PROBLEM #1 (25 Points): ***SOLUTION***

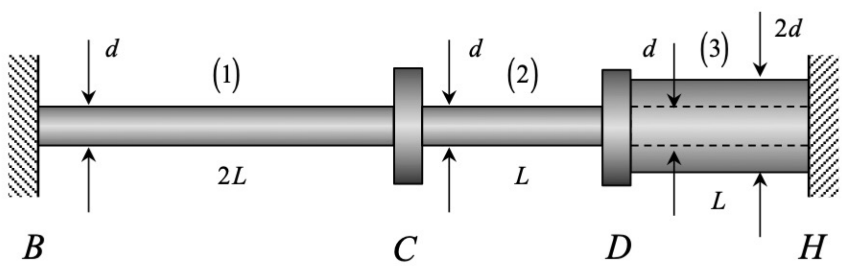
A rod is made up of elastic members (1), (2) and (3), with the material makeup of each member having a Young's modulus of E and a coefficient of thermal expansion of α. Members (1), (2) and (3) have lengths of $2L$, L and L, respectively. Members (1) and (2) have solid cross-sections with a diameter of d, whereas member (3) has a tubular cross-section with inner and outer diameters of d and $2d$, respectively. With the members being initially unstressed, the temperatures of (1) and (2) are increased by amounts of $2\Delta T$ and ΔT, respectively, while the temperature of (3) is held constant.

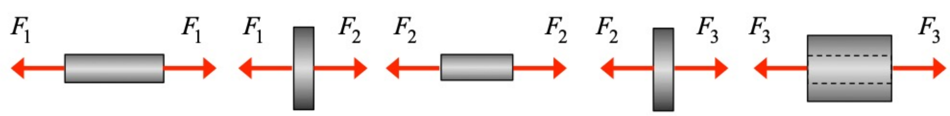
As a result of the temperature changes described above:

a) Determine the axial load (force) carried by each member. State whether each member is experiencing a compressive or tensile load.

b) Determine the axial strain in each member. Include an appropriate sign with each strain.

Leave your answers in terms of, at most, E, α, L, d and ΔT.





1. Equilibrium

$$(1)\ (C):\ \sum F = -F_1 + F_2 = 0 \Rightarrow F_1 = F_2$$

$$(2)\ (D):\ \sum F = -F_2 + F_3 = 0 \Rightarrow F_3 = F_2$$

2. Force/elongation

$$(3)\ e_1 = \frac{F_1(2L)}{EA_1} + \alpha(2\Delta T)(2L)\ ;\ A_1 = \pi\left(\frac{d}{2}\right)^2$$

$$(4)\ e_2 = \frac{F_2 L}{EA_2} + \alpha\Delta T L\ ;\ A_2 = \pi\left(\frac{d}{2}\right)^2$$

$$(5)\ e_3 = \frac{F_3 L}{EA_3}\ ;\ A_3 = \pi\left(\frac{2d}{2}\right)^2 - \pi\left(\frac{d}{2}\right)^2 = \frac{3}{4}\pi d^2$$

3. Compatibility

$$u_C = u_B + e_1 = e_1$$

$$u_D = u_C + e_2 = e_1 + e_2$$

$$(6)\ u_H = u_D + e_3 = e_1 + e_2 + e_3 = 0$$

4. Solve

$$(3)-(6) \Rightarrow 2\frac{F_1 L}{E\pi\left(d^2/4\right)}+4\alpha\Delta TL+\frac{F_2 L}{E\left(d^2/4\right)}+\alpha\Delta TL+\frac{F_3 L}{E\left(3d^2/4\right)}=0 \Rightarrow$$

$$8\frac{F_1}{E\pi d^2}+5\alpha\Delta TL+4\frac{F_2}{Ed^2}+\frac{4}{3}\frac{F_3}{Ed^2}=0 \Rightarrow$$

$$8F_1+4F_2+\frac{4}{3}F_3=-5\pi\alpha\Delta TLEd^2$$

Combining with (1) gives:

$$\left(8+4+\frac{4}{3}\right)F_1=-5\pi\alpha\Delta TLEd^2 \quad \Rightarrow \quad F_1=F_2=F_3=-\frac{3}{8}\pi\alpha\Delta TLEd^2 \quad (compression)$$

Strains

$$\varepsilon_1=\frac{e_1}{2L}=\frac{4F_1}{E\pi d^2}+2\alpha\Delta T=\frac{4F_1}{E\pi d^2}\left(-\frac{3}{8}\pi\alpha\Delta TLEd^2\right)+2\alpha\Delta T=\frac{1}{2}\alpha\Delta T$$

$$\varepsilon_2=\frac{e_2}{L}=\frac{4F_2}{E\pi d^2}+\alpha\Delta T=\frac{4}{E\pi d^2}\left(-\frac{3}{8}\pi\alpha\Delta TLEd^2\right)+\alpha\Delta T=-\frac{1}{2}\alpha\Delta T$$

$$\varepsilon_3=\frac{e_3}{L}=\frac{4F_3}{3E\pi d^2}=\frac{4}{3E\pi d^2}\left(-\frac{3}{8}\pi\alpha\Delta TLEd^2\right)=-\frac{1}{2}\alpha\Delta T$$

ME 323 – Mechanics of Materials
Examination #1
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PURDUE UNIVERSITY | Mechanical Engineering

Name (Print) SOLUTION
(Last) (First)

PROBLEM # 2 (25 points)

The critical components for the design of the planar truss in Figure 2a are considered to be member AB and the pin at C (shown in Figure 2b). The truss is subjected to a single downward force P at A. All members of the truss have a cross-sectional area of A=1 in^2. The cross-sectional area of the pin at C is A_C =0.5 in^2. The factor of safety (FS) against failure of AB by yielding is FS_{AB}= 3. The factor of safety against ultimate shear failure of the double-sided pin at C is FS_C = 4.

For member AB, σ_Y=36 ksi and for the pin material τ_U=48 ksi.

Find the largest P that can be applied without failure of the member AB and pin C.

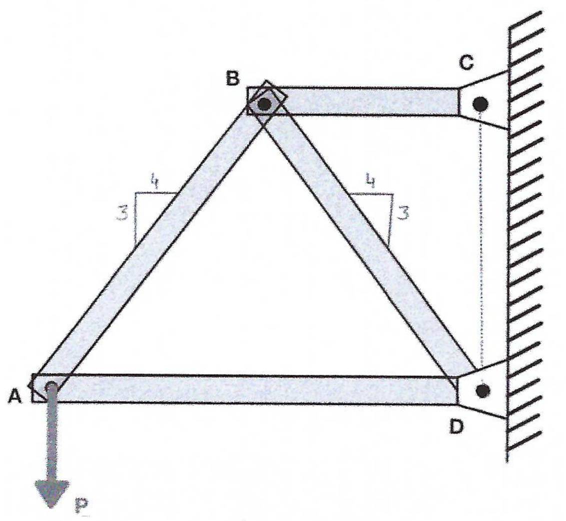


Figure 2(a)

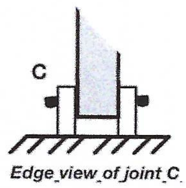
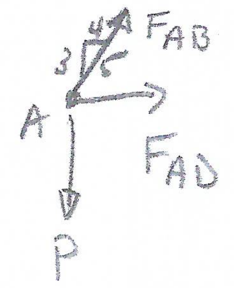


Figure 2(b)

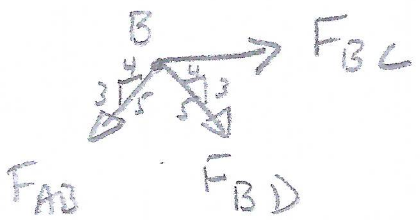
PROBLEM #2

Pin A



$$\Sigma F_y = \frac{3}{5} F_{AB} - P = 0 \qquad \boxed{F_{AB} = \frac{5P}{3}}$$

Pin B



$$\Sigma F_x = F_{BC} + \frac{4}{5} F_{BD} - \frac{4}{5} F_{AB} = 0$$

$$\Sigma F_y = -\frac{3}{5} F_{AB} - \frac{3}{5} F_{BD} = 0$$

$$F_{BD} = -F_{AB} = -\frac{5P}{3}$$

$$F_{BC} = \frac{4}{5} F_{AB} - \frac{4}{5} F_{BD}$$

$$= \frac{4}{\cancel{5}}\left[\frac{\cancel{5}}{3}P\right] - \frac{4}{\cancel{5}}\left[-\frac{\cancel{5}P}{3}\right]$$

$$\boxed{F_{BC} = \frac{8}{3}P}$$

For AB

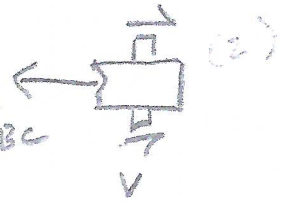
$$\sigma_{AB} = \frac{F_{AB}}{A} = \boxed{\frac{5P}{3} = \sigma_{AB}}$$

$$(FS)_{AB} = \frac{\sigma_y}{\sigma_{AB}} = 3 \Rightarrow \sigma_{AB} = \frac{\sigma_y}{3}$$

$$\boxed{\frac{5P}{3} = \frac{36\text{ ksi}}{3} = 12\text{ ksi}}$$

$$\boxed{P_{AB} = \frac{3}{5}(12) = 7200\text{ lb.}} = \frac{36}{5}$$

For pin C



$$V_{pin} = \frac{F_{BC}}{2} = \frac{\cancel{8}\,4}{(3)(\cancel{2})} P$$

$$V_{pin} = \frac{4}{3} P$$

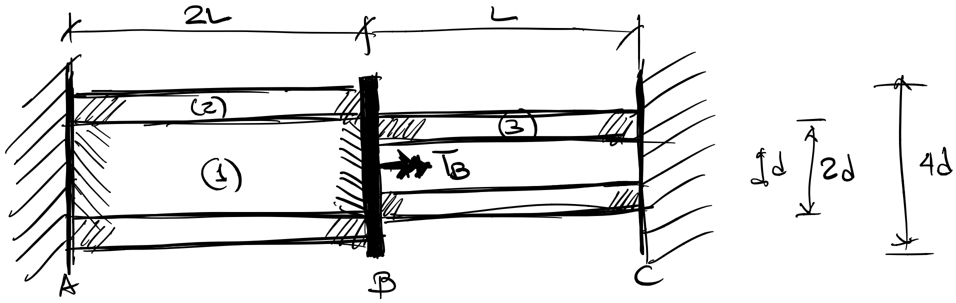
$$\tau_{pin} = \frac{V_{pin}}{A_p} = \frac{\frac{4}{3}P}{0.5} = \boxed{\frac{8}{3}P = \tau_{pin}}$$

$$(FS)_{pin} = \frac{\tau_U}{\tau_{pin}} \Rightarrow \tau_{pin} = \frac{\tau_U}{(FS)_{pin}} = \frac{48}{4} = 12\text{ ksi}$$

$$\frac{8}{3} P = 12\text{ ksi} \quad (.1)\ \boxed{P_{pin} = \frac{36\text{ ksi}}{8} = 4{,}500\text{ lb}}$$

$$\boxed{P_{allow} = 4{,}500\text{ lb}}$$

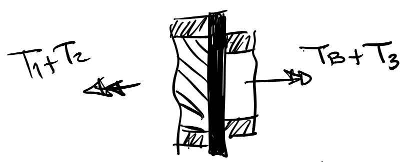
Smallest



- **FBD & Equilibrium**

$\rightarrow$ 1 equation & 3 unknowns

$\boxed{\text{STATICALLY INDETERMINATE}}$



$$T_B+T_3-T_1-T_2=0 \qquad (*3)$$

- **Compatibility conditions**

$\phi_A=0$ & $\phi_C=0$

$$\left.\begin{array}{l}\phi_1=\phi_B\\ \phi_2=\phi_B\\ \phi_3=-\phi_B\end{array}\right\}\quad \begin{array}{l}\phi_1=\phi_2\\ \phi_2=-\phi_3\end{array}\quad\Rightarrow$$

- **Torque-twist behavior**

$\Rightarrow$

$$T_1\frac{L_1}{G_1 I_{p1}}=T_2\frac{L_1}{G_2 I_{p2}} \qquad (*1)$$

$$T_2\frac{L_1}{G_2 I_{p2}}=-T_3\frac{L_3}{G_3 I_{p3}} \qquad (*2)$$

with...
$$I_{p1}=\frac{\pi\,(2d/2)^4}{2}=\frac{\pi d^4}{2}$$

$$I_{p2}=\frac{\pi}{2}\left[(4d/2)^4-(2d/2)^4\right]=\frac{\pi}{2}\left[16d^4-d^4\right]=\frac{\pi}{2}15d^4$$

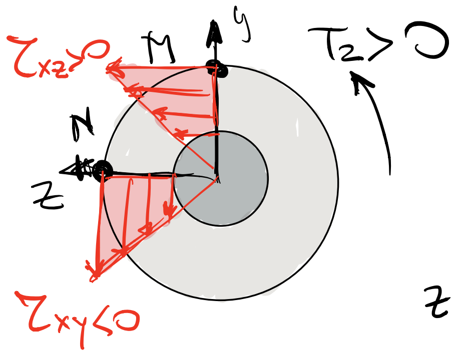
$$I_{p3}=\frac{\pi}{2}\left[(2d/2)^4-(d/2)^4\right]=\frac{\pi}{2}\left[d^4-\frac{1}{16}d^4\right]=\frac{\pi}{32}\left[16d^4-d^4\right]=\frac{\pi}{32}15d^4$$

- **Solve**

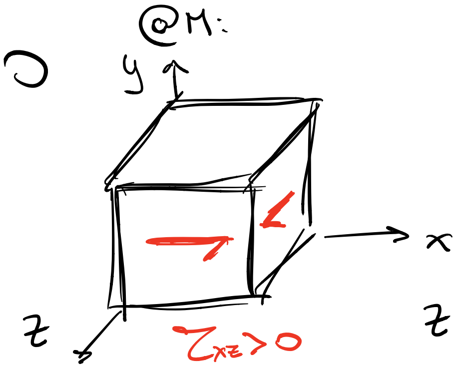
$$(*1)\Rightarrow\quad T_1=T_2\frac{G_1}{G_2 15}$$

$$(*2)\Rightarrow\quad T_3=-T_2\frac{L_1}{L_3}\frac{G_3}{G_2}\frac{1}{16}$$

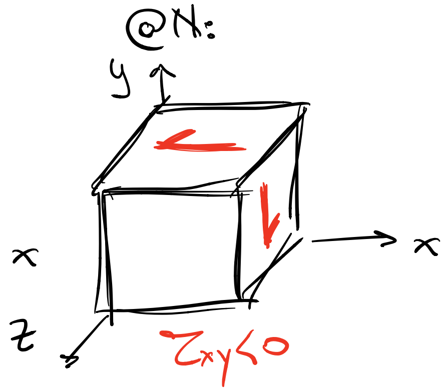
$$(*3)\Rightarrow\quad T_B=T_2\left[\frac{G_3L_1}{16G_2L_3}+\frac{G_1}{15G_2}+1\right]\Rightarrow\quad \boxed{\begin{array}{l}T_2=\frac{1}{3}T_B\\ T_1=\frac{1}{3}T_B\\ T_3=-\frac{1}{3}T_B\end{array}}$$



τxz>0
M
y
N
z
τxy<0
Tz>0



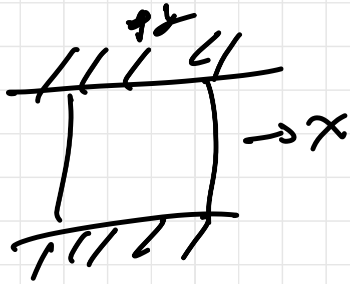
@M:
y
x
z
τxz>0



@N:
y
x
z
τxy<0

4. Part A

use Generalized Hooke's law



c) $\sigma_x = 0$, $\varepsilon_x = (1+\nu)\alpha\Delta T$

$\sigma_y = -\alpha E \Delta T$, $\varepsilon_y = 0$

4) Part B

a) F

b) T

c) F

d) F

e) F

f) F

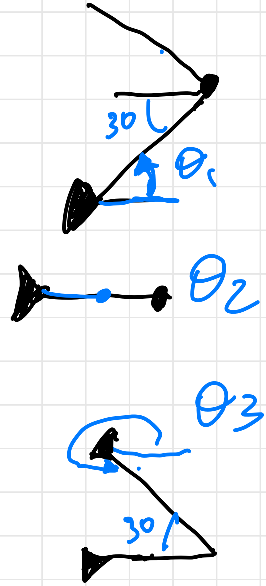
g) T

4) Part C

$\theta_1 = \pi/6$

$\theta_2 = 0$

$\theta_3 = \frac{11\pi}{6}$



4) Part D

(a) $e_1 = e_2$

(b) $e_1 = -e_2$

(c) $e_1 = e_2/2$

d) $e_1 + e_2 = 0$

e) none of the above