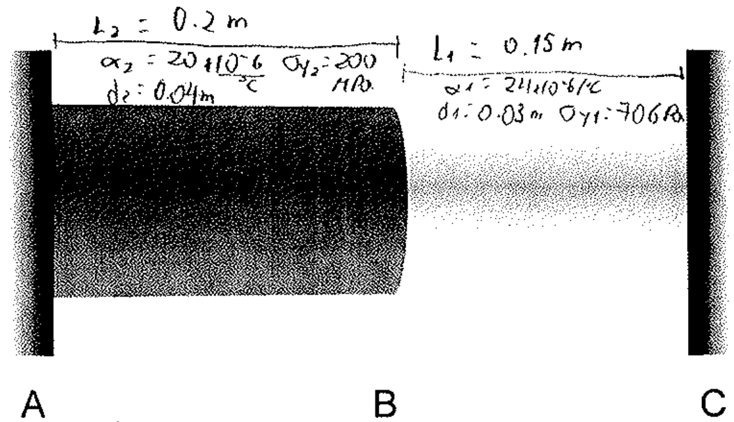
ME 323 Examination # 1 **Name** Solution Problem 1
(Print) **(Last)** **(First)**

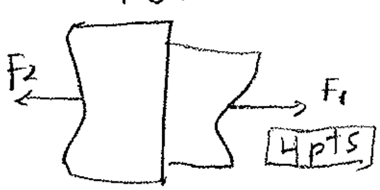
October 1, 2008 **Instructor** ____________________

PROBLEM #1 (33 points)

An assembly of two cylindrical rods (AB) and (BC) is restrained at both ends. Rod AB (length L= 200 mm, diameter 40 mm) is made of brass (modulus of elasticity E = 100 GPa, coefficient of thermal expansion $\alpha = 20\times10^{-6}/^\circ C$ and yield strength $\sigma_Y = 200$ MPa). Rod BC (length 150 mm, diameter 30 mm) is made of aluminum (modulus of elasticity E = 70 GPa, coefficient of thermal expansion $\alpha = 24\times10^{-6}/^\circ C$ and yield strength $\sigma_Y =$ 80 MPa). The system is heated by $100^\circ C$. Determine if the system is safe or will fail by plastic deformation.



FBD



$\sum F_x = 0$

$-F_2 + F_1 = 0$

$F_2 = F_1$

4pts

Geometry of Deformation:

$e_1 + e_2 = 0$

$e_1 = -e_2$ 8 pts

$$e_1 = \frac{F_1 L_1}{E_1 A_1} + L_1 \alpha_1 \Delta T$$

$$= \frac{F_1 (0.15m)}{70\times10^9 Pa \times \pi (0.015m)^2} + 0.15m\,(24\times10^{-6}/^\circ C)\times 100^\circ C$$

$$= 3.031\times10^{-9} F_1 + 0.00036\,m$$

$$e_2 = \frac{F_2 L_2}{E_2 A_2} + L_2 \alpha_2 \Delta T$$ 8 pts

$$= \frac{F_2 (0.2m)}{100\times10^9 Pa \times \pi (0.02)^2} + 0.2\,m \times \left(\frac{20\times10^{-6}}{^\circ C}\right)\times 100^\circ C$$

$$= 1.141\times10^{-9} F_2 + 0.0004\,m$$

using $e_1 = -e_2$ and $F_2 = F_1$

$$3.031\times10^{-9} F_1 + 0.00036\,m = -1.141\times10^{-9} F_1 - 0.0004$$

$$4.172\times10^{-9} F_1 = -0.00076$$ 5pts

$F_1 = -182.166$ KN

$F_2 = -182.166$ KN

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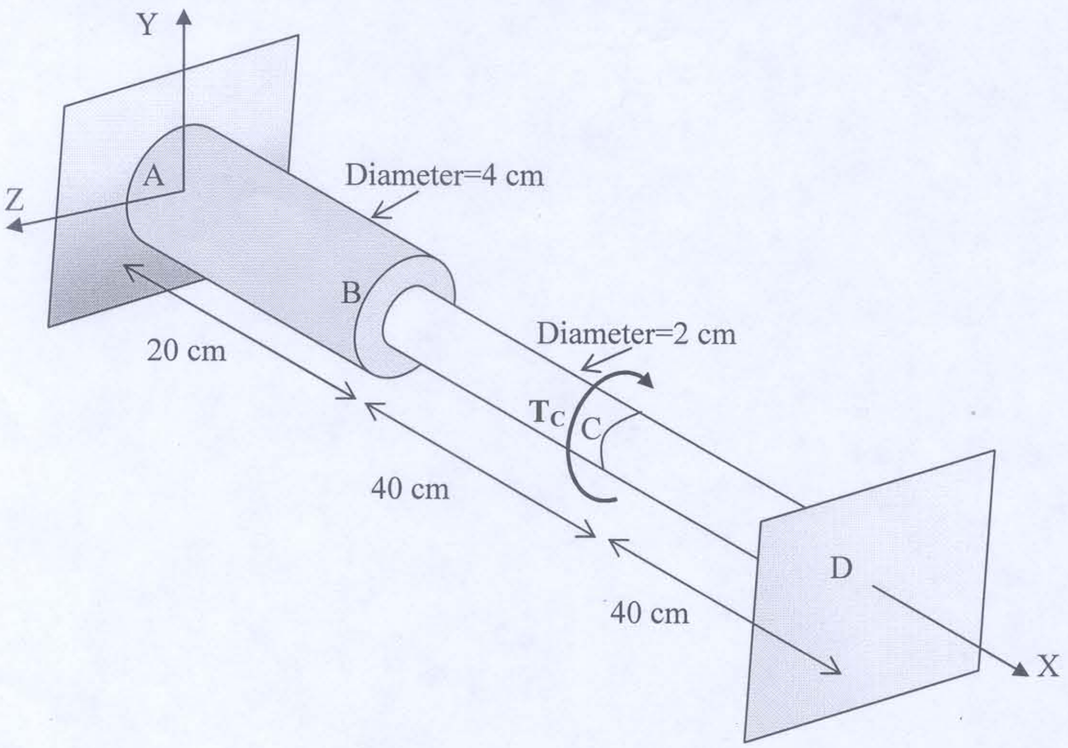
PROBLEM #4 (33 points)

The stepped rod shown below is made of Aluminum and bronze (shear modulus of aluminum G_{al}=25 GPa and shear modulus of bronze G_{br}=20 GPa) and is fixed to the walls at A and D. The aluminum section is 20 cm long and has a diameter of 4 cm. The bronze section is 80 cm long and has a diameter of 2 cm. An external torque T_C is applied as shown. The yield stress of the aluminum and bronze are τ_Y = 20 MPa and 10 MPa respectively. Determine the maximum allowable applied torque at C, T_C?



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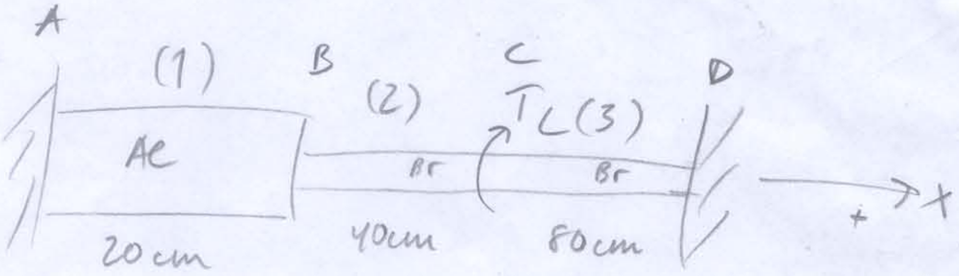
Prob. 2

FBD

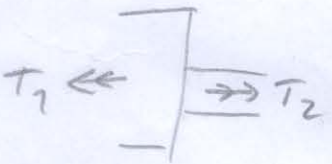


A:

$$T_1 - T_A = 0$$

$$T_1 = T_A$$

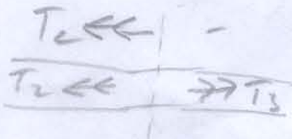
B:



$$T_2 - T_1 = 0$$

$$T_1 = T_2$$

C:



$$T_3 - T_2 - T_C = 0$$

$$T_C = T_3 - T_2$$

D:

$$T_D - T_3 = 0$$

$$T_3 = T_D$$

(8 pts)

$$I_{P\,Al} = \frac{\pi d_{Al}^4}{32} = 8 \cdot 10^{-8} \pi \ m^4$$

$$I_{P\,Br} = \frac{\pi \cdot d_{Br}^4}{32} = 5 \cdot 10^{-9} \pi \ m^4$$

(2 pts)

$$T_1 = \frac{\tau_{Al} \cdot I_{pAl}}{r_{Al}} = \frac{20 \cdot 10^6 \cdot 8 \cdot 10^{-8}\pi}{0{,}02} = 80\pi \text{ Nm}$$

$$T_2 = \frac{\tau_{Br} \cdot I_{pBr}}{r_{Br}} = \frac{10 \cdot 10^6 \cdot 5 \cdot 10^{-9}\pi}{0{,}01} = 5\pi \text{ Nm}$$

By FBD $T_1 = T_2 = 5\pi$ Nm

(9 pts)

$$\varphi_1 = \frac{T_1 L_{Al}}{G_{Al} I_{pAl}} = \frac{5\pi \cdot 0.2}{25 \cdot 10^9 \cdot 8 \cdot 10^{-8}\pi} = 0{,}005$$

$$\varphi_2 = \frac{T_2 \cdot L_{Br\,2}}{G_{Br} \cdot I_{pBr}} = \frac{5\pi \cdot 0{,}4}{20 \cdot 10^9 \cdot 5 \cdot 10^{-9}\pi} = 0.02$$

$$\varphi_3 = \frac{T_3 \cdot L_{Br\,3}}{G_{Br} \cdot I_{pBr}} = \frac{T_3 \cdot 0.8}{20 \cdot 10^{-9} \cdot 5 \cdot 10^{-9}\pi} = \frac{0.008}{\pi} T_3$$

Deformation geometry

$$\varphi_1 + \varphi_2 + \varphi_3 = 0$$

$$\Rightarrow 0{,}005 + 0{,}02 + \frac{0.008}{\pi} T_3 = 0$$

(12 pts)

$$T_3 = -2.5625\pi \text{ Nm}$$

$$T_C = T_3 - T_2 = -2.5625\pi - 5\pi = -7.5625\pi = -23.7583 \text{ Nm}$$

(3 pts)

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Check if section 1 or 2 fails:

$\sigma = \frac{F}{A}$ [2 pts]

$\sigma_1 = \frac{F_1}{A_1} = \frac{182\,166\ N}{\pi\,(0.015)^2} = 257.712\ MPa > 80\ MPa$ it fails!

$\sigma_2 = \frac{F_2}{A_2} = \frac{182\,166\ N}{\pi\,(0.02)^2} = 144.96\ MPa < 200\ MPa$ ok!

System fails since $\sigma_1 > \sigma_{Y1}$ [2 pts]

total = 33 Points

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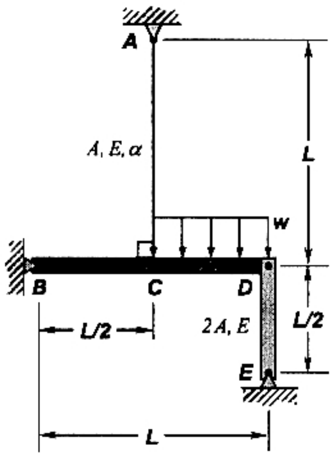
PROBLEM #5 (33 points)

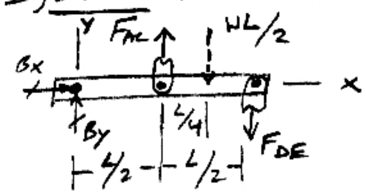
The rigid beam *BCD* shown below is supported by a frictionless pin at point *B*, vertical thin wire *AC* at point *C*, and bar *DE* at point *D*. Assume the following:

a) Prior to loading, beam *BCD* is horizontal
b) Prior to loading, there is no stress in wire *AC* or rod *DE*
c) Weight of beam *BCD* is negligible
d) There is very small displacement at point *D*
e) Wire *AC* has cross section area = *A*, modulus of elasticity, *E*
f) Bar *DE* has cross section area = 2*A*, and modulus of elasticity, *E*

The uniformly distributed load with intensity *w* per unit length is applied across section *CD*. Develop expressions for the axial forces in wire *AC* and bar *DE*.



10pts I) EQUIL FBD BCD



$+\circlearrowright (\Sigma M)_B = 0$

$-F_{AC}(L/2) + wL/2\left(\frac{3}{4}L\right) + F_{DE}(L) = 0$

$4F_{AC} - 8F_{DE} = 3wL$ (EQ 1)

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10pts II) FORCE - DEFORM BEHAVIOR

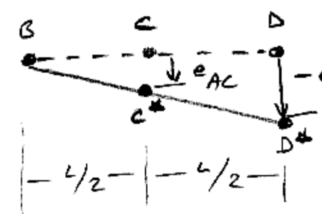
$e_{AC} = \left(\frac{FL}{AE}\right)_{AC}$; $L_{AC} = L$, $A_{AC} = A$, $E_{AC} = E$ $\Big\} = f$

$e_{AC} = \frac{F_{AC}L}{AE} = f F_{AC}$ (EQ 2)

$e_{DE} = \left(\frac{FL}{AE}\right)_{DE}$; $L_{DE} = L/2$, $A_{DE} = 2A$, $E_{DE} = E$ $\Big\} = \frac{f}{4}$

$e_{DE} = \frac{F_{DE}L}{4AE} = \frac{f}{4}F_{DE}$ (EQ 3)

10pts III) DEFORM GEOM (i.e. compatibility)



$\frac{e_{AC}}{L/2} = -\frac{e_{DE}}{L}$

$2e_{AC} + e_{DE} = 0$ (EQ 4)

3pts

• COMBINE EQ's 2 - 4

$2(fF_{AC}) + \frac{f}{4}F_{DE} = 0 \Rightarrow 8F_{AC} + F_{DE} = 0$ (EQ 5)

• SOLVE EQ's 1 & 5 SIMULTANEOUSLY

$-2[4F_{AC} - 8F_{DE} = 3wL]$

$8[8F_{AC} + F_{DE} = 0]$

$\to (4+64)F_{AC} = 3wL \Rightarrow$ $F_{AC} = \frac{3}{68}wL$

$\to (16+1)F_{DE} = -6wL \Rightarrow$ $F_{DE} = -\frac{6}{17}wL$