ME 375 X

Exam II

Measurement and Control Systems II

Friday April 3, 2015

Name Solution

Instructions:

(1) This is a closed book examination, but you are allowed one 8 ½ X11 one sided crib sheet.
(2) You have 50 minutes to work all three problems on the exam.
(3) You must show all your work to receive any credit.
(4) You must write neatly and should use a logical format to solve the problems. You are encouraged to really "think" about the problems before you start to solve them. Please write your name in the top right-hand corner of each page.
(5) A table of Laplace transform pairs and properties of Laplace transforms is attached at the end of this exam set.

Problem 1(33%) ____________________

Problem 2(33%) ____________________

Problem 3(34%) ____________________

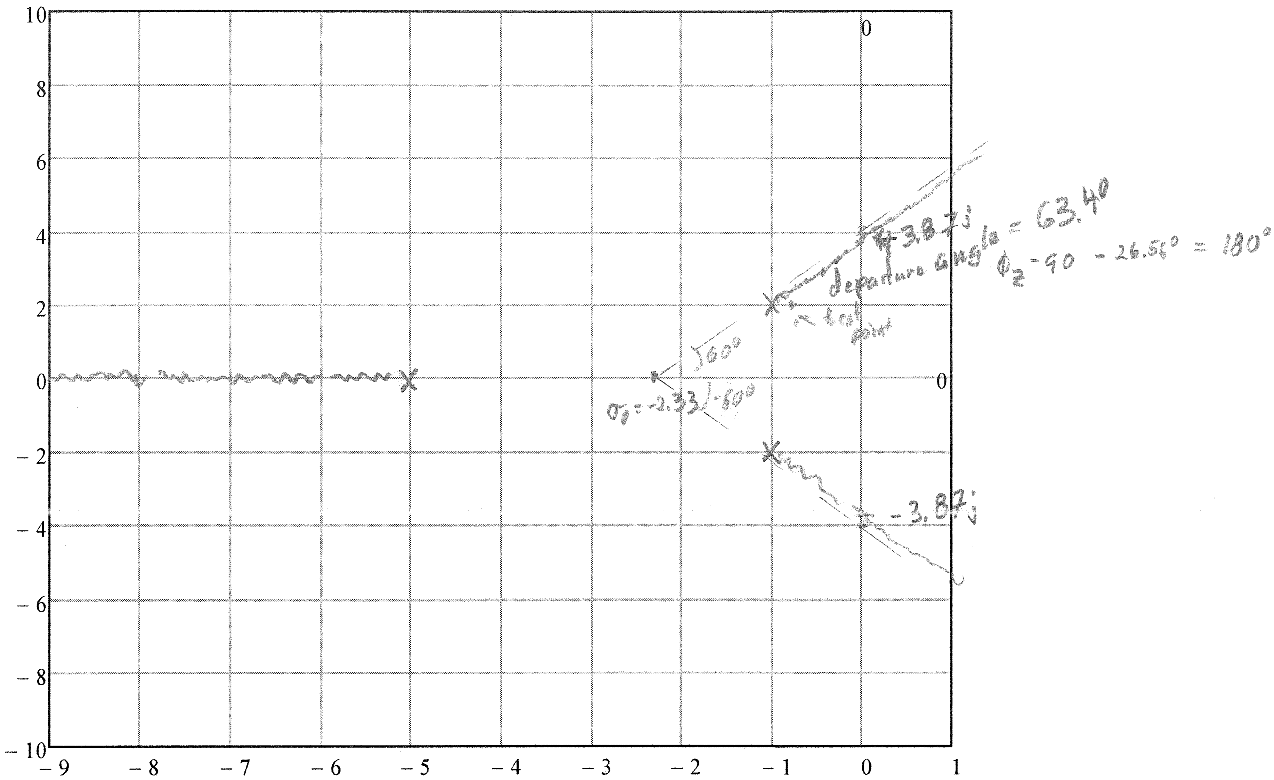
Name________________________________

Problem 1: (33%)

On the grid provided sketch the root locus of the transfer function:

$$G(s) = K\frac{25}{(s+5)(s^2+2s+5)} \quad s = -1 \pm 2j$$

$s^3 + 7s^2 + 15s + 25$



Answer the following (to aid with your sketch and for partial credit if needed.)

If certain questions do not apply to this root locus sketch just put NA (not applicable)

1). Finite open loop poles (number and locations)

3 poles at: $s = -5$, $s = -1 + 2j$, $s = -1 - 2j$

2). Finite open loop zeros (number and locations)

No finite zero

Name______________________________

Problem 1: (cont')

3). Number of branches

3 branches

4). Asymptote intersection with real axis

$$\sigma = \frac{-5 \; -1+2j \; -1-2j \; -0}{3-0} = \frac{-7}{3} \approx -2.33\overline{3}$$

5). Asymptote angles in the complex plane

$$\theta_k = \frac{(2K+1)\,180^\circ}{3-0} = +60^\circ,\ 180^\circ\ -60^\circ$$

6). Real axis Break-in or Break-out locations

NA

7). Angles of departure or arrival from complex poles or zeros

$$-\phi_z - 90 - 25.56 = 180$$

$$\phi_z = 63.4^\circ$$

8). Imaginary axis crossings (value of ω, and K)

$$T(s) = \frac{25K / s^3 + 7s^2 + 15s + 25}{1 + \frac{25K}{s^3+7s^2+15s+25}}$$

$$CE = s^3 + 7s^2 + 15s + 25 + 25K$$

$$(25 + 25K - 7\omega^2) + j(15\omega - \omega^3) = 0$$

$$\omega^2 = 15 \rightarrow \boxed{\omega = \sqrt{15} = 3.87}$$

$$25 + 25K - 7 \times 15 = 0$$

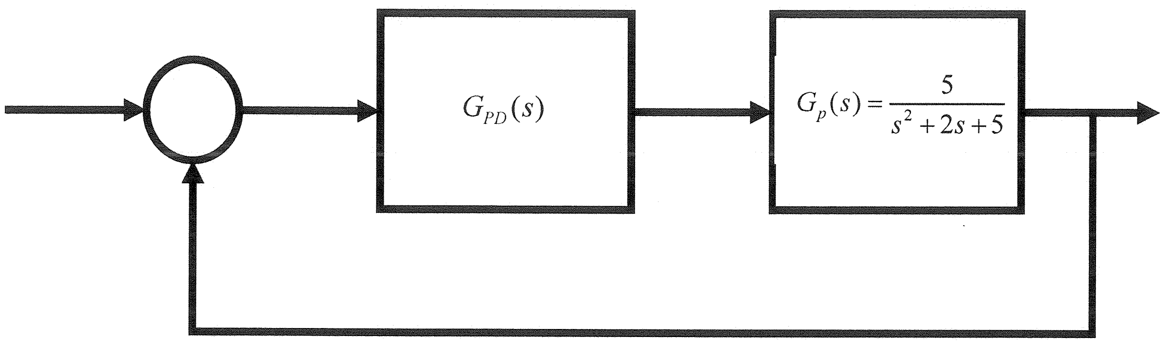
$$25K = 105 - 25$$

$$\boxed{K = 80/25 = 3.2}$$

Name________________________________

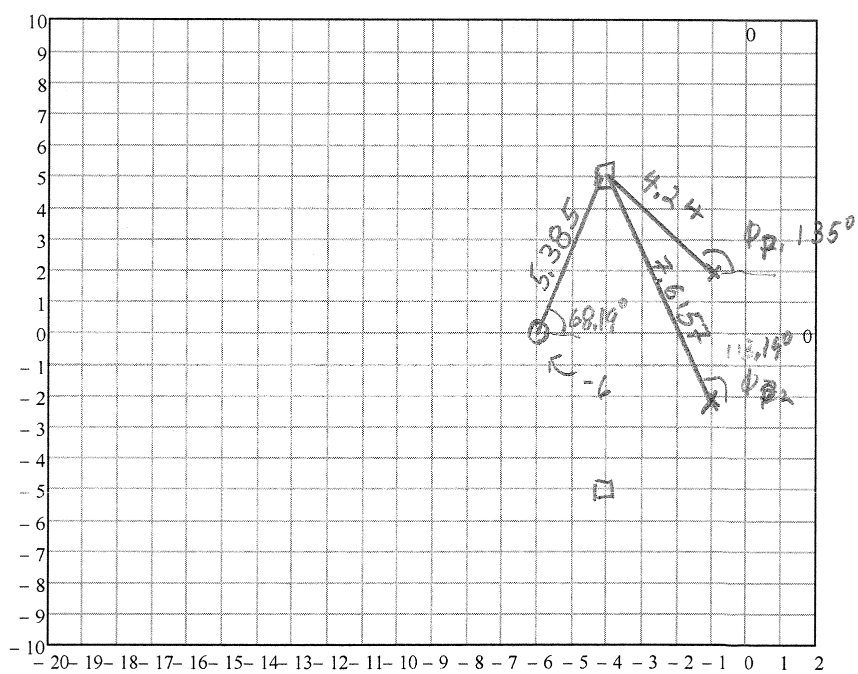
Problem 2:(33%)

For the control system shown in the figure below,



Design a PD (proportional derivative) controller to place the closed loop poles at $s = -4 \pm 5j$ (Calculate K and zero location.) Use the following sketch area to help you visualize your design.

Mark the open-loop poles and zeros, the desired closed-loop pole locations, plot and label line segments from the open loop poles and zeros to the operating points (label angles and line segment lengths)



Name______________________________

Problem 2:(cont')

$\phi_1 = 135^\circ$ $\phi_2 = 113.19^\circ$

4.2426 7.6157

$\phi_z - 135^\circ - 113.19^\circ = -180^\circ$

$\phi_z = 68.19$

$\tan 68.19 = \frac{5}{x}$ $x = \frac{5}{\tan 68.19} = 2.000$

$z = -(2+4) = -6$

5.385

$5K = \frac{(4.2426)(7.6157)}{(5.385)} = 6.000$

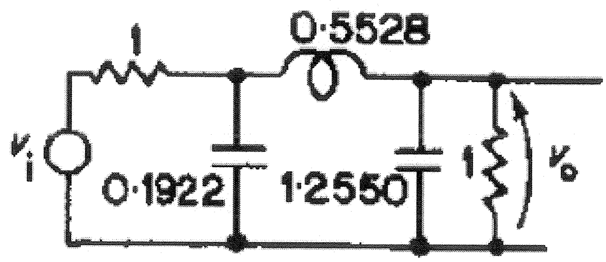
$K = \frac{6}{5} = 1.2$

$\boxed{G_c = 1.2(s+6)}$

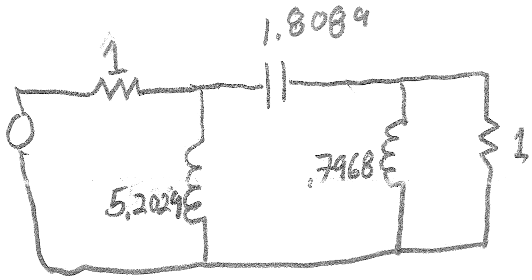
Name______________________________

Problem 3:(34%)

Design a 3rd order high pass Thompson filter to have a 600 Ω impedance and a break frequency of 10 kHz starting with the following low pass filter network having an impedance of 1 Ω and break frequency of 1 r/s .



1). Convert the low-pass network to a high-pass network with an impedance of 1 Ω and break frequency of 1 r/s. (Draw the new network and label values.)

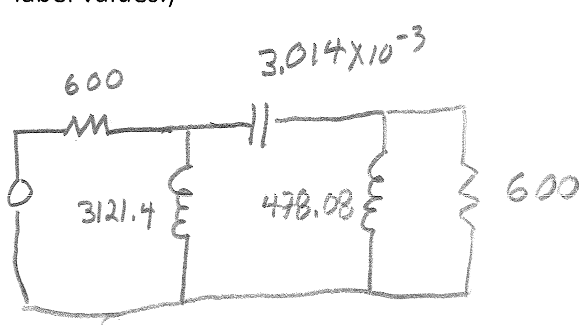


$\frac{1}{.1922} = 5.2029$

$\frac{1}{.5528} = 1.8089$

$\frac{1}{1.255} = .7968$

2). Scale the network impedance in part 1). From 1 Ω to 600 Ω. (Draw the new network and label values.)



$1 \times 600 = 600$

$\frac{1.8089}{600} = 3.014 \times 10^{-3}$

$5.2029 \times 600 = 3121.4$

$.7968 \times 600 = 478.08$

Name________________________________

Problem 3:(cont')

3). Scale the network in part 2). From a frequency of 1 r/s to 10 kHz. (Draw the new network and label the values.)

$$\frac{3121.4}{2\times10^4\times\pi} = 4.9678\times10^{-2}$$

$$\frac{3.014\times10^{-3}}{2\times10^4\times\pi} = 4.7969\times10^{-8}$$

$$\frac{478.08}{2\times10^4\times\pi} = 7.6088\times10^{-3}$$