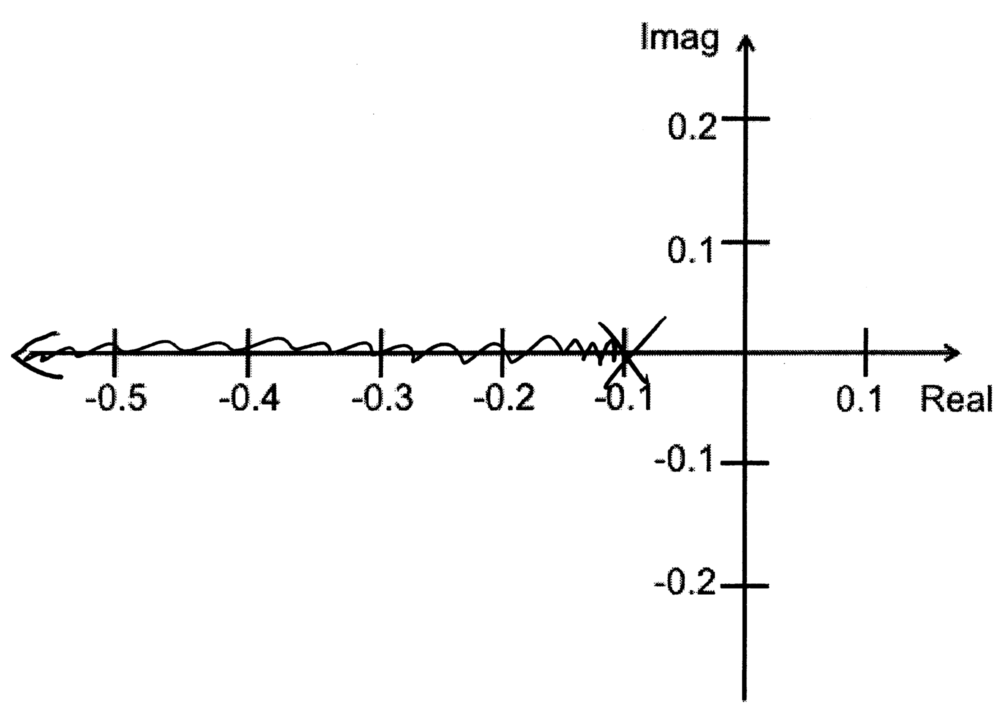
(a) For a proportional control law, $G_c(s) = K_p$

- draw the root locus for the closed loop characteristic equation as K_p increases;
- calculate where the root locus starts for $K_p = 0$ and indicate this on the diagram with an "X";
- describe what is happening to the transient response of the feedback control system as K_p increases.

char eq: $s + 0.1 + K_p = 0$

$s = -0.1 - K_p$

$s = -0.1$ when $K_p = 0$



as K_p increases, settling time decreases (system gets faster)

(b) For an integral control law, $G_c(s) = K_I/s$

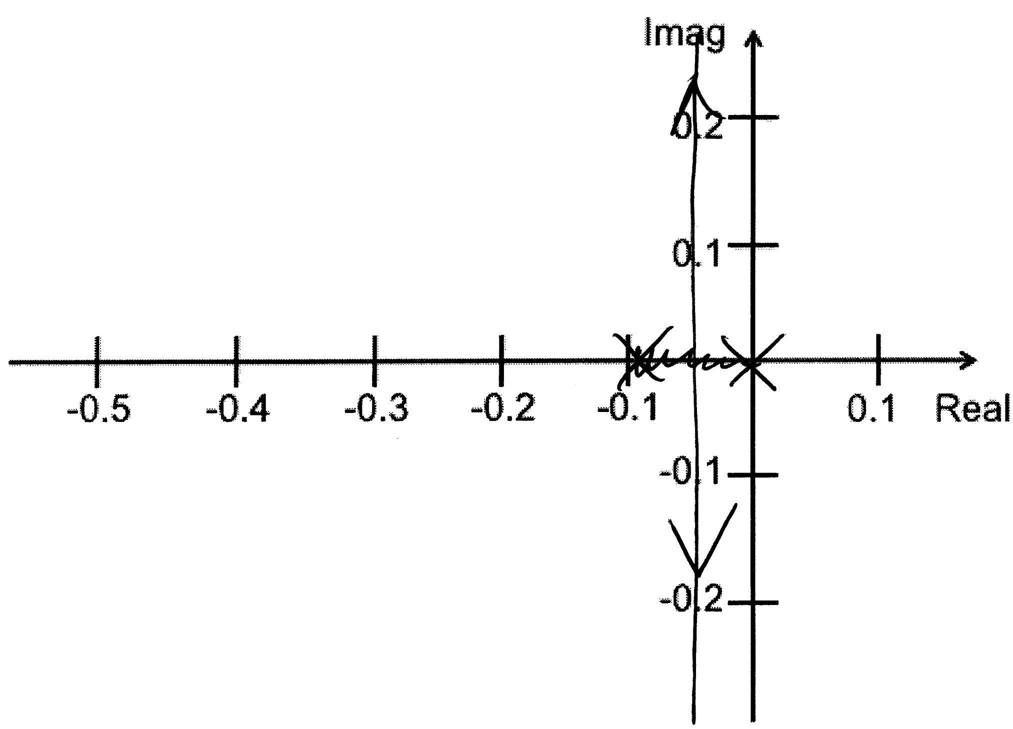
- draw the root locus for the closed loop characteristic equation as K_I increases;
- calculate where the root locus starts for $K_I = 0$ and indicate this on the diagram with an "X";
- describe what is happening to the transient response of the feedback control system as K_I increases.

char eq: $s^2 + 0.1s + K_I = 0$

$$s = -\frac{0.1}{2} \pm \frac{1}{2}\sqrt{0.01 - 4K_I}$$

for $K_I = 0$:

$$s = -\frac{0.1}{2} \pm \frac{1}{2} 0.1 = \begin{cases} -0.1 \\ 0 \end{cases}$$



as K_I increases, roots become complex, system becomes oscillatory, and damping decreases.