1 Problem 1

The figure below shows three flat plates, all of very large area. The two thin plates are made of insulating material and carry uniformly distributed surface charge densities of $\sigma_a = -2.5 \mu C/m^2$ and $\sigma_b = 8.5 \mu C/m^2$ respectively. The thick metal plate has a width $w = 3.0 \text{ cm}$, and is initially uncharged ($\sigma_{\text{metal}} = 0$). What is the magnitude of the electric field $E_A$ at the origin (the point marked A on the figure)? (Please use $d_1 = 4 \text{ cm}$, $d_2 = 12 \text{ cm}$.)

A. $3.3 \times 10^5 \text{N/C}$
B. $5.6 \times 10^5 \text{N/C}$
C. $1.6 \times 10^5 \text{N/C}$
D. $6.7 \times 10^5 \text{N/C}$
E. $19.1 \times 10^5 \text{N/C}$

Solution: Recall that the equation giving the electric field of an infinite plane is

$$E = \frac{\sigma}{2\epsilon_0}, \quad (1)$$
and also remember that this electric field points away from positive charges and towards negative charges. Since the electric field is constant, we ignore all the distances in this problem. At point $A$, the electric field due to the plane with charge density $\sigma_a$ points to the right (towards negative charge) and the electric field due to the plane with charge density $\sigma_b$ points to the left (away from positive charge). Thus the net electric field is

$$E_A = \frac{-\sigma_a + \sigma_b}{2\varepsilon_0} = -\frac{6 \mu C/m^2}{2\varepsilon_0} = -3.3 \times 10^5 N/C.$$  \hspace{1cm} (2)

We just need the magnitude though, so choose $A$.

## 2 Problem 2

What is the sign of the surface charge density $\sigma_{LHS}$ on the left-hand-side of the thick metal plate?

A. Not enough information.

B. $\sigma_{LHS} > 0$

C. $\sigma_{LHS} < 0$

D. $\sigma_{LHS} = 0$

**Solution:** We just calculated the net electric field at $A$ and saw that it was negative. That means that the electric field due to plate $A$ and $B$ inside the metal conductor is also negative. Thus, we need to cancel that out (since $E = 0$ within a conductor!) by having a positive charge on the left hand side (and a negative charge on the right hand side). So choose $B$. 
3 Problem 3

Points $R$ and $T$ are each a distance $d$ from each of the two particles with charges of equal magnitude and opposite signs as shown. If $k = 1/(4\pi \epsilon_0)$, the work required to move a particle with a negative charge $q$ from $R$ to $T$ is:

A. 0
B. $k\frac{qQ}{d^2}$
C. $k\frac{qQ}{d^2}$
D. $k\frac{qQ}{d}$
E. $k\frac{qQ}{\sqrt{2d}}$

Solution: Points $R$ and $T$ are at the same potential: $\frac{kQ}{d} + \frac{kQ}{d} = 0$. Thus, it takes $U = Vq = 0$ work to get from $R$ to $T$. Choose A.
4 Problem 4

Three charges $Q_1 (= +8.0 \mu C)$, $Q_2 (= -12.0 \mu C)$, and $Q_3 (= +5.0 \mu C)$ are arranged on the $y$-axis in the figure below. $Q_1$ is located at $(0, a)$, $Q_2$ is located at the origin, and $Q_3$ is located at $(0, -a)$; $a = 1.0$ cm. Find the energy stored in the charge configuration of $Q_1$, $Q_2$, and $Q_3$ (i.e., the work done that was necessary to assemble the three charges).

A. $-122$ J
B. $-13140$ J
C. 14940 J
D. 34.5 J
E. 158 J

Solution: Let’s do this the normal way - by starting off with the charges out at infinity, and then bringing them back together one-by-one. We can bring the first in for free - let’s bring in $Q_2$. Now let’s bring in $Q_1$. Its change in potential is

$$V_1 = \frac{kQ_2}{a} - 0,$$

since we’ve chosen infinity to be our reference point. The work associated with this move is

$$W_1 = \frac{kQ_1Q_2}{a}.$$
Now let’s bring in \( Q_3 \). Its change in potential is

\[
V_3 = \frac{kQ_2}{a} + \frac{kQ_1}{2a} - 0, \tag{5}
\]

so the work it takes to move this charge in is

\[
W_3 = \frac{kQ_2Q_3}{a} + \frac{kQ_1Q_3}{2a}. \tag{6}
\]

So, the total work is

\[
W = \frac{k}{a} \left( Q_1Q_2 + Q_2Q_3 + \frac{Q_1Q_3}{2} \right) = -122.4 J, \tag{7}
\]

so pick A.

5 Problem 5

A fourth charge \( q = +2 \mu C \) is now brought in from infinity, and placed on the \( x \)-axis at a distance of \( d = 3 \) cm from the origin. Find the \( y \)-component of the net force of \( Q_1, Q_2, \) and \( Q_3 \) on \( q \).

A. +17.1 N
B. −17.10 N
C. +54.0 N
D. 0 N
E. $-54.0 \text{ N}$

**Solution:** We just use Coulomb’s law:

$$\vec{F} = \frac{kQq}{r^2} \hat{r} = \frac{kQq}{r^3} \vec{r} \implies F_y = \frac{kQq}{r^3} r_y.$$  

(8)

Plugging in each charge gives

$$F_1y = \frac{kQ_1q}{\sqrt{a^2+d^2}}(-a)$$

$$F_2y = \frac{kQ_2q}{\sqrt{0^2+d^2}}(0)$$

$$F_3y = \frac{kQ_3q}{\sqrt{a^2+d^2}}(a),$$

(9)

so the total force is

$$F = \frac{kqQ}{\sqrt{a^2+d^2}}(-Q_1+Q_3) = -17.1 \text{ N},$$

(10)

so choose B.
6 Problem 6

A solid insulating sphere of radius $R$ contains positive charge that is distributed with a volume charge density that does not depend on angle but does increase with distance from the sphere’s center. Which of the graphs below might give the magnitude of $E$ of the electric field as a function of distance $r$ from the center of the sphere.

Solution: Remember one of the key results of Gauss’s law - whatever the charge looks like inside the sphere, outside of it it looks just like a point charge. Thus the electric field away outside of the radius of the sphere has to exhibit the $\frac{1}{r^2}$ behavior of a point charge, so we can ignore option D. Also, since right in the middle there is an equal amount of charge in all directions (since the volume charge density is spherically symmetric – does not depend on angle), the electric field has to be zero due to symmetry. So, A cannot be right. Next, picture a Gaussian surface inside the sphere. Clearly it will contain some charge, so the electric field cannot be zero. B must be incorrect, then. Finally, since we know that E gives the electric field for a constant density (the linear behavior gives that away), we can ignore E since we are told that the charge density is not constant. This just leaves C.
Problem 7

A particle with charge \( q = -1.6 \times 10^{-19} \text{C} \) and a mass \( m = 10^{-27} \text{kg} \), with an initial velocity \( v_x = 10^7 \text{ m/s} \), travels in a uniform electric field \( E_x = 500 \text{ V/m} \). Determine the time (in seconds) it takes for the particle to come to rest.

\[ v(t) = at + v_0 \]  \hspace{1cm} (11)

\[ F = ma \implies a = \frac{qE}{m}. \]  \hspace{1cm} (12)

Now we can just solve for \( t \):

\[ t = \frac{v(t) - v_0}{a} = \frac{m(0 - v_0)}{qE} = 0.000125, \]  \hspace{1cm} (13)

so option A.
8 Problem 8

The diagram shows the electric field lines due to two parallel metal plates. We conclude that

A. a proton at X would experience the same force if it were placed at Y
B. An electron at X could have its weight balanced by the electric force
C. a proton at X experiences a greater force than if it were placed at Z
D. the upper plate is positive and the lower plate is negative
E. a proton at X experiences less force than if it were placed at Z

Solution: Let’s eliminate options. Remember that in the center of the plane (where it can be treated as “infinite”) the electric field is independent of position. Thus, based on the locations of x and y, A is a reasonable answer. Consider B. Remember that electrons travel “backwards” along field lines. So, the force on them is down, in the same direction as gravity. So B cannot be right - the forces actually stack, not cancel out. Answer C cannot be right since the electric field is independent of distance. The upper plate has to be negative since the electric field lines are pointing towards it, so D cannot be wrong. E is wrong for the same reason C is long. Pick A!
9 Problem 9

Find the magnitude of the electric field, \( E \), in V/m at the point \((1.8\text{m}, -1.8\text{m})\) in the \( x - y \) plane for the potential function

\[
V(x, y) = (x^2y - y^2x - 10)\text{V}.
\]

(14)

A. 13.74
B. 188.95
C. 94.47
D. zero
E. 9.72

Solution: We can just calculate this based on the relationship between \( E \) and \( V \):

\[
\vec{E} = -\nabla V,
\]

(15)

so

\[
E_x(x, y) = -(2xy - y^2), \quad E_y(x, y) = -(x^2 - 2yx).
\]

(16)

Now we plug in \( x \) and \( y \) and calculate the magnitude!

\[
E = \sqrt{2(1.8)(-1.8) - (-1.8)^2}^2 + ((1.8)^2 - 2(-1.8)(1.8))^2 = 13.74.
\]

(17)

Pick A!
10 Problem 10

A particle with a charge of $5 \times 10^{-6}$ C and a mass of 20 grams moves uniformly with a speed of 5.6m/s in a circular orbit around a stationary particle with a charge of $-5 \times 10^{-6}$ C. The radius of the orbit (in meters) is:

A. 2
B. 0.35
C. 1
D. 0.17
E. zero

Solution: Another 172 question! Remember that particles in uniform circular motion have a centripetal acceleration to balance out the attractive force. Let’s set those equal:

$$\frac{kQ^2}{R^2} = \frac{mv^2}{R} \implies R = \frac{kQ^2}{mv^2} = 0.35,$$

answer B.
11 Problem 11

Consider an infinite line with charge density $\lambda_0 = +3 \mu C/m$, shown in the center of the figure below. Concentric with the line is a hollow thick-walled cylinder (shaded), made of conducting material. The hollow cylinder carries a charge per unit length of $\lambda = -3 \mu C/m$. Finally, a thin nonconducting cylindrical shell is concentric with the other two objects, and carries a charge per unit length of $\lambda_c = +6 \mu C/m$. The dimensions of the objects are shown in the figure; all three have infinite length. Which of the following graphs best represents the radial dependence of the electric field $E$?
Solution: We need to apply Gauss’s law a bunch. Let’s start on the outside. A gaussian surface around the entire wire setup encloses a positive charge $(6+3-3 = 6)$. Thus C cannot be right, since it says that the electric field outside of everything is negative. Now let’s make a Gaussian surface inside the outer shell but around the inner parts. The enclosed charge now is zero, so the electric field must be zero. We can then eliminate everything except for B.
12 Problem 12

Two identical conducting spheres $A$ and $B$ carry equal charge. They are separated by a distance much larger than their diameters. A third identical conducting sphere $C$ is uncharged. Sphere $C$ is first touched to $A$, then to $B$ and finally removed. As a result, the electrostatic force between $A$ and $B$, which was originally $F$, becomes:

A. $3F/8$
B. $F/2$
C. $F/16$
D. zero
E. $F/4$

Solution: When we touch $C$ to $A$, the charge distributes itself so that the conductors are equal potential surfaces (equivalent to saying that the electric field inside them is zero). This happens when the charge splits evenly. Now, when we touch $C$ to $B$, we split $\frac{3}{2}$ the original charge on $A$ or $B$ between the two, so that now they each have $\frac{3}{4}$ the initial charge.

To recap now: $A$ has charge $Q/2$, $B$ has charge $3Q/4$. The force, then, is

$$F = \frac{kQ_A Q_B}{r^2} = \frac{kQ^2}{r^2} \left( \frac{1}{2} \cdot \frac{3}{4} \right) = \frac{3}{8} F.$$  \hspace{1cm} (19)

Gotta pick A!
13 Problem 13

A uniformly charged sphere has a potential on its surface of 450 V. At a radial distance of 6.7 m from this sphere, the potential is 150 V. What is the radius of the sphere?

A. 2.2 m
B. 20.1 m
C. 3.35 m
D. 10.05 m
E. 13.4 m

Solution: Remember that, thanks to Gauss’s law, we know that the electric field / electric potential outside of a sphere is just like the electric field / electric potential of a point charge. And so:

\[ V_{\text{surface}} = \frac{kQ}{r}, \]  

(20)

and

\[ V_{\text{away}} = \frac{kQ}{r + R}. \]  

(21)

Let’s solve for \( Q \):

\[ Q = \frac{rV_{\text{surface}}}{k}, \]  

(22)

so

\[ V_{\text{away}} = \frac{k}{r + R} \frac{rV_{\text{surface}}}{k} \implies r + R = r \frac{V_{\text{surface}}}{V_{\text{away}}}. \]  

(23)

Finally,

\[ r + R = 3r \implies r = \frac{R}{2} = 3.35. \]  

(24)

Pick C!

14 Problem 14

What is the charge of the sphere?

A. 110 nC
B. 1006 nC
C. 503 nC
D. 167 nC
E. 670 nC

Solution: We just plug in $r$ and solve;

$$V_{\text{surface}} \cdot 3.35 = kQ \implies Q = 167.5 \times 10^{-9}. \tag{25}$$

Pick D!
15 Problem 15

Cube 1 and cube 2 are heavily charged plastic cubes that attract each other. Cube 3 is a conductor and is uncharged. Which of the following illustrates the force between cubes 1 and 3, and between cubes 2 and 3?

Solution: If cubes 1 and 2 attract each other, one is positively charged and the other is negatively charged. When we put a conducting uncharged cube near them, the positive cube will pull positive charges towards it and repel negative charges. Since the positive charges are closer than negative, the electrostatic attraction overcomes the electrostatic repulsion, so there will be a net attraction. The same thing happens for the negative cube. Thus, the cubes attract. We pick A.