Bending

Sign Convention:

Positive internal shear force (V)

Positive internal bending moment (M)

Methods to find Shear-Moment diagram:

1. Equilibrium method

   Strategy:

   → Find support reactions

   → Find V and M functions in each *continuous region* (draw the FBD of the beam within the continuous region and derive the V and M functions using static equilibrium equations)

   → Plot the shear and moment diagrams

2. Graphical Method to Draw the Shear and Moment Diagram

   Detailed strategy:

   → Find support reactions

   → Draw the shear force (V) diagram by making use of the following:

   - A point force causes a "jump" in the shear force diagram

     (↑ force = positive jump)

   - The value of distributed load (w) at x is equal to the slope of the shear diagram at that point

$$\frac{dV}{dx} = w(x)$$

   → Draw the bending moment (M) diagram by making use of the following:

   - A point moment causes a "jump" in the moment diagram

     (CCW moment = negative jump)

   - The value of shear force (V) at x is equal to the slope of the bending moment diagram at that point

$$\frac{dM}{dx} = V(x)$$

Important notes:

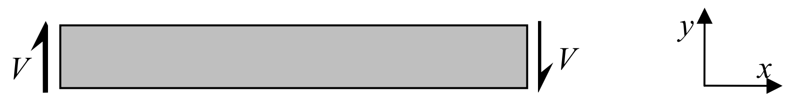
- To use the graphical method, the sign convention given in page 5-1 of this handout MUST be followed.
- You can use the following to check your work when using the graphical method:
  - Right before the beam starts ($x = 0_-$), the shear force and bending moment are zero
  - Right after the beam ends ($x = L_+$), the shear force and bending moment are zero

<u>Bending moment results in normal (flexural) stress</u>

$$\boxed{\sigma = -\frac{My}{I}}$$ ------------------------------ ***flexure formula***

where, M = internal bending moment on the cross sectional area
y = distance from the neutral axis
I = moment of inertia of the cross sectional area.

<u>Shear force results in shear stress</u>



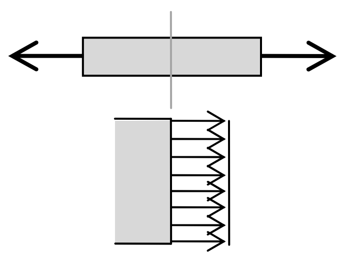
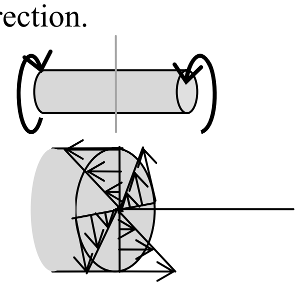
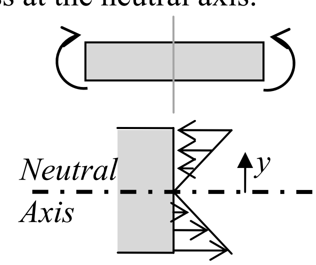
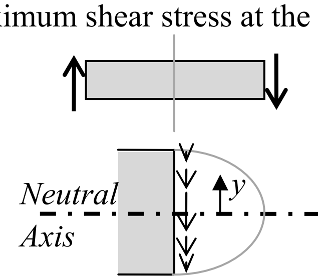
$$\boxed{\tau_{yx} = \tau_{xy} = \frac{QV_y}{I_z t}}$$

where, $Q = A'\overline{y}'$ = area of the cross section above that point * height from the neutral axis to the centroid above that point.
V = internal shear force
I_z = area moment of inertia
t = the width of the cross sectional area measured at the point where you want to find τ

Combined loading

Procedure to get the state of stress caused by combined loading:

1. Find internal resultants
2. Determine the stress component associated with each internal resultant. It is recommended to use inspection to determine direction of stress, instead of relying on the stress equations.

Normal Stress	**Shear Stress**
Axial Load *Equation*: $\sigma = \frac{F}{A}$ *Stress distribution*: uniform distribution 	**Torque** *Equation*: $\tau = \frac{Tr}{J}$ *Stress distribution*: Linear distribution in the radial direction. 
Bending Moment *Equation*: $\sigma = -\frac{My}{I}$ *Stress Distribution*: Linear distribution in y, a distance measured from the neutral axis. Zero stress at the neutral axis.  	**Shear Force** *Equation*: $\tau = \frac{VQ}{It}$ *Stress distribution*: quadratic distribution as a function of distance from the neutral axis. Maximum shear stress at the neutral axis. 
Special Case: Thin walled pressure vessel Cylindrical vessel: $\sigma_{hoop} = \frac{pr}{t}$ $\sigma_{longitudinal} = \frac{pr}{2t}$ Spherical vessel: $\sigma = \frac{pr}{2t}$	**Special Case: Shear force from direct shear** Equation: $\tau = \frac{V}{A}$ Stress Distribution: uniform distribution

3. Superposition (be careful with the direction of stress):
   - Sum the normal stresses together
   - Sum the shear stresses together

Stress transformation

In summary, the stress transformation equations are:

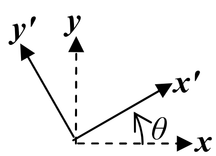
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = \frac{-\left(\sigma_x - \sigma_y\right)}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

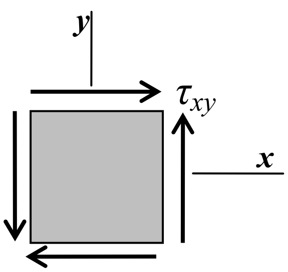
$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

Important:

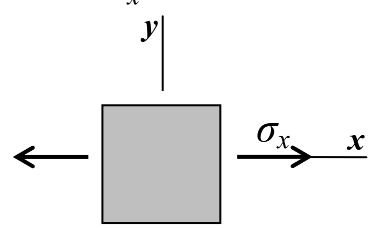
The angle θ is the angle measured counterclockwise from x to x':



Positive τ_{xy}:



Positive σ_x:



Principal Plane: The maximum and minimum normal stresses are called the "*principal stresses*".

The orientation angle of this plane, θ_p, is:

$$\tan\left(2\theta_p\right) = \frac{\tau_{xy}}{\left(\frac{\left(\sigma_x - \sigma_y\right)}{2}\right)}$$

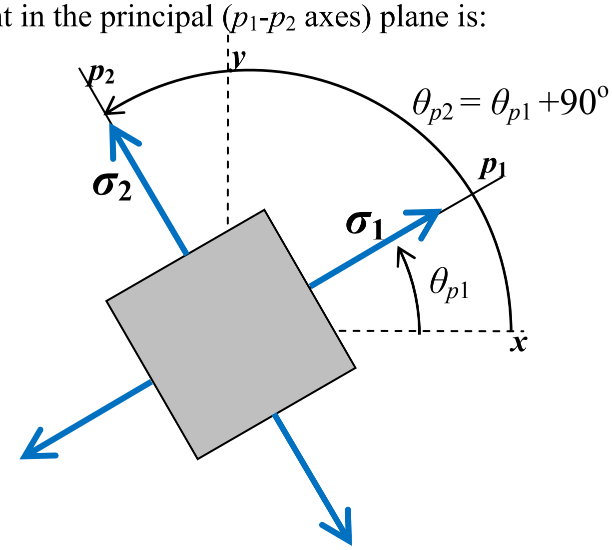
**See next page on how to use this eqn*

The stresses in this plane are:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

$$\tau_p = 0$$

Therefore, the stress element in the principal (p_1-p_2 axes) plane is:



Maximum in plane shear stress

The orientation angle of this plane, θs, is:

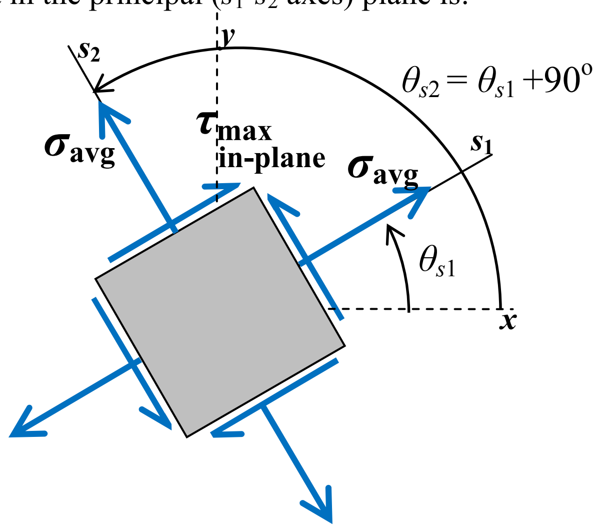
$$\boxed{\tan(2\theta_s) = \frac{\left(\frac{-(\sigma_x - \sigma_y)}{2}\right)}{\tau_{xy}}}$$

**See next page on how to use this eqn*

The stresses in this plane are:

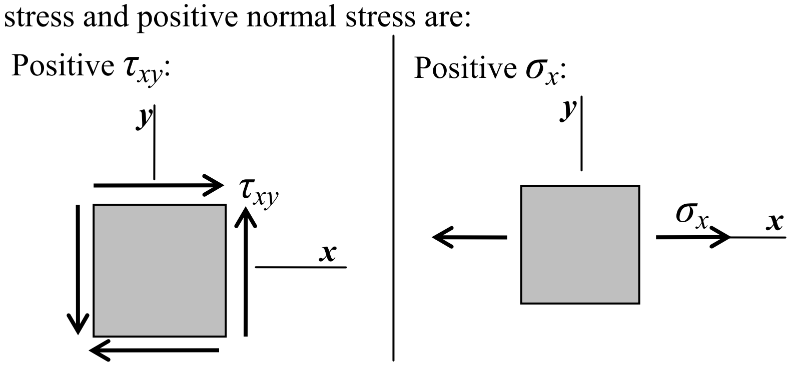
$$\boxed{\begin{aligned} \tau_{\substack{\max \\ in-plane}} &= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ \sigma_{s1} = \sigma_{s2} = \sigma_{avg} &= \frac{\sigma_x + \sigma_y}{2} \end{aligned}}$$

Therefore, the stress element in the principal (s_1-s_2 axes) plane is:



Important points:

1. θ_{p1} is measured CCW from x to p_1
2. θ_{s1} is measured CCW from x to s_1
3. positive shear stress and positive normal stress are:



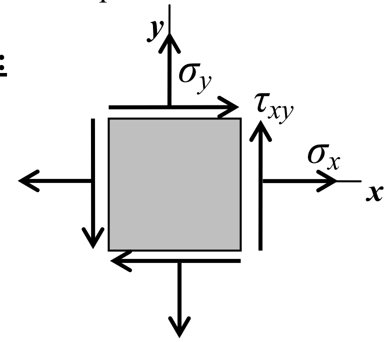
4. When calculating the angle from $\tan(2\theta_p) = \frac{\tau_{xy}}{\left(\frac{(\sigma_x - \sigma_y)}{2}\right)}$, we can get two angles, but we don't know which is θ_{p1} which is θ_{p2}. To find out, here is what we can do:

➔ Calculate $\sigma_{x'}$ using one of the angles. If $\sigma_{x'} = \sigma_1$, then that angle is θ_{p1}, if $\sigma_{x'} = \sigma_2$, then that angle is θ_{p2}. (See example 9.3 on page 448 of textbook.)

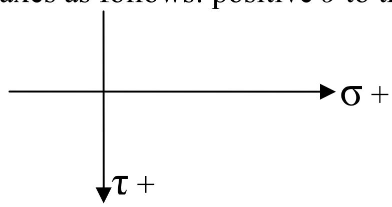
Mohr's Circle

Mohr's circle is a graphical representation of the stress transformation equations.

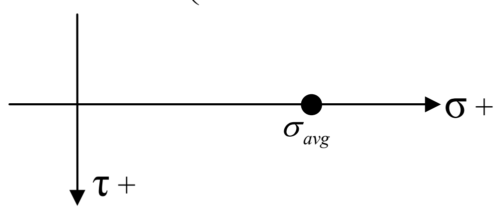
Procedure to construct a Mohr's Circle, given a stress state:



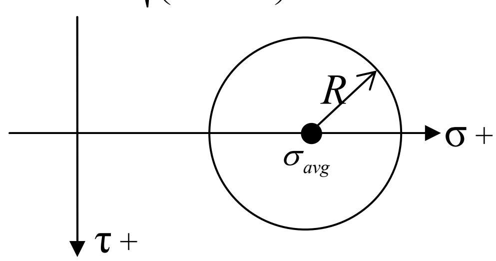
1. Draw the σ and τ axes as follows: positive σ to the right and positive τ down:



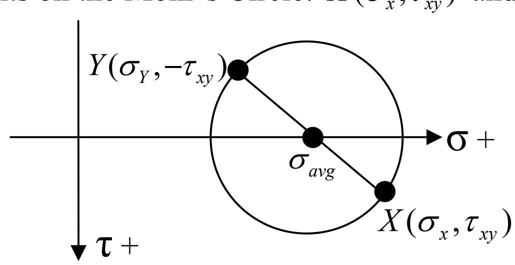
2. Position the center of the circle at $\left(\sigma = \sigma_{avg} = \dfrac{\sigma_x + \sigma_y}{2}; \quad \tau = 0\right)$



3. The radius of the circle is $R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$

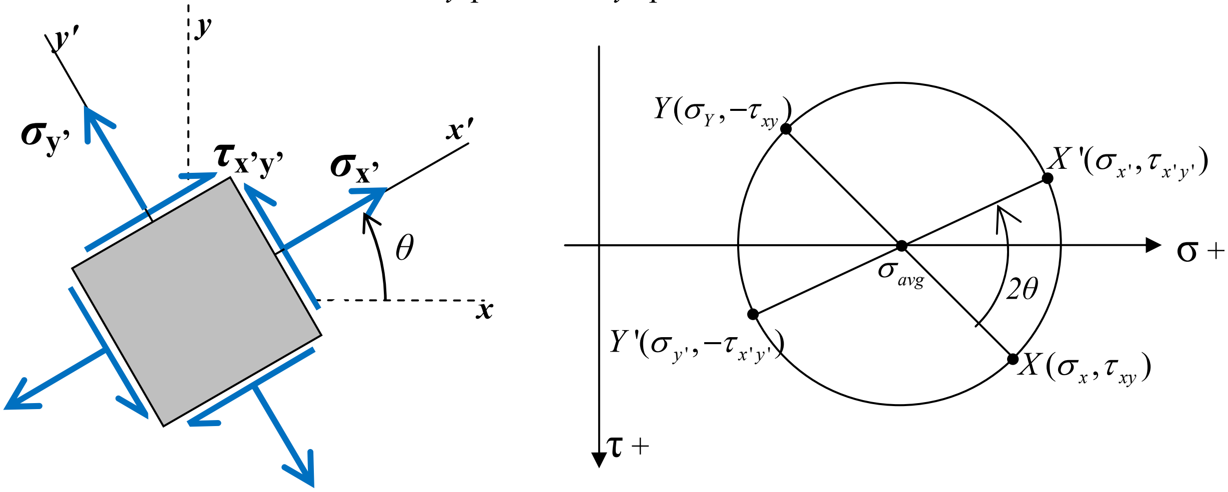


4. The stress state in the x-y plane shown above are represented by a straight line connecting two points on the Mohr's Circle: $X(\sigma_x, \tau_{xy})$ and $Y(\sigma_Y, -\tau_{xy})$



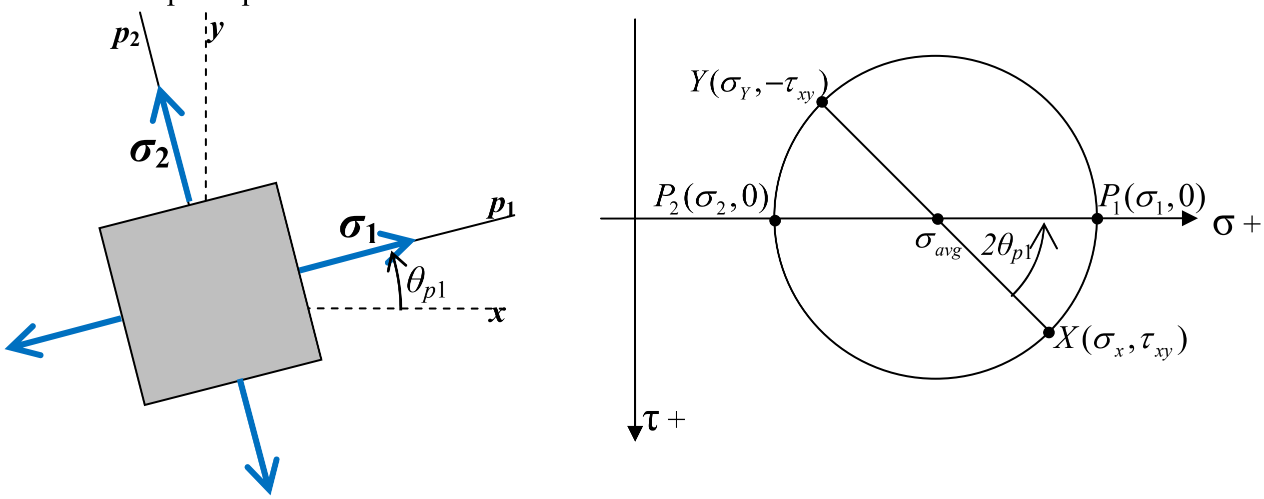
<u>How to use the Mohr's Circle</u>

1. Stress transformation from *x*-*y* plane to *x*'-*y*' plane in a Mohr's Circle:



***θ* CCW rotation (*x* to *x*') in stress element = 2*θ* CCW rotation in Mohr's Circle (*X* to *X*')**

2. Plane of principal stress



3. Plane of maximum in-plane shear stress