ME 375 X

Final Exam

Measurement and Control Systems II

Wednesday May 6, 2015

Name Solution

Instructions:

(1) This is a closed book examination, but you are allowed three 8 ½ X11 one sided crib sheets.
(2) You have 120 minutes to work all four problems on the exam.
(3) You must show all your work to receive any credit.
(4) You must write neatly and should use a logical format to solve the problems. You are encouraged to really "think" about the problems before you start to solve them. Please write your name in the top right-hand corner of each page.
(5) A table of Laplace transform pairs and properties of Laplace transforms is attached at the end of this exam set.

Problem 1: (25%) ______________

Problem 2: (25%) ______________

Problem 3: (25%) ______________

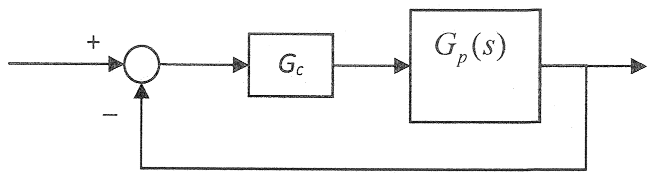
Problem 4: (25%) ______________

Total: ______________

Name Solution

Problem 1: (25%)

For the system shown in the figure, design a compensator using **Direct Pole Placement.**



Where $G_p(s) = \frac{4}{(s+1)(s+2)}$, and the form of the controller is $C(s) = \frac{as+b}{cs+d}$. The close-loop pole of the system should be located at : $s = -10, \quad s = -3 \pm 4j$.

(a) Determine the desired closed-loop characteristic polynomial.

$(s^2 + 6s + 25)(s+10) = \boxed{s^3 + 16s^2 + 85s + 250}$

(b) Determine the characteristic equation of the closed-loop system.

$(cs+d)(s+1)(s+2) + 4(as+b) = cs^3 + (3c+d)s^2 + (2c+3d+4a)s + 4b+2d$

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Problem 1: (cont')

(c) Equate like powers of 's' between the polynomials in parts (a) and (b) to determine the values of the coefficients a, b, c, and d and fill the those values in the blanks below.

a = ___11___, b = ___56___, c = ___1___, d = ___13___.

$c = 1$

$3(1) + d = 16 \Rightarrow d = 16 - 3 = 13$

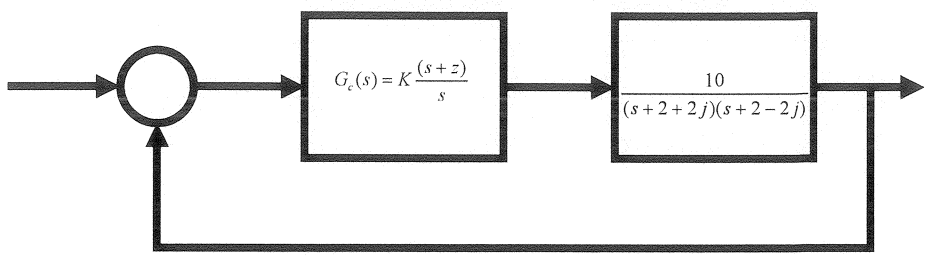
$2(1) + 3(13) + 4a = 85$

$2 + 39 + 4a = 85 \Rightarrow a = \frac{44}{4} = 11$

$4b + 2(13) = 250 \Rightarrow b = \frac{224}{4} = 56$

Problem 2: (25%)

A compensator $G_c(s)$ is to be designed for the following system:

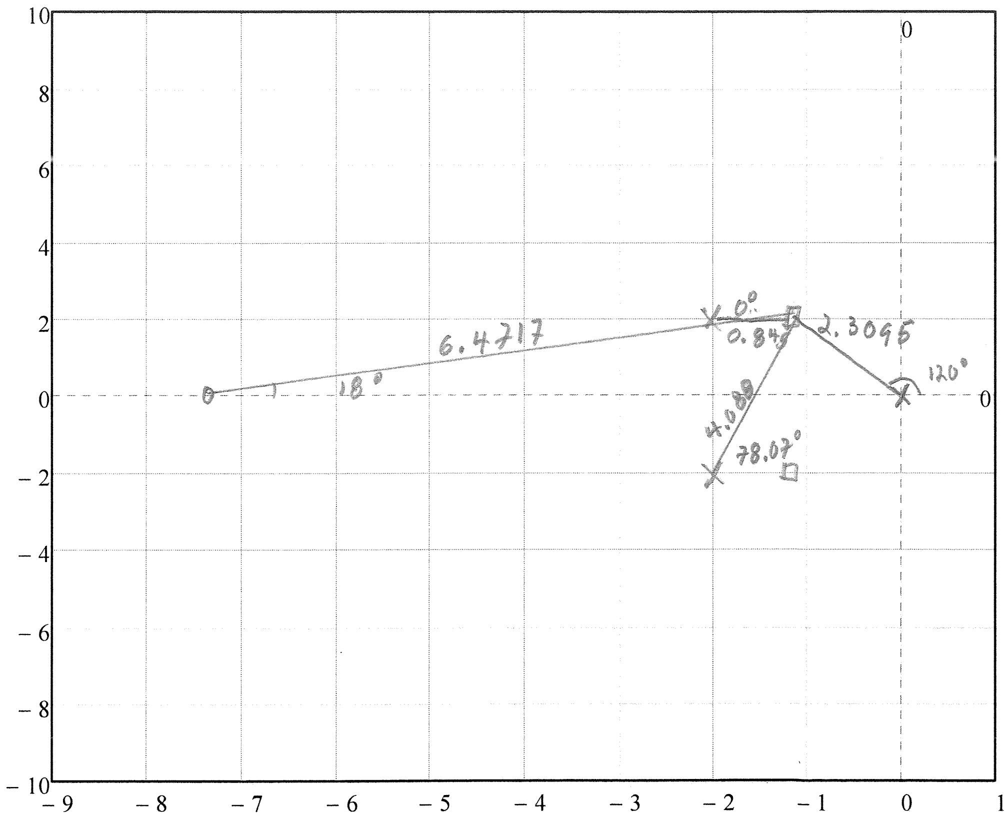


Using **Root Locus Techniques**, design a PI controller ($G_c(s) = K\frac{(s+z)}{s}$) to meet the following design specifications

(i) Zero steady-state error to unit step reference input

(ii) Damping ratio of the desired roots is 0.5

(iii) Damped natural frequency of the desired roots is 2 rad/sec.



Use the area above to show the open – loop poles and zeros, the desired closed – loop poles, the angles for the open loop poles and zero to the desired closed –loop poles and the line segment lengths from the open loop poles and zeros to the closed – loop poles.

Name________________________________

(a) Show or explain why the steady – state error requirement is satisfied.

The integrator in the PI controller will make the system type 1

$\Rightarrow$ error to unit step is zero

(b) Determine the location of the desired closed – loop pole(s) and plot on the graph area.

$\omega_d = \omega_n \sqrt{1-\zeta^2} = 2$

$\omega_n = \frac{2}{\sqrt{1-\zeta^2}}$

$\zeta \omega_n = \frac{\zeta 2}{\sqrt{1-\zeta^2}} = 1.155$

desired pole location is $-1.155 \pm 2j$

(c) Determine the "defect" angle using the "Angle Criterion" and plot angles on the graph area.

$\phi_d - 0° - 78° - 120° = -180$ $\phi_d = 18°$

(d) Determine the location of the controller zero and plot on the graph area.

$\tan 18° = \frac{2}{z - 1.155} \Rightarrow z = 7.31$

zero at $s = -7.31$

Name____________________________

Problem 2: (cont')

(e) Determine the line segment length between the poles and zeros and the desired operating point and plot them on the graph area.

$\sqrt{-1.155^2+2^2} = 2.3095$

$(-1.155 \quad 2) - (-2 \quad 2) = .845$

$|(-1.155 \quad 2) - (-2 \quad -2)| = 4.088$

$|(-1.155 \quad 2) - (-7.31 \quad 0)| = 6.47$

(e) Using the "magnitude criterion" determine the controller gain.

$10K = \frac{(6.845)(2.31)(4.088)}{(6.47)} = 1.23 \Rightarrow K = 0.123$

(f) What is the controller transfer function?

$G_c = 0.123 \frac{(s+7.31)}{s}$

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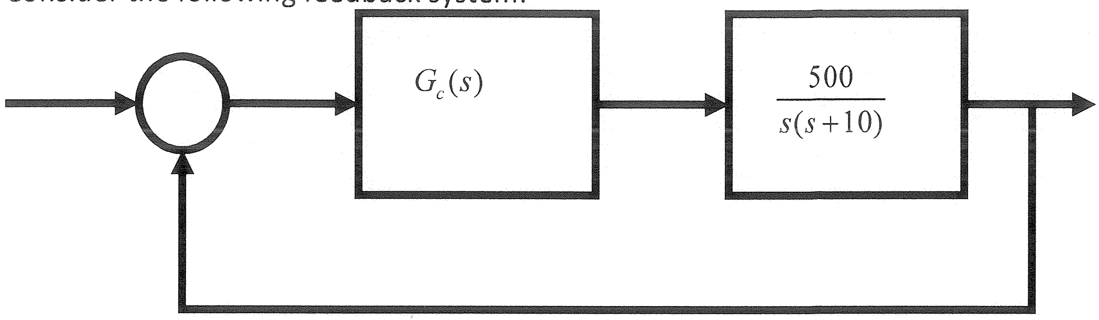
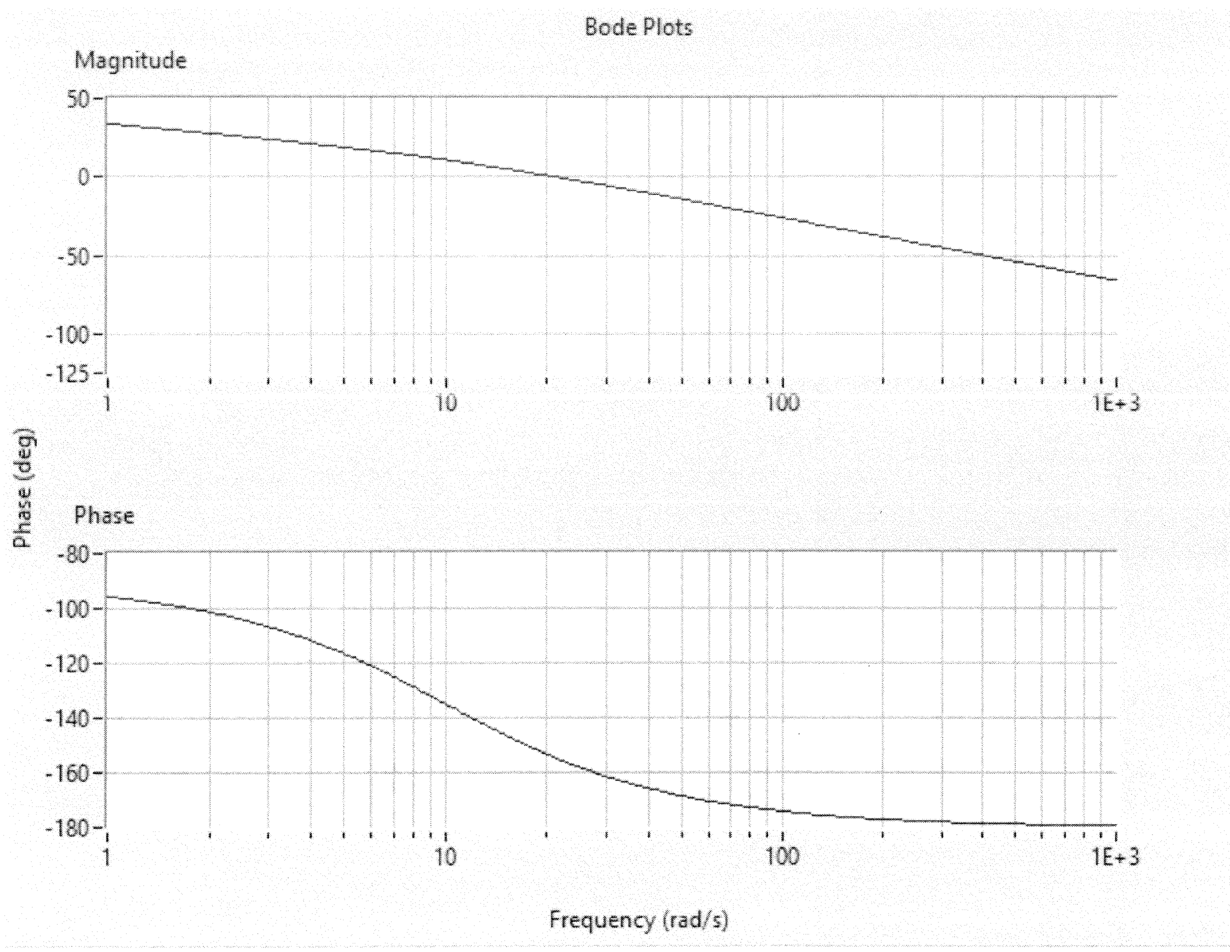
Problem 3: (25%)

Consider the following feedback system:



The Bode plot of this system is shown below:



Design using frequency domain methods a compensator to meet the following specifications

i) Steady – State Error to unit ramp ≤ 0.02

ii) Percent overshoot to a step input ≤ 20%

iii) Bandwidth ≥ 20 rad/sec.

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Problem 3: (cont')

(a) Show that the steady – state error requirement is just met without additional gain.

$$e_{ss} = \frac{1}{K_v} \qquad K_v = \lim_{s \to 0} s\,G(s) = \lim_{s \to 0} s \frac{500}{s(s+10} = 50$$

$$e_{ss} = \frac{1}{50} = 0.02 \checkmark$$

(b) Determine the damping ratio value to meet requirement ii).

$$\zeta = \frac{-\ln(.2)}{\sqrt{\pi^2 + \ln^2(.2)}} = .4559$$

(c) Determine the approximant phase margin from the damping ratio determined in part (b)

$$PM \simeq 100\zeta \cong 46^\circ$$

(d) What gain crossover frequency should you design for?

want Band width ≥ 20 rad/sec

ω_{gc} < Bandwidth < 2 ω_{gc}

so choose ω_{gc} = 20 rad/sec

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Problem 3: (cont')

(e) Design for the phase margin required from part (c) at the gain crossover frequency in part (d).

$$\text{phase}(20\ \text{rad/sec}) = -90 - \tan^{-1}\frac{\omega}{10} = -153.43^\circ$$

$PM = 26.56^\circ$ want $PM \cong 46^\circ$

design $\phi_m = 46 - 26.56^\circ \cong 20^\circ$

$$\alpha = \frac{1+\sin 20}{1-\sin 20} = 2.0396$$

$20 = \omega_1\sqrt{\alpha}$

$\omega_1 = 14.004$ rad/sec

$\omega_2 = 28.56$ rad/sec

$$G_c = \frac{1 + s/14.004}{1 + s/28.56}$$

(f) Design the lag compensator to correct the overall system gain.

plant gain at 20 rad/sec $\frac{500}{20\sqrt{20^2+10^2}} = 1.118$

lead gain $= \sqrt{\alpha} = 1.4281$

total gain $= 1.5966 = \beta$

need $\omega_2 \leq \frac{1}{10}\omega_{gc} = .2$

$1.5966 = \frac{.2}{\omega_p} \Rightarrow \omega_p = \frac{.2}{1.5966} = .125$

$$G_c = \frac{1 + s/.2}{1 + s/.125}$$

(g) What is the transfer function of the lag/lead compensator?

$$\boxed{G_{c_{total}} = \frac{1+s/.2}{1+s/.125} \cdot \frac{1+s/14.004}{1+s/28.56}}$$

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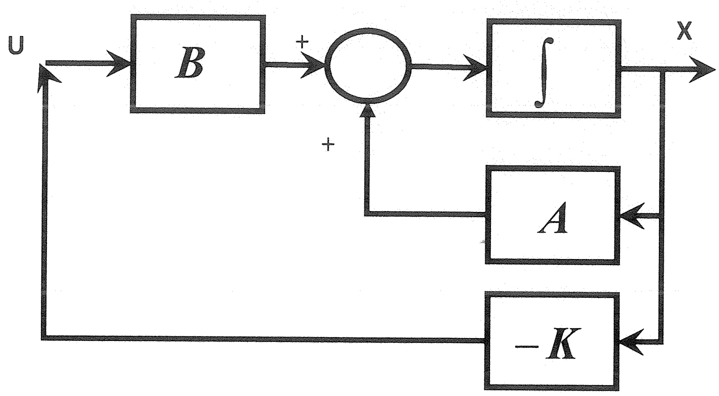
Problem 4: (25%)

Consider the regulator system shown below:



Where:

$\dot{x} = Ax + Bu$ and $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. The system uses state feedback control: $u = -Kx$.

The desired closed – loop poles are at $s = -2 \pm 4j, \quad s = -10$

Find the gain values $K = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}$

(a) Determine the desired characteristic equation.

$(s+2+4j)(s+2-4j)(s+10) = s^3 + 14s^2 + 60s + 200$

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Problem 4: (cont')

(b) Determine the system characteristic equation. ($|sI - A + BK|$)

$$\left| \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix} \right|$$

$$= \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1+k_1 & 5+k_2 & s+6+k_3 \end{vmatrix}$$

$$= s^3 + (6+k_3)s^2 + (5+k_2)s + 1+k_1$$

(c) Equate like powers of 's' between the polynomials in parts (a) and (b) to determine the values of the coefficients k_1, k_2, and k_3 and fill the those values in the blanks below.

k_1 = 199, k_2 = 55, k_3 = 8.

$1+k_1 = 200 \qquad k_1 = 199$

$5+k_2 = 60 \qquad k_2 = 55$

$6+k_3 = 14 \qquad k_3 = 8$