1. Demonstrate that the minimum cation-to-anion radius ratio for a coordination number of 8 is 0.732.

Solution

This problem asks us to show that the minimum cation-to-anion radius ratio for a coordination number of 8 is 0.732. From the cubic unit cell shown below

![Cubic Unit Cell Diagram]

the unit cell edge length is $2r_A$, and from the base of the unit cell

$$x^2 = (2r_A)^2 + (2r_A)^2 = 8r_A^2$$

Or

$$x = 2r_A\sqrt{2}$$

Now from the triangle that involves $x$, $y$, and the unit cell edge

$$x^2 + (2r_A)^2 = y^2 = (2r_A + 2r_C)^2$$

$$4r_A^2 \sqrt{2} + 4r_A^2 = (2r_A + 2r_C)^2$$

Which reduces to

$$2r_A(\sqrt{3} - 1) = 2r_C$$

Or

$$\frac{r_C}{r_A} = \sqrt{3} - 1 = 0.732$$
2. Compute the atomic packing factor for the rock salt crystal structure in which $r_C/r_A = 0.414$.

Solution

This problem asks that we compute the atomic packing factor for the rock salt crystal structure when $r_C/r_A = 0.414$. From Equation 3.2

$$\text{APF} = \frac{V_S}{V_C}$$

With regard to the sphere volume, $V_S$, there are four cation and four anion spheres per unit cell. Thus,

$$V_S = (4)\left(\frac{4}{3} \pi r_A^3\right) + (4)\left(\frac{4}{3} \pi r_C^3\right)$$

But, since $r_C/r_A = 0.414$

$$V_S = \frac{16}{3} \pi r_A^3 \left[1 + (0.414)^3\right] = (17.94) r_A^3$$

Now, for $r_C/r_A = 0.414$ the corner anions in Table 12.2 just touch one another along the cubic unit cell edges such that

$$V_C = a^3 = [2(r_A + r_C)]^3$$

$$= [2(r_A + 0.414r_A)]^3 = (22.62)r_A^3$$

Thus

$$\text{APF} = \frac{V_S}{V_C} = \frac{(17.94) r_A^3}{(22.62) r_A^3} = 0.79$$
3. A hypothetical AX type of ceramic material is known to have a density of 2.65 g/cm$^3$ and a unit cell of cubic symmetry with a cell edge length of 0.43 nm. The atomic weights of the A and X elements are 86.6 and 40.3 g/mol, respectively. On the basis of this information, which of the following crystal structures is (are) possible for this material: rock salt, cesium chloride, or zinc blende? Justify your choice(s).

Solution

We are asked to specify possible crystal structures for an AX type of ceramic material given its density (2.65 g/cm$^3$), that the unit cell has cubic symmetry with edge length of 0.43 nm (4.3 × 10^{-8} cm), and the atomic weights of the A and X elements (86.6 and 40.3 g/mol, respectively). Using Equation 12.1 and solving for $n'$ yields

$$n' = \frac{\rho V_C N_A}{\sum A_C + \sum A_A}$$

$$= \frac{(2.65 \text{ g/cm}^3)[(4.30 \times 10^{-8} \text{ cm})^3/\text{unit cell}][6.022 \times 10^{23} \text{ formula units/mol}]}{(86.6 + 40.3) \text{ g/mol}}$$

$$= 1.00 \text{ formula units/unit cell}$$

Of the three possible crystal structures, only cesium chloride has one formula unit per unit cell, and therefore, is the only possibility.
4. A three-point bending test is performed on a glass specimen having a rectangular cross section of height \(d = 5\, \text{mm}\) (0.2 in.) and width \(b = 10\, \text{mm}\) (0.4 in.); the distance between support points is 45 mm (1.75 in.).

(a) Compute the flexural strength if the load at fracture is 290 N (65 lb).

(b) The point of maximum deflection \(\Delta y\) occurs at the center of the specimen and is described by

\[
\Delta y = \frac{FL^3}{48EI}
\]

where \(E\) is the modulus of elasticity and \(I\) is the cross-sectional moment of inertia. Compute \(\Delta y\) at a load of 266 N (60 lb).

**Solution**

(a) For this portion of the problem we are asked to compute the flexural strength for a glass specimen that is subjected to a three-point bending test. The flexural strength (Equation 12.7a) is just

\[
\sigma_{fs} = \frac{3FL}{2bd^2}
\]

for a rectangular cross-section. Using the values given in the problem statement,

\[
\sigma_{fs} = \frac{(3)(290\, \text{N})(45 \times 10^{-3}\, \text{m})}{(2)(10 \times 10^{-3}\, \text{m})(5 \times 10^{-3}\, \text{m})^2} = 7.83 \times 10^7\, \text{N/m}^2 = 78.3\, \text{MPa} \quad (10,660\, \text{psi})
\]

(b) We are now asked to compute the maximum deflection. From Table 12.5, the elastic modulus \((E)\) for glass is 69 GPa \((10 \times 10^6\, \text{psi})\). Also, the moment of inertia for a rectangular cross section (Figure 12.32) is just

\[
I = \frac{bd^3}{12}
\]

Thus,

\[
\Delta y = \frac{FL^3}{48E\left(\frac{bd^3}{12}\right)} = \frac{FL^3}{4Ebd^3}
\]

\[
= \frac{(266\, \text{N})(45 \times 10^{-3}\, \text{m})^3}{(4)(69 \times 10^9\, \text{N/m}^2)(10 \times 10^{-3}\, \text{m})(5 \times 10^{-3}\, \text{m})^3}
\]

\[
= 7.0 \times 10^{-5}\, \text{m} = 7.0 \times 10^{-2}\, \text{mm} \quad (2.5 \times 10^{-3}\, \text{in.})
\]
5. A three-point bending test was performed on an aluminum oxide specimen having a circular cross section of radius 3.5 mm (0.14 in.); the specimen fractured at a load of 950 N (215 lb) when the distance between the support points was 50 mm (2.0 in.). Another test is to be performed on a specimen of this same material, but one that has a square cross section of 12 mm (0.47 in.) length on each edge. At what load would you expect this specimen to fracture if the support point separation is 40 mm (1.6 in.)?

Solution

For this problem, the load is given at which a circular specimen of aluminum oxide fractures when subjected to a three-point bending test; we are then asked to determine the load at which a specimen of the same material having a square cross-section fractures. It is first necessary to compute the flexural strength of the aluminum oxide, Equation 12.7b, and then, using this value, we may calculate the value of $F_f$ in Equation 12.7a. From Equation 12.7b

$$\sigma_{fs} = \frac{F_f L}{\pi R^3}$$

$$= \frac{(950 \text{ N})(50 \times 10^{-3} \text{ m})}{(\pi)(3.5 \times 10^{-3} \text{ m})^3} = 352 \times 10^6 \text{ N/m}^2 = 352 \text{ MPa} \quad (50,000 \text{ psi})$$

Now, solving for $F_f$ from Equation 12.7a, realizing that $b = d = 12 \text{ mm}$, yields

$$F_f = \frac{2 \sigma_{fs} d^3}{3L}$$

$$= \frac{(2)(352 \times 10^6 \text{ N/m}^2)(12 \times 10^{-3} \text{ m})^3}{(3)(40 \times 10^{-3} \text{ m})} = 10,100 \text{ N} \quad (2165 \text{ lb}_f)$$
6. *Cite the two desirable characteristics of glasses.*

**Solution**

Two desirable characteristics of glasses are optical transparency and ease of fabrication.
7. (a) Explain why residual thermal stresses are introduced into a glass piece when it is cooled.

(b) Are thermal stresses introduced upon heating? Why or why not?

Solution

(a) Residual thermal stresses are introduced into a glass piece when it is cooled because surface and interior regions cool at different rates, and, therefore, contract different amounts; since the material will experience very little, if any deformation, stresses are established.

(b) Yes, thermal stresses will be introduced because of thermal expansion upon heating for the same reason as for thermal contraction upon cooling.
8. Borosilicate glasses and fused silica are resistant to thermal shock. Why is this so?

Solution

Borosilicate glasses and fused silica are resistant to thermal shock because they have relatively low coefficients of thermal expansion; therefore, upon heating or cooling, the difference in the degree of expansion or contraction across a cross-section of a ware that is constructed from these materials will be relatively low.
9. Compute repeat unit molecular weights for the following: (a) poly(vinyl chloride), (b) poly(ethylene terephthalate), (c) polycarbonate, and (d) polydimethylsiloxane.

Solution

(a) For poly(vinyl chloride), each repeat unit consists of two carbons, three hydrogens, and one chlorine (Table 14.3). If \( A_C, A_H \) and \( A_{Cl} \) represent the atomic weights of carbon, hydrogen, and chlorine, respectively, then

\[
m = 2(A_C) + 3(A_H) + (A_{Cl})
\]

\[
= (2)(12.01 \text{ g/mol}) + (3)(1.008 \text{ g/mol}) + 35.45 \text{ g/mol} = 62.49 \text{ g/mol}
\]

(b) For poly(ethylene terephthalate), from Table 14.3, each repeat unit has ten carbons, eight hydrogens, and four oxygens. Thus,

\[
m = 10(A_C) + 8(A_H) + 4(A_O)
\]

\[
= (10)(12.01 \text{ g/mol}) + (8)(1.008 \text{ g/mol}) + (4)(16.00 \text{ g/mol}) = 192.16 \text{ g/mol}
\]

(c) For polycarbonate, from Table 14.3, each repeat unit has sixteen carbons, fourteen hydrogens, and three oxygens. Thus,

\[
m = 16(A_C) + 14(A_H) + 3(A_O)
\]

\[
= (16)(12.01 \text{ g/mol}) + (14)(1.008 \text{ g/mol}) + (3)(16.00 \text{ g/mol})
\]

\[
= 254.27 \text{ g/mol}
\]

(d) For polydimethylsiloxane, from Table 14.5, each repeat unit has two carbons, six hydrogens, one silicon and one oxygen. Thus,

\[
m = 2(A_C) + 6(A_H) + (A_{Si}) + (A_O)
\]

\[
= (2)(12.01 \text{ g/mol}) + (6)(1.008 \text{ g/mol}) + (28.09 \text{ g/mol}) + (16.00 \text{ g/mol}) = 74.16 \text{ g/mol}
\]
10. *The number-average molecular weight of a polypropylene is 1,000,000 g/mol. Compute the degree of polymerization.*

**Solution**

We are asked to compute the degree of polymerization for polypropylene, given that the number-average molecular weight is 1,000,000 g/mol. The repeat unit molecular weight of polypropylene is just

\[
m = 3(A_C) + 6(A_H)
\]

\[
= (3)(12.01 \text{ g/mol}) + (6)(1.008 \text{ g/mol}) = 42.08 \text{ g/mol}
\]

Now it is possible to compute the degree of polymerization using Equation 14.6 as
11. Make comparisons of thermoplastic and thermosetting polymers (a) on the basis of mechanical characteristics upon heating, and (b) according to possible molecular structures.

Solution

(a) Thermoplastic polymers soften when heated and harden when cooled, whereas thermosetting polymers, harden upon heating, while further heating will not lead to softening.

(b) Thermoplastic polymers have linear and branched structures, while for thermosetting polymers, the structures will normally be network or crosslinked.
12. (a) Is it possible to grind up and reuse phenol-formaldehyde? Why or why not?

(b) Is it possible to grind up and reuse polypropylene? Why or why not?

Solution

(a) It is not possible to grind up and reuse phenol-formaldehyde because it is a network thermoset polymer and, therefore, is not amenable to remolding.

(b) Yes, it is possible to grind up and reuse polypropylene since it is a thermoplastic polymer, will soften when reheated, and, thus, may be remolded.
13. Explain briefly why the tendency of a polymer to crystallize decreases with increasing molecular weight.

Solution

The tendency of a polymer to crystallize decreases with increasing molecular weight because as the chains become longer it is more difficult for all regions along adjacent chains to align so as to produce the ordered atomic array.
14. Briefly explain how each of the following influences the tensile or yield strength of a semicrystalline polymer and why:

(a) Molecular weight
(b) Degree of crystallinity
(c) Deformation by drawing
(d) Annealing of an undeformed material

Solution

(a) The tensile strength of a semicrystalline polymer increases with increasing molecular weight. This effect is explained by increased chain entanglements at higher molecular weights.

(b) Increasing the degree of crystallinity of a semicrystalline polymer leads to an enhancement of the tensile strength. Again, this is due to enhanced interchain bonding and forces; in response to applied stresses, interchain motions are thus inhibited.

(c) Deformation by drawing increases the tensile strength of a semicrystalline polymer. This effect is due to the highly oriented chain structure that is produced by drawing, which gives rise to higher interchain secondary bonding forces.

(d) Annealing an undeformed semicrystalline polymer produces an increase in its tensile strength.
15. *Make a schematic plot showing how the modulus of elasticity of an amorphous polymer depends on the glass transition temperature. Assume that molecular weight is held constant.*

**Solution**

For an amorphous polymer, the elastic modulus may be enhanced by increasing the number of crosslinks (while maintaining the molecular weight constant); this will also enhance the glass transition temperature. Thus, the modulus-glass transition temperature behavior would appear as
16. A continuous and aligned fiber-reinforced composite having a cross-sectional area of 1130 mm$^2$ (1.75 in.$^2$) is subjected to an external tensile load. If the stresses sustained by the fiber and matrix phases are 156 MPa (22,600 psi) and 2.75 MPa (400 psi), respectively, the force sustained by the fiber phase is 74,000 N (16,600 lb) and the total longitudinal strain is $1.25 \times 10^{-3}$, determine
(a) the force sustained by the matrix phase
(b) the modulus of elasticity of the composite material in the longitudinal direction, and
(c) the moduli of elasticity for fiber and matrix phases.

Solution

(a) For this portion of the problem we are asked to calculate the force sustained by the matrix phase. It is first necessary to compute the volume fraction of the matrix phase, $V_m$. This may be accomplished by first determining $V_f$ and then $V_m$ from $V_m = 1 - V_f$. The value of $V_f$ may be calculated since, from the definition of stress (Equation 6.1), and realizing $V_f = A_f / A_c$ as

\[ \sigma_f = \frac{F_f}{A_f} = \frac{F_f}{V_f A_c} \]

Or, solving for $V_f$

\[ V_f = \frac{F_f}{\sigma_f A_c} = \frac{74,000 \text{ N}}{(156 \times 10^6 \text{ N/m}^2)(1130 \text{ mm}^2)(1 \text{ m/1000 mm})^2} = 0.420 \]

Also

\[ V_m = 1 - V_f = 1 - 0.420 = 0.580 \]

And, an expression for $\sigma_m$ analogous to the one for $\sigma_f$ above is

\[ \sigma_m = \frac{F_m}{A_m} = \frac{F_m}{V_m A_c} \]

From which

\[ F_m = V_m \sigma_m A_c = (0.580)(2.75 \times 10^6 \text{ N/m}^2)(1.13 \times 10^{-3} \text{ m}^2) = 1802 \text{ N (406 lbf)} \]

(b) We are now asked to calculate the modulus of elasticity in the longitudinal direction. This is possible realizing that $E_c = \frac{\sigma_c}{\varepsilon}$ (from Equation 6.5) and that $\sigma_c = \frac{F_m + F_f}{A_c}$ (from Equation 6.1). Thus
\[ E_c = \frac{\sigma_c}{\varepsilon} = \frac{A_c}{\varepsilon A_c} = \frac{F_m + F_f}{\varepsilon A_c} \]

\[ = \frac{1802 \, \text{N} + 74,000 \, \text{N}}{(1.25 \times 10^{-3}) (1130 \, \text{mm}^2)(1 \, \text{m}/1000 \, \text{mm})^2} = 53.7 \times 10^9 \, \text{N/m}^2 = 53.7 \, \text{GPa} \ (7.77 \times 10^6 \, \text{psi}) \]

(c) Finally, it is necessary to determine the moduli of elasticity for the fiber and matrix phases. This is possible assuming Equation 6.5 for the matrix phase—i.e.,

\[ E_m = \frac{\sigma_m}{\varepsilon_m} \]

and, since this is an isostrain state, \( \varepsilon_m = \varepsilon_c = 1.25 \times 10^{-3} \). Thus

\[ E_m = \frac{\sigma_m}{\varepsilon_c} = \frac{2.75 \times 10^6 \, \text{N/m}^2}{1.25 \times 10^{-3}} = 2.2 \times 10^9 \, \text{N/m}^2 \]

\[ = 2.2 \, \text{GPa} \ (3.2 \times 10^5 \, \text{psi}) \]

The elastic modulus for the fiber phase may be computed in an analogous manner:

\[ E_f = \frac{\sigma_f}{\varepsilon_f} = \frac{\sigma_f}{\varepsilon_c} = \frac{156 \times 10^6 \, \text{N/m}^2}{1.25 \times 10^{-3}} = 1.248 \times 10^{11} \, \text{N/m}^2 \]

\[ = 124.8 \, \text{GPa} \ (18.1 \times 10^6 \, \text{psi}) \]
17. (a) Compute the electrical conductivity of a 5.1-mm (0.2-in.) diameter cylindrical silicon specimen 51 mm (2 in.) long in which a current of 0.1 A passes in an axial direction. A voltage of 12.5 V is measured across two probes that are separated by 38 mm (1.5 in.).

(b) Compute the resistance over the entire 51 mm (2 in.) of the specimen.

Solution

This problem calls for us to compute the electrical conductivity and resistance of a silicon specimen.

(a) We use Equations 18.3 and 18.4 for the conductivity, as

\[ \sigma = \frac{1}{\rho} = \frac{II}{Vd^2} = \frac{II}{V\pi \left( \frac{d}{2} \right)^2} \]

And, incorporating values for the several parameters provided in the problem statement, leads to

\[ \sigma = \frac{(0.1 \text{ A})(38 \times 10^{-3} \text{ m})}{(12.5 \text{ V})(\pi) \left( \frac{5.1 \times 10^{-3} \text{ m}}{2} \right)^2} = 14.9 \quad (\Omega \cdot \text{m})^{-1} \]

(b) The resistance, \( R \), may be computed using Equations 18.2 and 18.4, as

\[ R = \frac{\rho l}{A} = \frac{l}{\sigma A} = \frac{l}{\sigma \pi \left( \frac{d}{2} \right)^2} \]

\[ = \frac{51 \times 10^{-3} \text{ m}}{\left[ 14.9 \quad (\Omega \cdot \text{m})^{-1} \right] (\pi) \left( \frac{5.1 \times 10^{-3} \text{ m}}{2} \right)^2} = 168 \quad \Omega \]
18. *Describe the composition of wood cells in terms of the strong and weaker phases.*

**Solution**
The cell core is filled with lumen and the strong phase is composed of cellulose while the weaker phase is composed on lignin.
19. Answer the following questions about wood:
(a) Which wood cut shrinks uniformly during drying?
(b) Is wood easier to cut perpendicular or parallel to the cells?
(c) What is the effect of drying on wood density and modulus of elasticity?

**Solution**
(a) The quarter-sawn cuts shrinks uniformly during drying due to the orientation of the wood rings.
(b) Wood is easier to cut parallel to the cells.
(c) Both the density and modulus of elasticity increase after drying.
20. *In terms of electron energy band structure, discuss reasons for the difference in electrical conductivity between metals, semiconductors, and insulators.*

**Solution**

For metallic materials, there are vacant electron energy states adjacent to the highest filled state; thus, very little energy is required to excite large numbers of electrons into conducting states. These electrons are those that participate in the conduction process, and, because there are so many of them, metals are good electrical conductors.

There are no empty electron states adjacent to and above filled states for semiconductors and insulators, but rather, an energy band gap across which electrons must be excited in order to participate in the conduction process. Thermal excitation of electrons will occur, and the number of electrons excited will be less than for metals, and will depend on the band gap energy. For semiconductors, the band gap is narrower than for insulators; consequently, at a specific temperature more electrons will be excited for semiconductors, giving rise to higher conductivities.
21. (a) Calculate the drift velocity of electrons in germanium at room temperature and when the magnitude of the electric field is 1000 V/m. (b) Under these circumstances, how long does it take an electron to traverse a 25-mm (1-in.) length of crystal?

Solution

(a) The drift velocity of electrons in Ge may be determined using Equation 18.7. Since the room temperature mobility of electrons is 0.38 m²/V·s (Table 18.3), and the electric field is 1000 V/m (as stipulated in the problem statement),

\[ v_d = \mu_e E \]

\[ = (0.38 \text{ m}^2/\text{V} \cdot \text{s})(1000 \text{ V/m}) = 380 \text{ m/s} \]

(b) The time, \( t \), required to traverse a given length, \( l (= 25 \text{ mm}) \), is just

\[ t = \frac{l}{v_d} = \frac{25 \times 10^{-3} \text{ m}}{380 \text{ m/s}} = 6.6 \times 10^{-5} \text{ s} \]