ME 323 Examination # 1 **Name** ______________________________

February 21, 2018 **Instructor** ______________________________

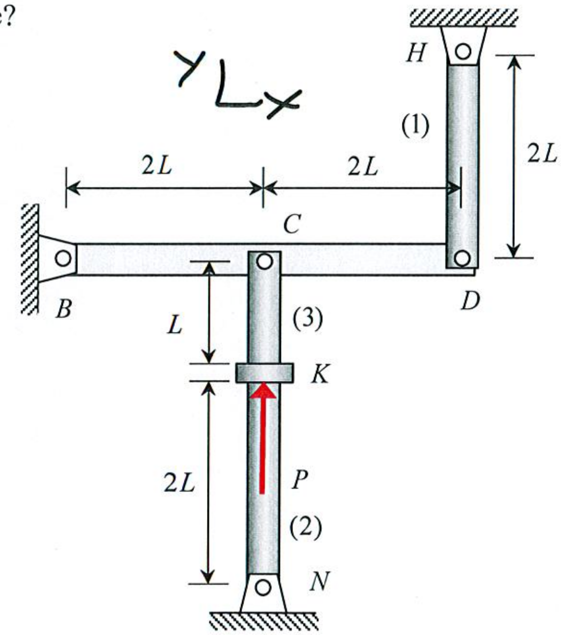
PROBLEM NO. 1 – 25 points max.

A *rigid* bar BD (of length $4L$) is pinned to ground at end B. This bar is also supported by an elastic element (1) and an elastic element assembly (2)+(3) at pins D and C, respectively, as shown in the figure. Each element is made up of a material with a Young's modulus of E and a coefficient of thermal expansion of α, and has a cross-sectional area of A. The temperature of element (1) is increased by an amount ΔT, whereas the temperature of (2) and (3) remain unchanged. A vertical force P acts at rigid connector K that joins elements (2) and (3). Let the temperature change and applied loading be related through $\alpha\Delta T = 2P/EA$. Assume that bar BD undergoes small rotations resulting from the applied load and temperature change.

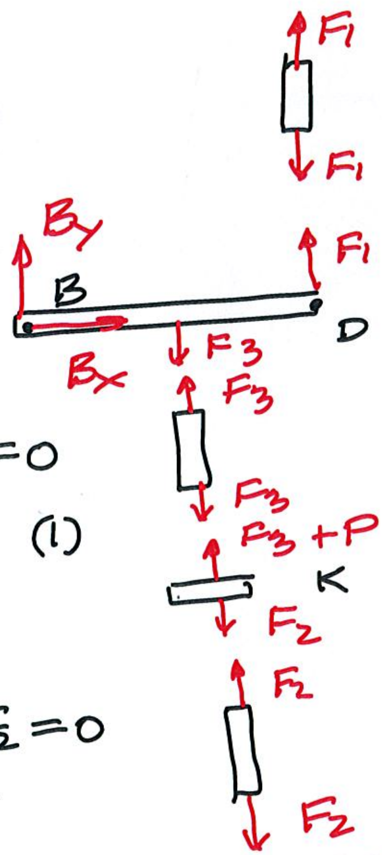
a) Determine the load (force) carried each of the elements (1), (2) and (3). Your answers should be expressed as a fraction of the applied load P.

b) Is the *stress* in element (1) tensile or compressive?

c) Is the *strain* in element (1) extensive or contractive?



1. Equilibrium



BD:

$\sum M_B = F_3(2L) - F_1(4L) = 0$

↳ $\boxed{F_3 = 2F_1}$ (1)

K:

$\sum F_y = F_3 + P - F_2 = 0$

↳ $F_2 - F_3 = P$

↳ $F_2 - 2F_1 = P \Rightarrow \boxed{F_2 = 2F_1 + P}$ (2)

INDETERMINATE!

2. Load/deformation

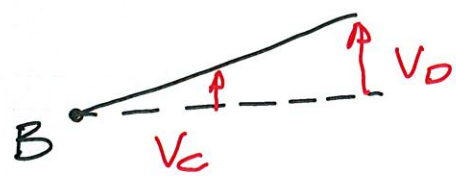
(3) $e_1 = \dfrac{F_1(2L)}{EA} + \alpha\Delta T(2L)$

(4) $e_2 = \dfrac{F_2(2L)}{EA}$

(5) $e_3 = \dfrac{F_3(L)}{EA}$

3. Compatibility

For small angle of rotation θ:



$$\left.\begin{aligned} V_C &= (2L)\theta \\ V_D &= (4L)\theta \end{aligned}\right\} \Rightarrow \theta = \frac{V_C}{\cancel{2}\cancel{L}} = \frac{V_D}{\underset{2}{\cancel{4}}\cancel{L}} \Rightarrow V_D = 2V_C$$

Also $V_C = e_2 + e_3$

$V_D = -e_1$

(6) $\therefore\ -e_1 = 2(e_2 + e_3)$

4. Solve

(3) – (6): $-\left[\dfrac{2F_1\cancel{L}}{EA} + 2\alpha\Delta T\cancel{L}\right] = 2\left[\dfrac{2F_2\cancel{L}}{EA} + \dfrac{F_3\cancel{L}}{EA}\right]$

(7) $\hookrightarrow\ -2F_1 - 2\alpha\Delta TEA = 4F_2 + 2F_3$

(1), (2), (7): $-2F_1 - 2\alpha\Delta TEA$

$= 4[2F_1 + P] + 2[2F_1]$

$$\hookrightarrow \quad (8+4+2)F_1 = -2\alpha\Delta T EA - 4P$$

$$\hookrightarrow \quad F_1 = -\frac{1}{7}\alpha\Delta T\, EA + \frac{2}{7}P \;;\; \alpha\Delta T = \frac{2P}{EA}$$

$$= -\frac{1}{7}\left(\frac{2P}{EA}\right)EA - \frac{2}{7}P = -\frac{4}{7}P$$

comp. stress

$$\therefore \quad F_2 = 2\left[-\frac{4}{7}P\right] + P = -\frac{P}{7}$$

$$F_3 = 2\left[-\frac{4}{7}P\right] = -\frac{8}{7}P$$

$$e_1 = \frac{2F_1L}{EA} + 2\alpha\Delta T L$$

$$= \frac{2L}{EA}\left(-\frac{4}{7}P\right) + 2\left(\frac{2P}{EA}\right)L$$

$$= \left(-\frac{8}{7}+4\right)\frac{PL}{EA} = \frac{20}{7}\frac{PL}{EA}$$

$\hookrightarrow$ extensive strain

PROBLEM NO. 2 – 25 points max.

A composite shaft is made of a brass sleeve (1) ($G_1 = 40\ GPa$) over an aluminum core (2) ($G_2 = 25\ GPa$). The sleeve (1) and core (2) are each attached to a fixed wall at B and to a rigid connector at A. A torque $T = 200\ N \cdot m$ is applied to connector A. The diameter of the aluminum core is $d_2 = 50\ mm$, and the length of the shaft is $L = 0.5\ m$.

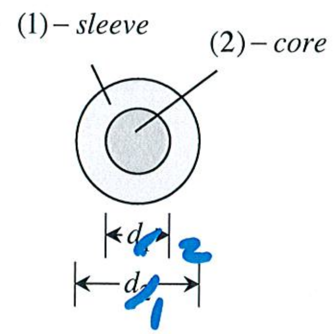
$d_2 = 0.05\ m$

a) Determine the thickness of the brass sleeve to obtain an angle of twist $\phi_A = 0.0015\ rad$ at end A.

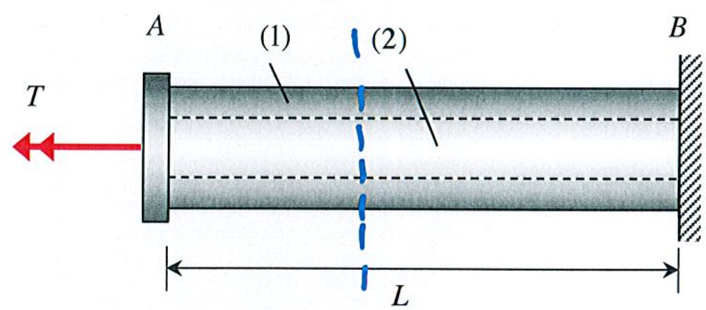
b) Determine the torque load carried by each shaft.

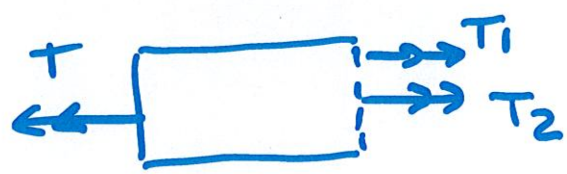
c) Calculate the maximum *shear stress* for the shaft and show its location on the cross-section.

d) Calculate the maximum *shear strain* for the shaft and show its location on the cross-section.



cross-sectional view





$$T_1 + T_2 = T = 200\ N.m$$

$$\phi_A = \phi_1 = \phi_2 = \frac{T_1 L}{G_1 I_{p_1}} = \frac{T_2 L}{G_2 I_p}$$

(1) $\phi_A = 0.0015 = \dfrac{T_1 L}{G_1 I_{p_1}}$ $\quad I_{p_1} = \dfrac{\pi}{32}(d_1^4 - d_2^4)$

PROBLEM NO. 2 (continued)

$$\phi_A = 0.0015 = \frac{T_2 L}{G_2 I_{p2}} \qquad I_{p2} = \frac{\pi}{32} d_2^4$$

$$\Rightarrow T_2 = \frac{0.0015 \cdot 25 \; 10^9 \cdot \pi (25 \cdot 10^{-3})^4}{0.5 \cdot 2}$$

b) $\boxed{T_2 = 46 \text{ Nm}}$ $\quad T_1 = 200 - 46$

$$\boxed{T_1 = 154 \text{ N.m}}$$

Using (1) $\quad I_{p1} = \frac{T_1 L}{G_1 \cdot 0.0015}$

$$(d_1^4) = (d_2)^4 + \frac{T_1 L \cdot 32}{G_1 \cdot 0.0015 \cdot \pi}$$

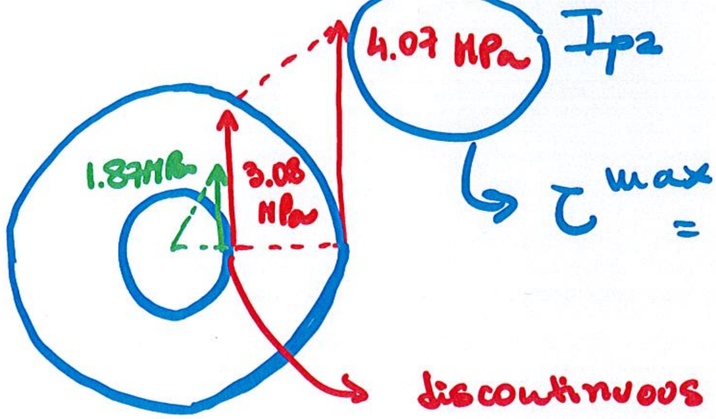
$$\boxed{d_1 = 0.066 \text{ m}}$$

a) $\quad t = \frac{d_1 - d_2}{2} = \frac{0.016}{2} \text{ m}$

$$\boxed{t = 0.008 \text{ m}}$$

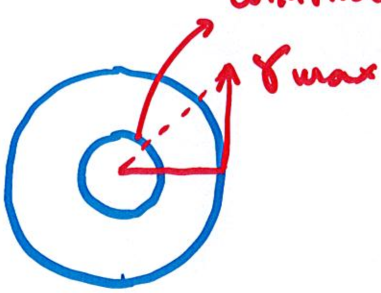
c) $\tau_{(1)}^{max} = \dfrac{T_1\, d_1/2}{I_{p1}} = 4.07\,MPa$

$\tau_{(2)}^{max} = \dfrac{T_2\, d_2/2}{I_{p2}} = 1.87\,MPa$



$\tau^{max} = 4.07\,MPa$

d)



$\gamma_{max} = \dfrac{d_1}{2}\,\dfrac{0.0015}{L}$

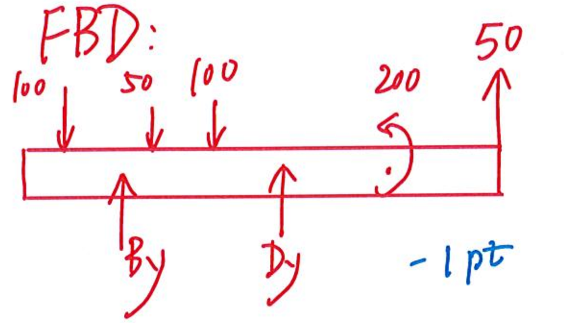
$= 0.000099$

PROBLEM NO. 3 – 25 points max.

A simply-supported beam AF is 12 *m* long and is supported by a pine joint at B and and a roller support at D. It is subjected to the following loads as shown.

Construct shear force and bending moment diagrams for the beam. Mark numerical values for points A through F along with max/min values on these diagrams for the shear, and moment diagrams respectively. Provide justification for the numbers of the shear force and bending moment diagrams and label the function types: *constant*, *linear*, *quadratic*, *cubic*, etc.

FBD:



-1 pt

$\Sigma M_B = 0 \Rightarrow$

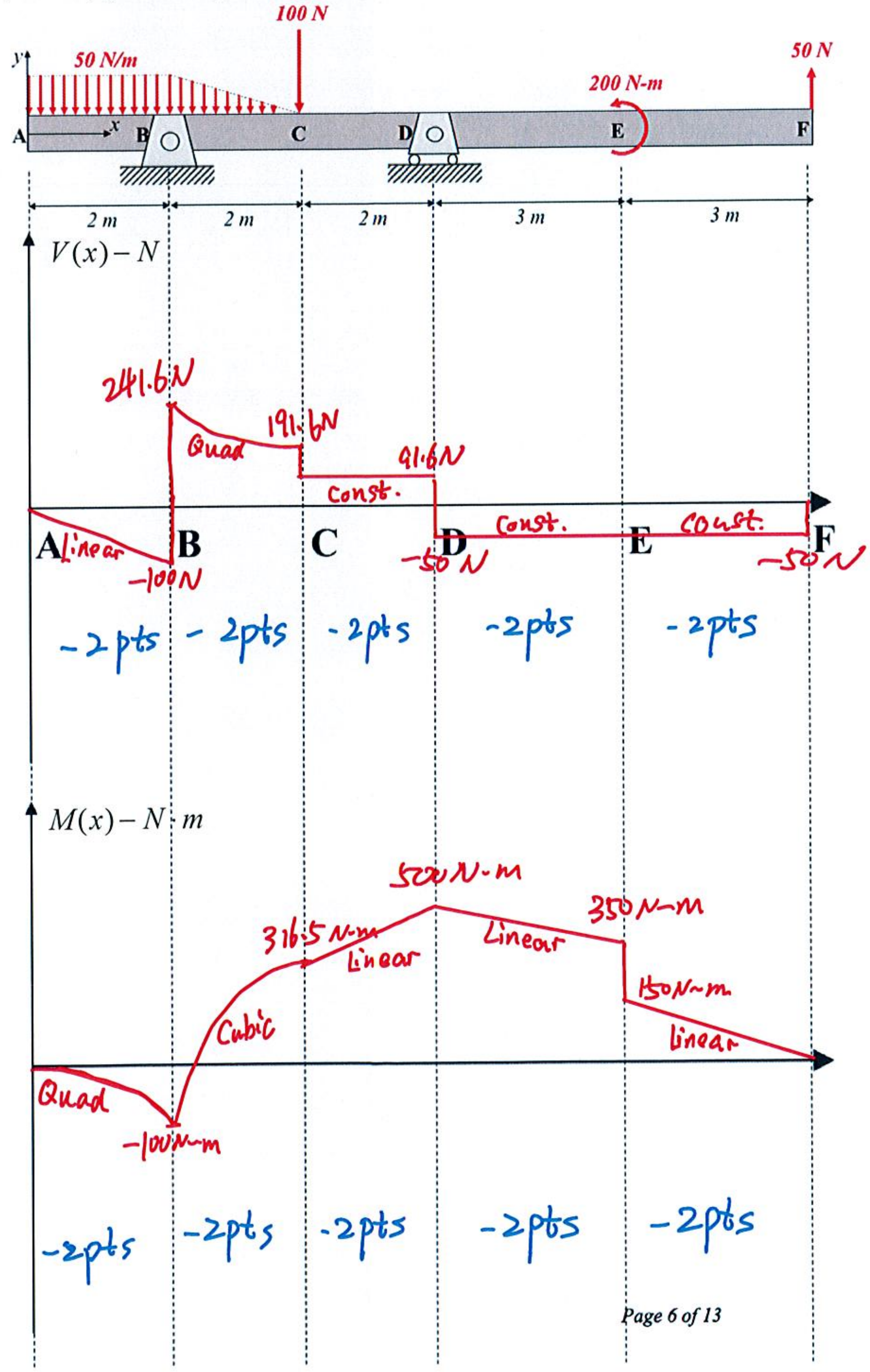
$50 \times 10 + 200 + D_y \cdot 4 - 100 \times 2 - 50 \times \frac{2}{3} + 100 \times 1 = 0$

$D_y = -141.6\ N$ -2pt

$\Sigma F_y = 0 \Rightarrow$

$50 + D_y - 100 - 50 + B_y - 100 = 0$

$B_y = 341.6\ N$ -2pt



PROBLEM NO. 3 (continued)

① Section AB:

$V(0^+) = 0$

$V(2^-) = V(0) - 50\times2 = -100\ N$

$M(0^+) = 0$

$M(2^-) = 0 - \frac{1}{2}\times2\times100 = -100\ N\text{-}m$

② Section BC:

$V(2^+) = V(2^-) + B_y = -100 + 341.6 = 241.6\ N$

$V(x^-) = V(2^+) + \int_2^x -50 + \frac{50}{2}(x-2)\,dx.$

$= 241.6 + \left(-100x + \frac{50}{4}x^2\right)\Big|_2^x$

$= 391.6 - 100x + \frac{50}{4}x^2$

$V(4^-) = 391.6 - 100\times4 + \frac{50}{4}\times16$

$= 191.6\ N$

$M(2^+) = M(2^-) = -100\ N\text{-}m.$

$M(x^-) = M(2^+) + \int_2^x (391.6 - 100x + \frac{50}{4}x^2)\,dx$

$= -100 + \left(391.6x - 50x^2 + \frac{50}{12}x^3\right)\Big|_2^x$

$= -716.5 + 391.6x - 50x^2 + \frac{25}{6}x^3$

$M(4^-) = 316.6\ N\text{-}m$

③ Section CD:

$V(4^+) = V(4^-) - 100 = 91.6\ N$

$V(6^-) = V(4^+) = 91.6\ N$

$M(4^+) = M(4^-) = 316.6\ N\text{-}m$

$M(6^-) = M(4^+) + 91.6\times2$

$= 500\ N$

④ Section DE:

$V(6^+) = V(6^-) + D_y$

$= 91.6 - 141.6 = -50\ N$

$V(9^-) = V(6^+) = -50\ N$

$M(6^+) = M(6^-) = 500\ N\text{-}m$

$M(9^-) = M(6^+) - 50\times3 = 350\ N\text{-}m$

⑤ Section EF:

$V(9^+) = V(9^-) = -50\ N$

$V(12^-) = V(9^+) = -50\ N$

$M(9^+) = M(9^-) - 200 = 150\ N\text{-}m$

$M(12^-) = M(9^+) - 50\times3 = 0\ N\text{-}m$

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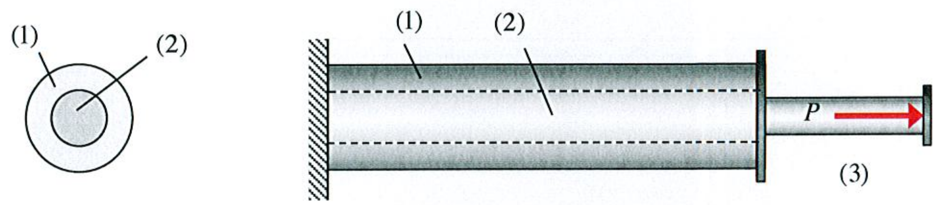
YOU ARE NOT REQUIRED TO SHOW YOUR WORK FOR ANY PART OF PROBLEM NO. 4.

PROBLEM NO. 4 - PART A – 2 points max.

TRUE or *FALSE*: The stress in element (3) depends on the Young's moduli of the three elements.

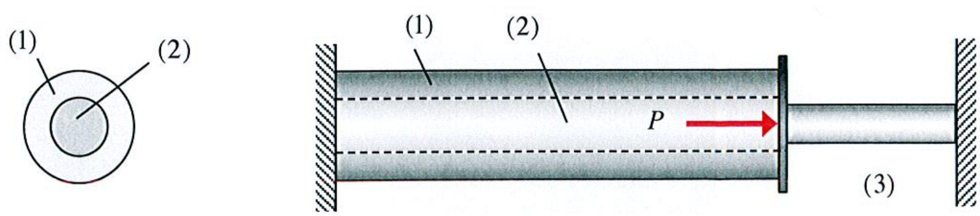


cross-sectional view

Determinate shaft ⇒ internal resultants depend only on equilibrium
⇒ does not depend on material properties

PROBLEM NO. 4 - PART B – 2 points max.

TRUE or *FALSE*: The stress in element (3) depends on the Young's moduli of the three elements.



cross-sectional view

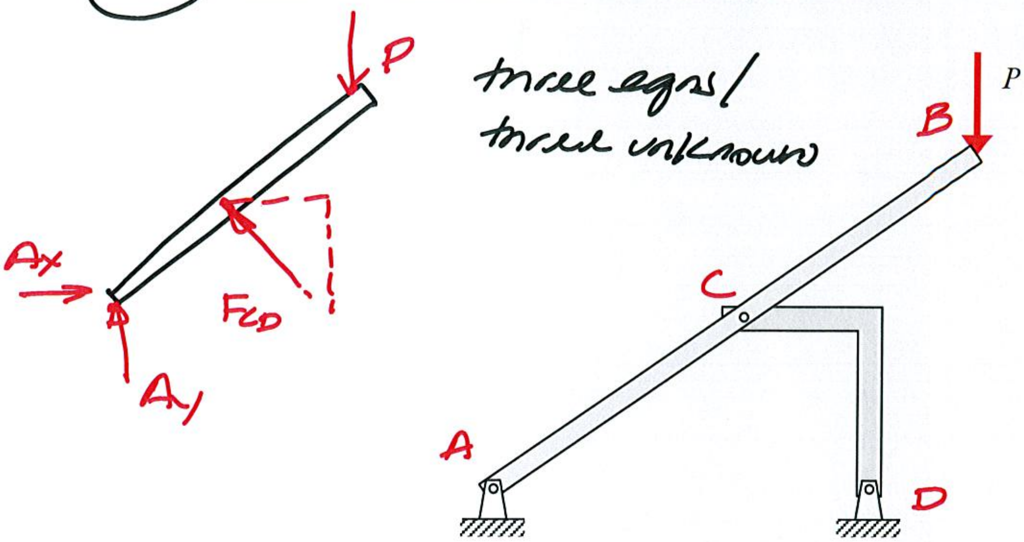
Indeterminate shaft ⇒ depends on both equilibrium and deformation

⇒ does depend on material

PROBLEM NO. 4 - PART C – 2 points max.

TRUE or FALSE: The structure shown below is *statically determinate*.

three eqns/
three unknowns



PROBLEM NO. 4 - PART D – 2 points max.

Through a uni-axial loading test, the material is known to have a yield strength and an ultimate strength of:

$$\sigma_Y = 42\ ksi$$

$$\sigma_U = 62\ ksi$$

respectively. A structural member made up of this material experiences a uni-axial loading, producing an axial component of normal stress of $\sigma = 50\ ksi$. Circle a *single* response below:

a) The material of the structural member has failed.

b) The material of the structural member has not failed.

c) The failure criteria depend on the criteria that are specified for the design.

Failure definition depends on application. The material has failed if yielding is considered failure; has not failed if failure is defined as necking.

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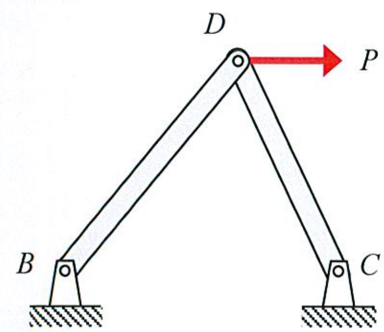
PROBLEM NO. 4 - PART E – 2 points max.

For the loading shown, the pin at C has been designed for a factor of safety of $FS = 1.2$ against failure in shear. If the diameter of the pin at C is doubled, what is the new factor of safety?



$\tau = \frac{V}{A}$; $A = \pi \left(\frac{d}{2}\right)^2$

If d is doubled, τ goes down by a factor of 4

$\Rightarrow$ FS goes up by a factor of 4

PROBLEM NO. 4 - PART F – 2 points max.

The edges of a rectangular plate (made up of a material with Young's modulus E and Poisson's ratio ν) experience uniform tensile, normal stresses throughout, with $\sigma_x \neq 0$ and $\sigma_y \neq 0$. For this loading, the plate has x and y components of strain of ε_x and ε_y. Suppose that the x-component of normal stress is increased to $2\sigma_x$, doubling the original value, and with σ_y remaining unchanged. After this increase, what is the new value of the x-component of strain (circle a *single* response below)?

a) $2\varepsilon_x$

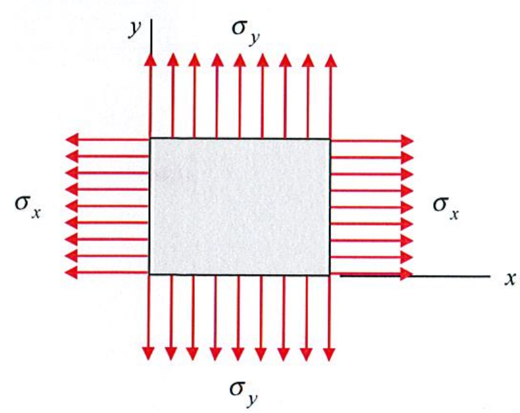
b) $-2\nu\frac{\sigma_y}{E}$

c) ε_x

d) $\frac{\varepsilon_x}{2}$

e) $-\nu\frac{\sigma_y}{2E}$

f) None of the above



(1) $\varepsilon_{x_1} = \frac{1}{E}\left[\sigma_x - \nu\sigma_y\right]$

(2) $\varepsilon_{x_2} = \frac{1}{E}\left[2\sigma_x - \nu\sigma_y\right]$ ← Need to know σ_y

PROBLEM NO. 4 - PART G – 2 points max.

A block is situated between two fixed walls, as shown below. The block is given a uniform temperature increase of ΔT. As a result of this temperature increase, the y-components of stress and strain are such that (circle a *single* response below):

a) $\sigma_y > 0$ *and* $\varepsilon_y > 0$

b) $\sigma_y > 0$ *and* $\varepsilon_y = 0$

c) $\sigma_y > 0$ *and* $\varepsilon_y < 0$

d) $\sigma_y < 0$ *and* $\varepsilon_y > 0$

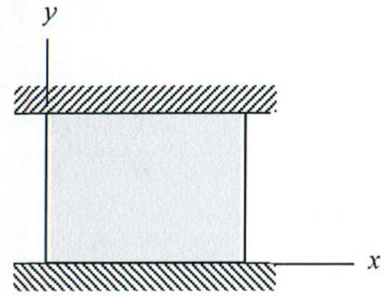
e) $\sigma_y < 0$ *and* $\varepsilon_y = 0$

f) $\sigma_y < 0$ *and* $\varepsilon_y < 0$

g) $\sigma_y = 0$ *and* $\varepsilon_y < 0$

h) $\sigma_y = 0$ *and* $\varepsilon_y = 0$

i) $\sigma_y = 0$ *and* $\varepsilon_y > 0$



- Fixed walls $\Rightarrow \varepsilon_y = 0$
- As temperature is increased, block tries to expand between the two fixed walls $\Rightarrow \sigma_y < 0$

PROBLEM NO. 4 - PART H – 3 points max.

For the loading on the truss shown, joint C has a total displacement of $d > 0$. Let e_2 represent the elongation of member (2). Circle the a *single* response below relating the size of e_2 to d:

a) $e_2 > d$

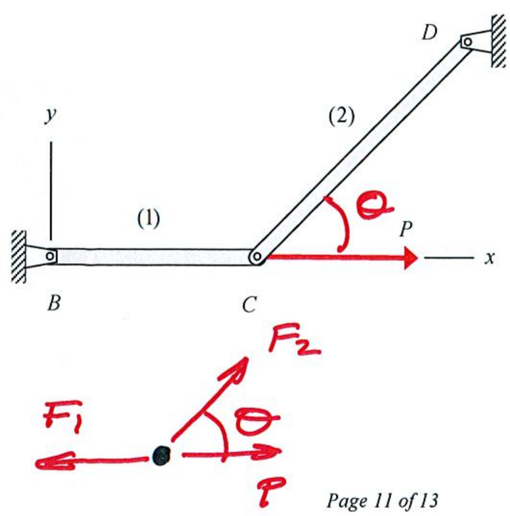
b) $e_2 = d$

c) $0 < e_2 < d$

d) $e_2 = 0$

e) $e_2 < 0$

f) more information is needed about the problem in order to answer this question



$$\sum F_y = F_2 \sin\theta = 0 \Rightarrow F_2 = 0$$

$$\Rightarrow e_2 = \frac{F_2 L_2}{E_2 A_2} = 0$$

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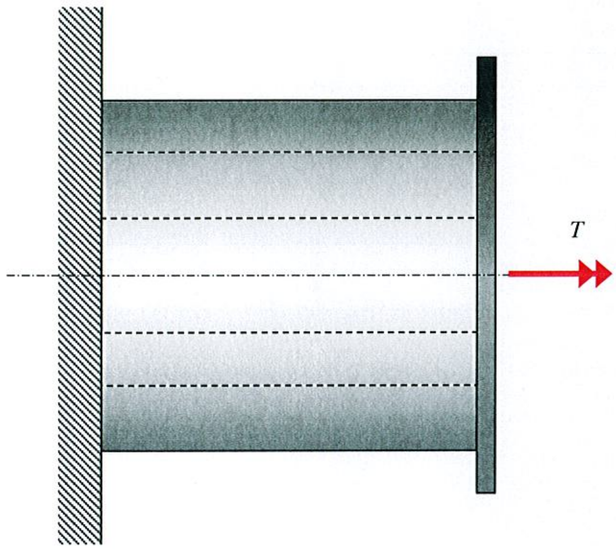
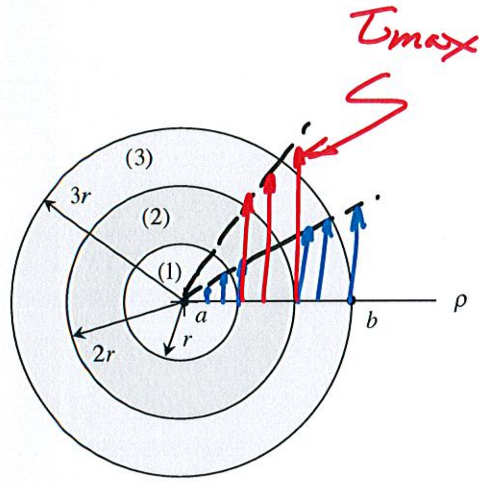
PROBLEM NO. 4 - PART I – 4 points max.

A shaft is made up of concentrically-joined elements (1), (2) and (3), where (1) and (2) are tubes, and (3) is a core element. Elements (1) and (3) have Young's moduli of $E_1 = E_3 = E$ and element (2) has a Young's modulus of $E_2 = 2E$. All elements are attached to a fixed wall at their left ends, and are attached to rigid connector at their right ends. A torque T is applied to the connector, as shown below. On the cross-section view of the shaft shown below right:

a) make a drawing of the distribution of shear stress along the radial line ab on the cross section.

b) indicate the location of the maximum shear stress on the cross section of the shaft.





END VIEW

$$\gamma = \frac{d\phi}{dx}\rho$$

$$\tau = G\gamma = \left(G\frac{d\phi}{dx}\right)\rho$$

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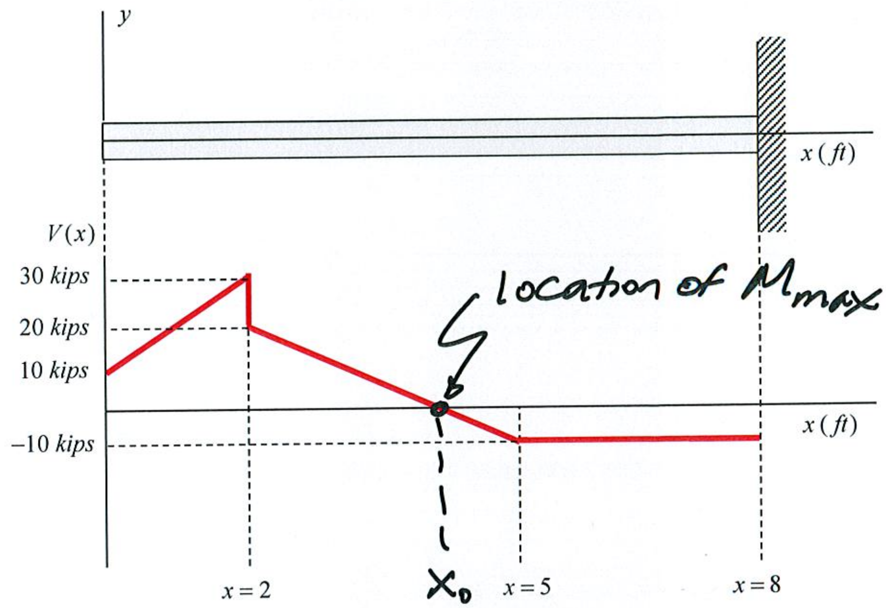
PROBLEM NO. 4 - PART J – 4 points max.

A cantilevered beam having a length of 8 feet is acted upon by line loads and concentrated point loads along its length (the loadings are not shown on the figure below), with no concentrated couples acting. The shear force $V(x)$ for this loading is provided below the figure of the beam. From this information, determine the *location* of the *maximum bending moment* in the beam.



$$V = \frac{dM}{dx} \Rightarrow \frac{dM}{dx} = 0 \text{ when } V = 0$$

$$V(x_0) = 0 = \cancel{V(x=2)}^{\,20} + \left(\frac{-30}{3}\right)(x_0 - 2)$$

↳ $x_0 = 4\ ft$