1. True and False

Circle either TRUE or FALSE

i: When a polycrystalline metal is cooled (e.g., in liquid nitrogen), its x-ray diffraction peaks will shift to slightly higher two theta angles compared to the room temperature x-ray diffraction pattern. **TRUE**  **FALSE**

ii: The slope at r=r₀ of the potential energy vs. separation distance curve is directly proportional to Young’s modulus (E). **TRUE**  **FALSE**

iii: Short chain polymers like waxes have poor mechanical properties because of their low degree of crystallinity. **TRUE**  **FALSE**

Waxes have a high degree of crystallinity due to their short chain length, so that is not the cause of their poor mechanical properties.

iv: Polymers for which \( \bar{M}_w \) and \( \bar{M}_n \) are approximately equal have a narrow molecular weight distribution. **TRUE**  **FALSE**

v: Values for coefficient of thermal expansion are typically larger for polymers than for metals and ceramics. **TRUE**  **FALSE**
2. Planes, Directions and slip systems.

a) Draw the following inside the cubes provided.

b) The two drawings below show the arrangement of atoms in the \{110\} and \{100\} of the same crystal structure. **First**, circle whether this is the FCC or BCC structure. **Second**, on each drawing use arrows to label the \(<100>\) in each plane.

The structure is

\[
\text{FCC} \quad \text{BCC}
\]

\[
\{110\} \quad \{100\}
\]

(5) c) In the space provided to the right, state whether the pairs of planes and directions below are possible slip systems for: a FCC metal (write “FCC”), for a BCC metal (write “BCC”), or are not possible slip systems for either FCC or BCC (write “N”).

i) \((\bar{1}00)[0\bar{1}1]\)  

   _____ N ____ (100) not the slip plane

ii) \((1\bar{1}0)[\bar{1}1\bar{1}]\)  

   _____ N ___ dot product not equal to zero

iii) \((1\bar{1}\bar{1})[\bar{1}0\bar{1}]\)  

   ____FCC____

iv) \((10\bar{1})[11\bar{1}]\)  

   ____BCC____

v) \((1\bar{1}\bar{1})[01\bar{1}]\)  

   _____N____ dot product not equal to zero
(15) 3. Multiple Choice, circle one answer for each problem, no partial credit.

The next page is left blank for your use, but no partial will be given for anything written there.

(5) a) The potential energy between the two atoms/ions is given by:  
\[ E = -\frac{A}{r} + \frac{B}{r^n} \]
where \( E \) is the potential energy, \( r \) the separation distance between atoms, and \( A, B, \) and \( n \) are constants. Given that \( A=3.0\times10^{-15} \) J*m, \( B=3.21\times10^{-94} \) J*m^9, and \( n=9 \), the separation distance at which the force between the two atoms is equal to zero (in units of nm) is:

i) 0.255 nm  
ii) 0.177 nm  
iii) 0.135 nm  
iv) 0.230 nm  
v) 0.145 nm

(5) b) Molybdenum has a BCC crystal structure. X-ray diffraction is performed on a polycrystalline molybdenum sample using Cu(K\(\alpha\)) x-rays (\(\lambda=0.154 \) nm). If the (200) reflection for molybdenum is observed at 58.7\(^\circ\) two theta, the atomic radius of molybdenum is:

i) 0.314 nm  
ii) 0.078 nm  
iii) 0.153 nm  
iv) 0.180 nm  
v) 0.136 nm

(5) c) X-ray diffraction is performed on a polycrystalline sample of a FCC metal using Cu(K\(\alpha\)) x-rays (\(\lambda=0.154 \) nm). The (200) reflection is observed at 46.1\(^\circ\) two theta, and from this you determine that the lattice parameter of the FCC metal is 0.393 nm. As one moves to larger two theta values, the next observed peak will be closest to ____ two theta:

i) 67.3\(^\circ\)  
ii) 52.0\(^\circ\)  
iii) 62.5\(^\circ\)  
iv) 57.4\(^\circ\)  
v) 85.5\(^\circ\)
Problem 5

a) \( E = - \frac{A}{r^2} + \frac{B}{r^n} \)
\( F = \frac{dE}{dr} = -\frac{A}{r^2} - \frac{nB}{r^{n+1}} = 0 \)
\( A = 3.0 \times 10^{-15} \text{ J} \) \( \frac{\text{C}}{\text{m}} \)
\( B = 3.21 \times 10^{-94} \text{ J} \) \( \frac{\text{C}}{\text{m}} \)
\( n = 9 \)

\[ \frac{A}{r^2} = \frac{nB}{r^{10}} \Rightarrow \frac{r^8}{A} = \frac{nB}{r^{10}} \]
\[ r = \sqrt[10]{\frac{nB}{A}} = \sqrt[10]{\frac{8}{9} \left( \frac{3.21 \times 10^{-94}}{3.0 \times 10^{-15}} \right) \text{ m}^2} \]

\[ r = 1.77 \times 10^{-10} \text{ m} = 0.177 \text{ nm} \]

b) Mo, BCC crystal \( r = \frac{\sqrt{3} a}{4} \)
\( a = 0.154 \text{ nm} \) \( \{200\} @ 58.7^\circ \theta \)

\( \theta = 29.35^\circ \) \( \lambda = 2d\sin\theta \) \( d = \frac{\lambda}{2\sin\theta} = \frac{0.154 \text{ nm}}{2\sin 29.35^\circ} \)
\( d = 0.157 \text{ nm} \) \( a = d\sqrt{h^2+k^2+l^2} = d\sqrt{2^2} = 0.314 \text{ nm} \)

\[ r = \frac{\sqrt{3} a}{4} = 0.136 \text{ nm} \]
5c) FCC metal  (200) @ 46.12°

Next peak is @ (220)

\[
d = \frac{\lambda}{2 \sin \theta} = \frac{0.154}{2 \sin (23.05°)} = 0.197 \text{nm}
\]

\[
a = \phi 2d = 0.393 \text{nm} = 0.139
\]

\[
\sin \theta = \frac{\lambda}{2d} = 33.6
\]

\[
\therefore \quad 2\theta = 67.2°
\]
(8) 4. Elastic Deformation

Bars of aluminum alloy 6061 (yield stress = 55 MPa) have a thickness of 2 mm, a width of 5 mm, and a length of 20 mm. The following set of load-displacement data was collected during uniaxial tensile testing.

<table>
<thead>
<tr>
<th>Load (N)</th>
<th>Displacement (mm)</th>
<th>Stress (MPa)</th>
<th>Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>3.33x10^-3</td>
<td>10</td>
<td>1.67x10^-4</td>
</tr>
<tr>
<td>200</td>
<td>6.67x10^-3</td>
<td>20</td>
<td>3.33x10^-4</td>
</tr>
<tr>
<td>300</td>
<td>1x10^-2</td>
<td>30</td>
<td>5.0x10^-4</td>
</tr>
<tr>
<td>400</td>
<td>1.33x10^-2</td>
<td>40</td>
<td>6.65x10^-4</td>
</tr>
<tr>
<td>500</td>
<td>1.67x10^-2</td>
<td>50</td>
<td>8.35x10^-4</td>
</tr>
<tr>
<td>600</td>
<td>1.9x10^-2</td>
<td>60</td>
<td>9.5x10^-4</td>
</tr>
<tr>
<td>700</td>
<td>2.0x10^-2</td>
<td>70</td>
<td>10x10^-4</td>
</tr>
</tbody>
</table>

(5) a) Using the load-displacement data calculate the Young’s modulus of the aluminum alloy.

Hooke’s law: $\sigma = \varepsilon E$

In the elastic region Young’s modulus can be calculated by taking the ratio of stress and strain. Stress is calculated from the force divided by cross-sectional area, and strain is the displacement divided by the original sample length. These calculations are performed below for the data taken at a load of 400 N.

$$\sigma (400N) = \frac{400 \text{ N}}{(2 \times 10^{-3} \text{ m})(5 \times 10^{-3} \text{ m})} = 40 \text{ MPa}$$

$$\varepsilon = \frac{1.33 \times 10^{-2} \text{ mm}}{20 \text{ mm}} = 6.65 \times 10^{-4}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{4 \times 10^6 \text{ N/m}^2}{6.65 \times 10^{-4}} = 6.02 \times 10^{10} \text{ Pa} = 60 \text{ GPa}$$

(3) b) Are there any data points in the Table above that you should not use to calculate Young’s modulus? Briefly explain.

One should not use the load-displacement data for at 600 and 700N. The stress associated with this data is above the yield stress. Therefore, the data is no longer in the linear elastic region and does not obey Hooke’s law.
(6) 5. A polycrystalline bar of a metallic alloy has a Young’s modulus $E = 100$ GPa, a yield stress $\sigma_{\text{yield}}=270$ MPa and a Poisson ratio $\nu=0.30$.

A tensile stress of 50 MPa is applied in the x-direction and a tensile stress of 175 MPa is applied in the y-direction. Calculate the resulting strains in the x-, y-, and z-directions. Please write your answers in the spaces provided below.

\[
\varepsilon_x = \frac{\sigma_x}{E} - \nu \left( \frac{\sigma_y}{E} + \frac{\sigma_z}{E} \right) = \frac{50 \text{ MPa}}{100 \times 10^3 \text{ MPa}} - 0.30 \left( \frac{175 \text{ MPa}}{100 \times 10^3 \text{ MPa}} \right) - 0 = 5.0 \times 10^{-4} - 5.25 \times 10^{-4} = -2.5 \times 10^{-5}
\]

\[
\varepsilon_y = \frac{\sigma_y}{E} - \nu \left( \frac{\sigma_x}{E} + \frac{\sigma_z}{E} \right) = \frac{175 \text{ MPa}}{100 \times 10^3 \text{ MPa}} - 0.30 \left( \frac{50 \text{ MPa}}{100 \times 10^3 \text{ MPa}} \right) - 0 = 1.75 \times 10^{-3} - 1.5 \times 10^{-4} = 1.6 \times 10^{-3}
\]

\[
\varepsilon_z = \frac{\sigma_z}{E} - \nu \left( \frac{\sigma_x}{E} + \frac{\sigma_y}{E} \right) = 0 - 0.30 \left( \frac{50 \text{ MPa} + 175 \text{ MPa}}{100 \times 10^3 \text{ MPa}} \right) = -6.75 \times 10^{-4}
\]

\[
\varepsilon_x = -2.5 \times 10^{-5}
\]

\[
\varepsilon_y = 1.6 \times 10^{-3}
\]

\[
\varepsilon_z = -6.75 \times 10^{-4}
\]
(10) 6. Yield stress of undeformed aluminum alloy 6061: \( \sigma_{\text{yield}} = 55 \text{ MPa} \), \( \sigma_{\text{UTS}} = 124 \text{ MPa} \).

Strain Hardening Constants for undeformed aluminum alloy 6061: \( K = 224 \text{ MPa} \), \( n = 0.21 \)

(2) a) You have three rectangular plates of aluminum alloy 6061. Plate 1 is undeformed, Plates 2 and 3, starting in the undeformed state, have been strain hardened at room temperature to the equivalent of a true strain of: 0.05 for Plate 2 and 0.20 for Plate 3. Calculate the true stress of Plates 2 and 3 required to reach their respective true strains, and write your results in the Table below.

\[
\sigma_T = K \varepsilon^n
\]

<table>
<thead>
<tr>
<th>Plate</th>
<th>True strain ((\varepsilon_T))</th>
<th>True stress ((\sigma_T)) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.05</td>
<td>119.4</td>
</tr>
<tr>
<td>3</td>
<td>0.20</td>
<td>159.8</td>
</tr>
</tbody>
</table>

(8) b) Test bars are cut from each of Plates 1, 2 and 3, and subject to mechanical testing in uniaxial tension. In the plot below, draw for each Plate an engineering stress-strain curve reflecting their expected mechanical behavior when subjected to uniaxial tensile testing. Your plots should be quantitatively correct where possible, e.g., using the data provided and calculated in part a) above. Clearly label which curve belongs to which Plate.

Quantitative: Plate 1: the yield stress is 55 MPa and the tensile strength is 124 MPa from the problem statement. Plates 2 and 3: the yield stress is 119.4 and 159.8 MPa respectively due to the strain hardening caused by rolling.

Qualitative: Due to strain hardening the tensile strength should increase and strain to failure decrease as the amount of strain hardening increases. Also, the Young’s modulus should stay the same because it is unaffected by strain hardening.