Name: _____________, _______________ Instructor: Susilo Koslowski Raman
(Print) (Last) (First) (Circle one)

ME 323 MIDTERM # 1: SOLUTION

FALL SEMESTER 2011

Time allowed: 1hour

Instructions

1. Begin each problem in the space provided on the examination sheets. If additional space is required, use the yellow paper provided. Work on one side of each sheet only, with only one problem on a sheet.
2. Each problem is of equal value.
3. To obtain maximum credit for a problem, you must present your solution clearly. Accordingly:
    a. Identify coordinate systems
    b. Sketch free body diagrams
    c. State units explicitly
    d. Clarify your approach to the problem including assumptions
4. If your solution cannot be followed, it will be assumed that it is in error.
5. When handing in the test, make sure that ALL SHEETS are in the correct sequential order. Remove the staple and restaple, if necessary.

Prob. 1 _______________________

Prob. 2 _______________________

Prob. 3 _______________________

Total _________________________

Name: ______________, ________________ Instructor: Susilo Koslowski Raman
(Print) (Last) (First) (Circle one)

Equations

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \quad \gamma_{xz} = \frac{1}{G}\tau_{xz} \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\gamma = \rho\frac{d\phi}{dx}$$

$$\tau = G\rho\frac{d\phi}{dx}$$

$$T = G\frac{d\phi}{dx}\int_{cross\,section}\rho^2 dA$$

$$T = GI_p\frac{d\phi}{dx}$$

$$\tau = \frac{Tr}{I_p}$$

$$\varphi = \frac{TL}{GI_p}$$

$$I_{p\_Circular\_Cross\_Section} = \frac{\pi d^4}{32}$$

$$I_{p\_Hollow\_Circular\_Cross\_Section} = \frac{\pi(d_o{}^4 - d_i{}^4)}{32}$$

$$\varepsilon = \frac{L_f - L_i}{L_i}$$

$$e = \frac{FL}{EA} + L\alpha\Delta T$$

$$F = \frac{EA}{L}(e - L\alpha\Delta T)$$

$$FS = \frac{Failure\ Stress}{Allowable\ Stress}, \frac{Yield\ Strength}{State\ of\ Stress}$$

$$e = u\cos(\theta) + v\sin(\theta)$$

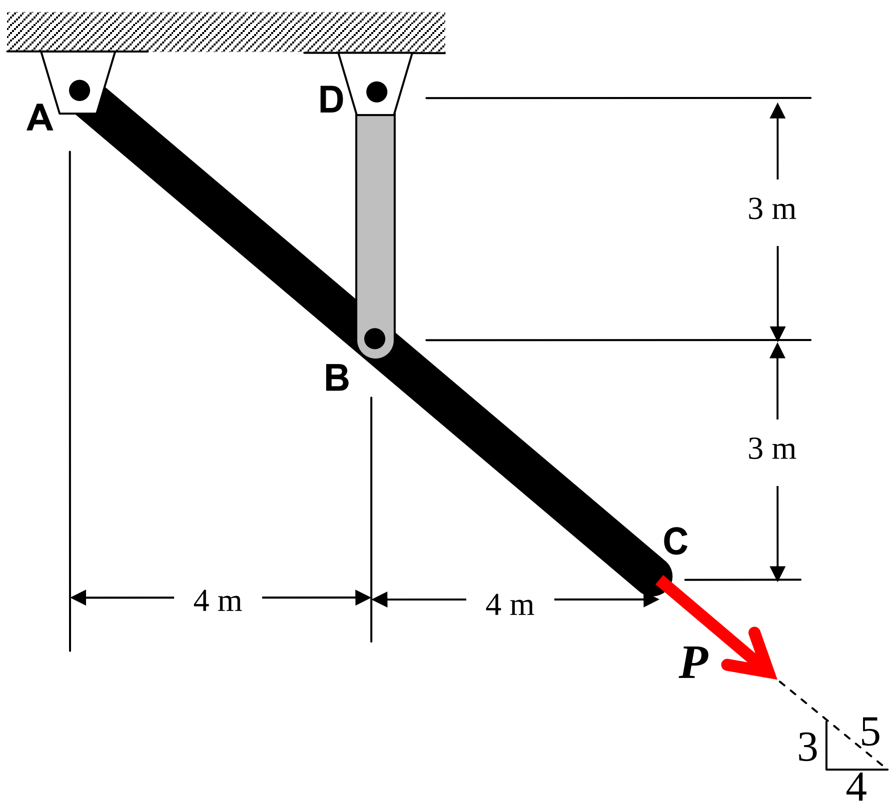
Name: ________________, __________________ Instructor: Susilo Koslowski Raman
(Print) (Last) (First) (Circle one)

Problem 1

In the structure shown below the Rod **BD** is made of aluminum ($\sigma_Y = 60 \times 10^3$ ksi) with a cross-sectional area $A_{BD} = 3\ in^2$. The pin at **A** is made of brass ($\tau_Y = 15 \times 10^3$ ksi), has a diameter $d_{pin} = 0.25$ in and holds the structure in a single shear joint.

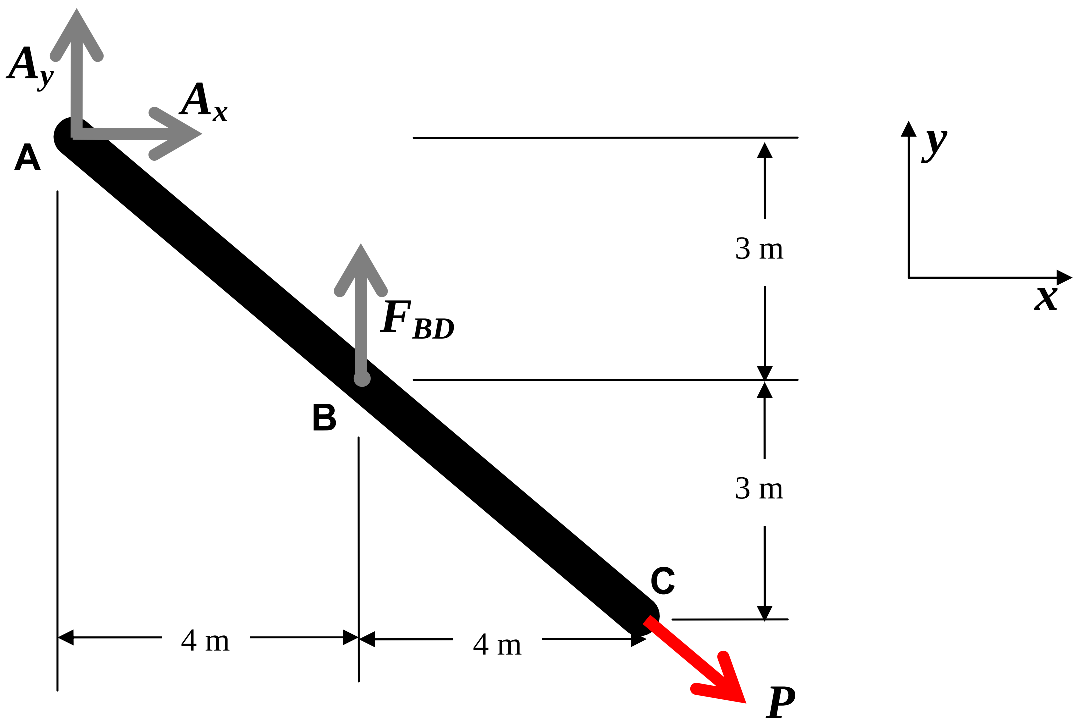
Find the maximum force $\boldsymbol{P_{max}}$ that can be applied at point **C** given that a **Factor of Safety of 2.0** is required for both rod **BD** and pin **A**.

Assume that bar **ABC** is rigid and that the diameter of pins **B** and **C** are large enough that they do not need to be considered. Only consider rod **BD** and the pin **A** in your calculation to find $\boldsymbol{P_{max}}$.



Name: ______________, ________________ Instructor: Susilo Koslowski Raman
(Print) (Last) (First) (Circle one)

<u>Static equilibrium analysis to relate internal forces to P</u>



$$\sum M_A = 0 \rightarrow (4\,ft)F_{BD} + (6\,ft)\left(\frac{4}{5}P\right) - (8\,ft)\left(\frac{3}{5}P\right) = 0$$

$$\therefore F_{BD} = 0$$

$$\sum F_x = 0 \rightarrow A_x = -\frac{3}{5}P$$

$$\sum F_y = 0 \rightarrow A_y = \frac{4}{5}P - F_{BD} = \frac{4}{5}P$$

<u>Considering bar BD:</u>

$$\sigma_{allow} = \frac{\sigma_{failure}}{FS} = \frac{60\times10^3 ksi}{2} = 30\times10^3 ksi$$

$$F_{allow} = \sigma_{allow} A_{BD} = \left(30\times10^3 ksi\right)\left(3in^2\right) = 90\times10^3 kips$$

From static equilibrium analysis, $F_{BD} = 0$, which means that F_{BD} is independent of P and will never exceed F_{allow} Fallow for any value of P.

<u>Considering Pin A:</u>

$$\tau_{allow} = \frac{\tau_{failure}}{FS} = \frac{15\times10^3 ksi}{2} = 7.5\times10^3 ksi$$

$$V_{allow} = \tau_{allow} A_{BD} = \left(7.5\times10^3 ksi\right)\left(\frac{\pi}{4}0.25^2 in^2\right) = 386.15 kips$$

Resultant force on pin: $F_A = \sqrt{\left(A_x\right)^2 + \left(A_y\right)^2}$

Since this is a single shear pin, $V_{pin} = F_A$

Therefore, the maximum shear force on the pin is, $V_{allow,pin} = 386.15 kips$

$$V_{allow,pin} = 386.15 kips = \sqrt{\left(-\frac{3}{5}P_{\max}\right)^2 + \left(\frac{4}{5}P_{\max}\right)^2} = P_{\max}$$

$$\therefore P_{\max} = 386.15 kips$$

Name: ______________, ________________ Instructor: Susilo Koslowski Raman
(Print) (Last) (First) (Circle one)

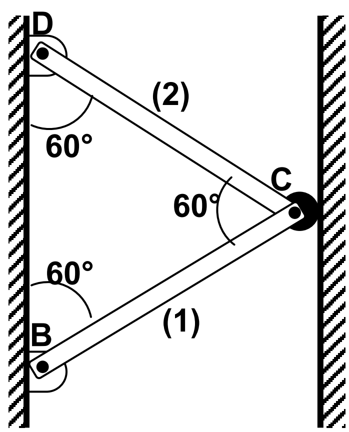
Problem 2

A mechanism for a thermal switch is shown below. The mechanism consists of two elastic bars of length $\mathbf{L}$, elastic modulus $\mathbf{E}$, coefficient of thermal expansion $\boldsymbol{\alpha}$, cross section area $\mathbf{A}_{\mathbf{bar}}$. The mechanism is pinned to a rigid wall at **B** and **D** and at **C** the mechanism is pinned to a rigid roller. Both the bars are uniformly heated or cooled by the surrounding air temperature. Gravity is not to be considered in the problem.

Assume a starting condition when the temperature is at $\mathbf{T_c}$ and the gap is just closed, and the roller is just touching the wall (as shown),

(a) Let the temperature be <u>increased</u> to a final temperature $\mathbf{T_f}$ so that $\mathbf{T_f - T_c > 0}$ so the roller presses firmly against the wall. Using the method of sections draw a free body diagram. What additional equations do you need to solve for the internal axial forces? Derive an expression for the axial normal stress $\boldsymbol{\sigma_1, \sigma_2}$ in the two rods in terms of $\boldsymbol{\alpha}, \mathbf{L}, \mathbf{E}, \mathbf{A}_{bar}, (\mathbf{T_f} - \mathbf{T_c})$.

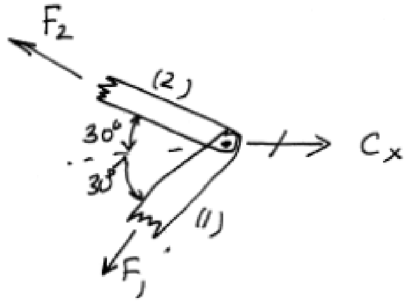
(b) Next let the temperature of the bars be <u>decreased</u> to a final temperature $\mathbf{T_f}$ so that $\mathbf{T_f - T_c} < \mathbf{0}$ and a gap opens up between the roller and the rigid wall. Using the method of sections draw a free body diagram and find the internal axial forces in the two bars. What is the axial normal stress $\boldsymbol{\sigma_1, \sigma_2}$ in the two rods?



Name: ______________, ________________ Instructor: Susilo Koslowski Raman
(Print) (Last) (First) (Circle one)

Problem 2

(a) FBD w/ sections



Note: since all Joints are pin Joints, this is a truss with no shear force or bending moment resultants

$+\uparrow \sum F_y = 0 \Rightarrow +F_2 \sin 30° - F_1 \sin 30° = 0 \Rightarrow \therefore \boxed{F_1 = F_2}$ ①

$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow -F_1 \cos 30° - F_2 \cos 30° + C_x = 0$

$\Rightarrow \boxed{C_x - F_1 \frac{\sqrt{3}}{2} - F_2 \frac{\sqrt{3}}{2} = 0}$ ②

Note: There was no need to assume symmetry to prove this

We have 2 eqns ① & ② but 3 unknowns C_x, F_1 & F_2

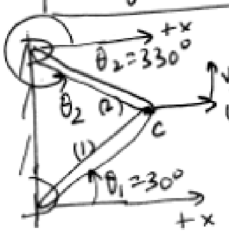
By combining ① & ②, we have $F_1 = F_2 = F$

$C_x - \sqrt{3} F = 0$ ③ i.e 1 eqn & 2 unknowns C_x & F

So we need a compatability condition!

Note: We should not assume $e_1 = e_2 = 0$; instead we need to use the given statement that $u_c = 0$

Remember sign convention while using this formula



$e_1 = \overset{0}{\cancel{u_c}} \cos\theta_1 + v_c \sin\theta_1 = 0.5 v_c$

$e_2 = \overset{0}{\cancel{u_c}} \cos\theta_2 + v_c \sin\theta_2 = -0.5 v_c$

Combining the two, $\boxed{e_1 + e_2 = 0}$! ④

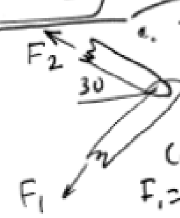
But $e_1 = \frac{F_1 L}{EA} + \alpha L \Delta T$ & $e_2 = \frac{F_2 L}{EA} + \alpha L \Delta T$

and since $F_1 = F_2$ (see eqn ①) $\therefore$ $\boxed{e_1 = e_2}$ ⑤

From ④ & ⑤, we have $e_1 = e_2 = 0$

$\therefore \boxed{\sigma_1 = \frac{F_1}{A} = -E\alpha\Delta T = \sigma_2}$

(b)



$+\uparrow \sum F_y = 0 \Rightarrow F_2 \sin 30 = F_1 \sin 30$

$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow -F_1 \cos 30 - F_2 \cos 30 = 0$

Combining, we find $F_1 = F_2 = 0 \Rightarrow \therefore \sigma_1 = \sigma_2 = 0$

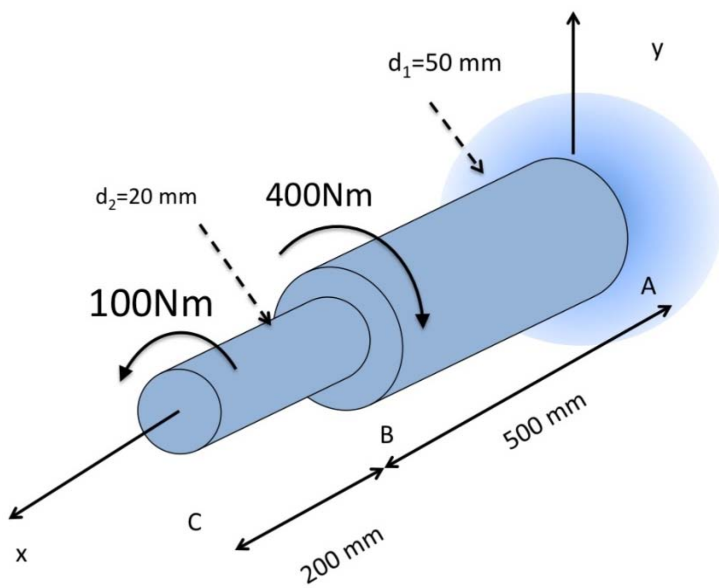
Name: ________________, __________________ Instructor: Susilo Koslowski Raman
(Print) (Last) (First) (Circle one)

Problem 3

The shaft **AC** is subjected to torsional loads at sections **B** and **C**. The diameters are d_1=50 mm and d_2=20mm.

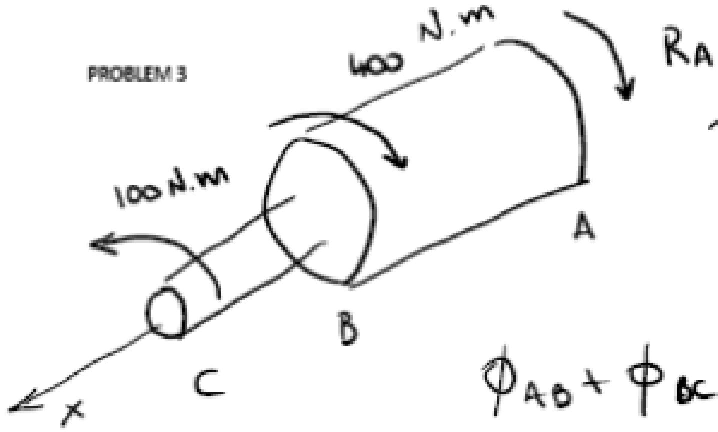
(a) If the twist angle at point **C** is ϕ_C**=0.06rad** determine the shear modulus (***G***) of the material of the shaft**.**

(b) Determine the magnitude of the maximum shear stress in the shaft and indicate the location of maximum shear stress along the shaft, and location on the cross section.



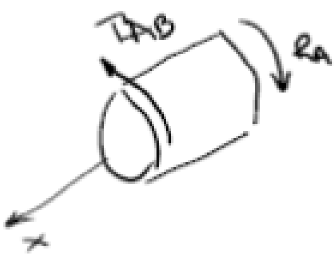
Name: ______________, ________________ **Instructor: Susilo Koslowski Raman**
(Print) **(Last)** **(First)** **(Circle one)**

PROBLEM 3



$R_A + 400 - 100 = 0$
$= -300$ Nm.

$\phi_{AB} + \phi_{BC} = 0.06$ rad



$$\frac{T_{AB} L_{AB}}{G I_{p_1}} + \frac{T_{BC} L_{BC}}{G I_{p2}} = 0.06$$

$T_{AB} = -300$ N.m

$T_{BC} = 100$ N.m

$$I_{p_1} = \frac{\pi d_1^4}{32} = 6.1\ 10^{-7}\ m^4$$

$$I_{p2} = \frac{\pi d_2^4}{32} = 1.6\ 10^{-8}$$

$G = 17$ GPa

b) Max between B and C, outer radius.

$\tau^{AB} = 12.2$ MPa

$\tau^{BC} = 63.$ MPa