ME 323 – Mechanics of Materials
Midterm #1
February 23rd, 2022

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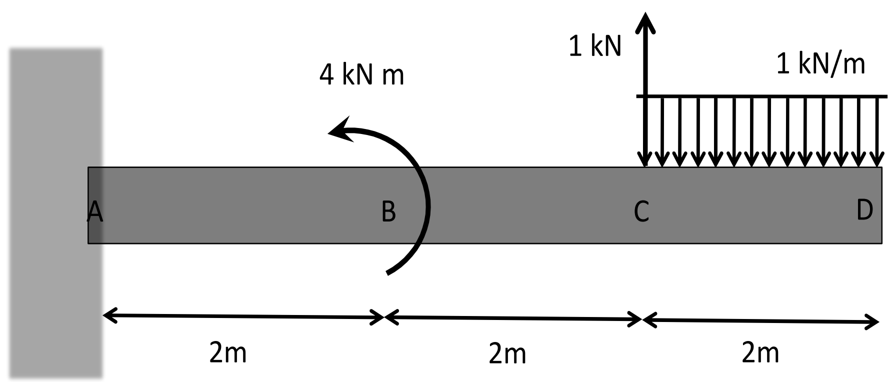
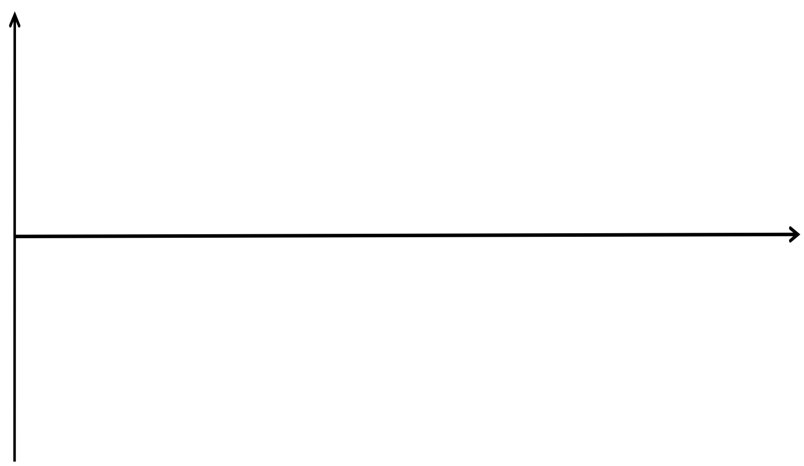
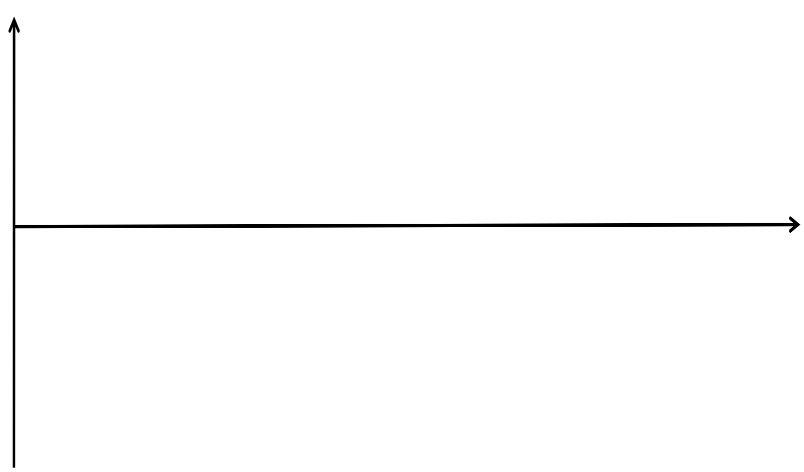
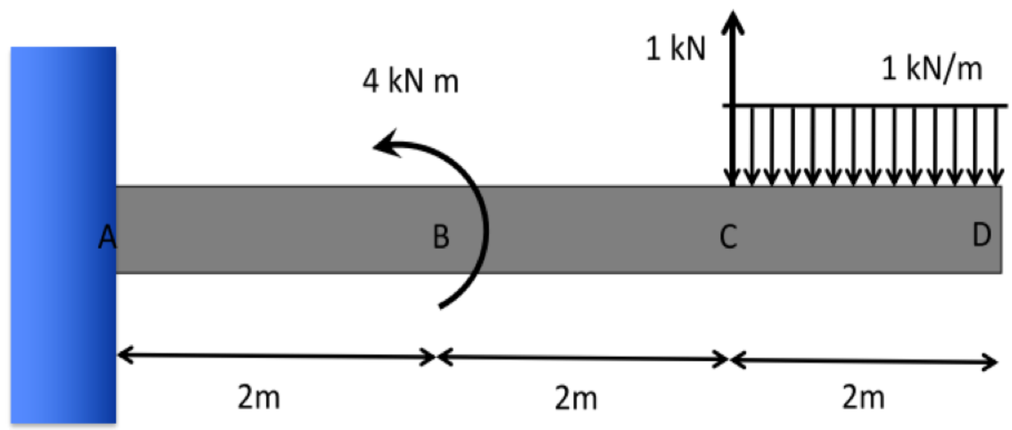
PROBLEM # 1 (25 points)

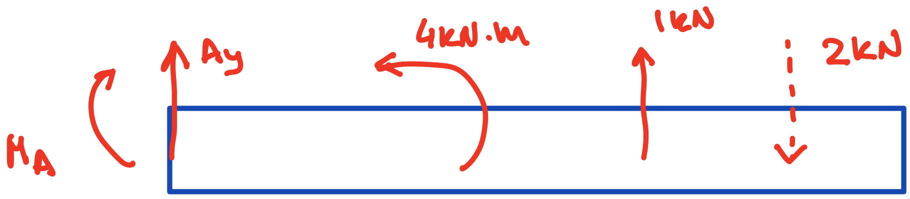
Construct the shear and bending moment diagrams for the beam AD. Clearly write all the information in the diagrams. Include values of local maximum and local minimum of V and M, the x coordinate where V and M are zero, and indicate the order (constant, linear, etc.) of V and M in different sections of beam.











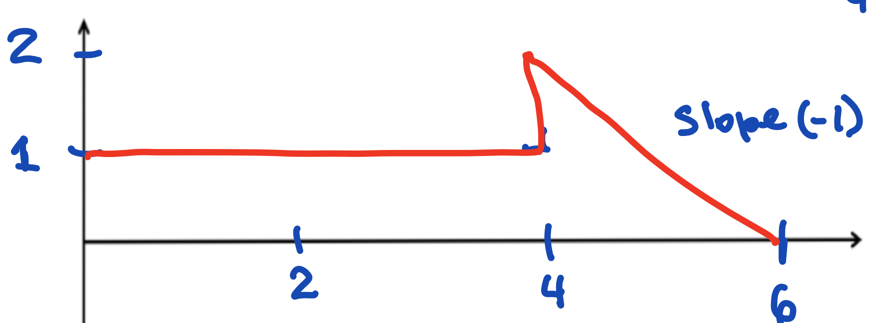
$$\Sigma F_y = 0 \qquad A_y + 1 - 2 = 0$$

$$A_y = 1 \text{ kN}$$

$$\Sigma M_A = 0 \qquad -M_A + 4 + 1 \cdot 4 - 2 \cdot 5$$

$$M_A = -2 \text{ kN.m}$$

V[kN]



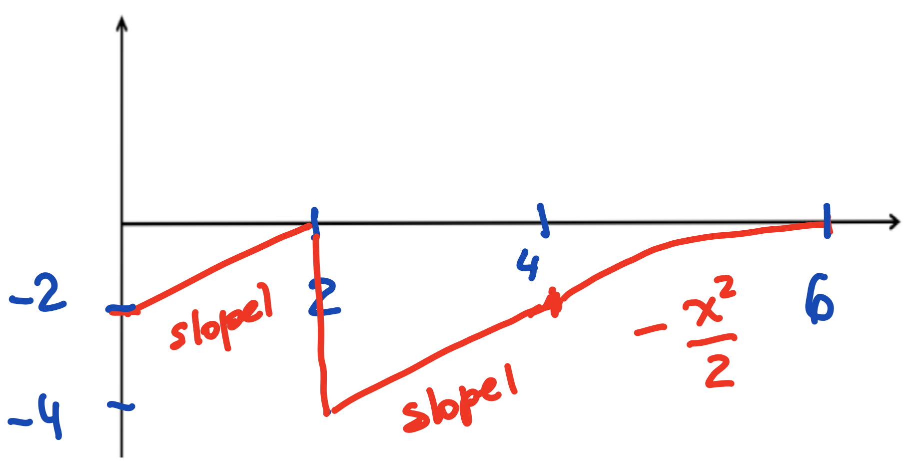
$4 \leq x \leq 6$

$$V(x) = V(4) + \int_4^x (-1)\, d\xi$$

$$V(x) = 2 + (-x) + 4$$

$$V(x) = 6 - x$$

M[kN·m]



$0 < x \leq 2$

$$M(x) = M(0) + \int_0^x 1\, d\xi$$

$$M(x) = -2 + x$$

$2 < x \leq 4$

$$M(x) = M(2) + \int_2^x 1\, d\xi$$

$$M(x) = -4 + x - 2$$

$$M(x) = x - 6$$

$4 \le x \le 6$

$$M(x) = M(4) + \int_4^x \left(6 - \xi\right) d\xi$$

$$= -2 + \left(6\xi - \frac{\xi^2}{2}\right)\Big|_4^x$$

$$= -2 + 6x - \frac{x^2}{2}$$

$$- \left(24 - 8\right)$$

$$M(x) = -18 + 6x - \frac{x^2}{2}$$

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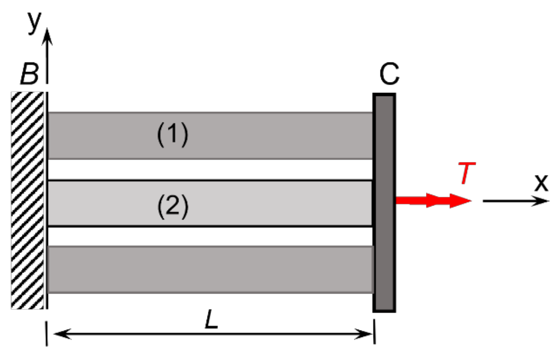
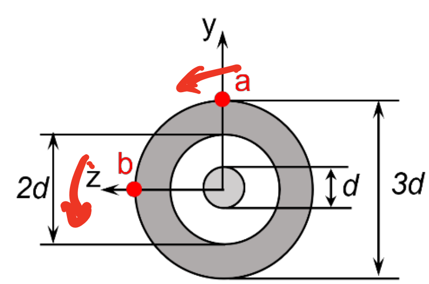
PROBLEM # 2 (25 points)

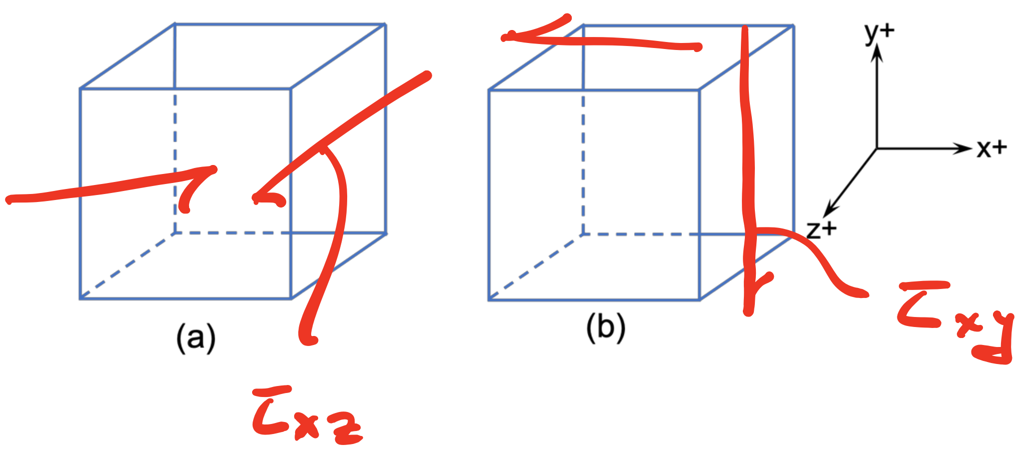
A shaft is made up of members (1) and (2), with the members joined by a rigid connector C. Member (1) is a hollow cylinder, while member (2) has a solid, circular cross section. The end B at x = 0 for both members are fixed to a wall. A torque T is applied to the rigid connector C as shown. The two shafts have a shear modulus of $G_1=G$ and $G_2=2G$.

a) Determine if the assembly is statically determinate or indeterminate.
b) Determine the value of the internal torques T_1 and T_2 as a function of applied torque T.
c) Calculate the maximum shear stress in each member.
d) Consider the points "a" and "b" on the outer radius of shaft (1). Draw on the stress elements given, the stress states at "a" and "b".

Provide your answers in terms of T, d, L and G.







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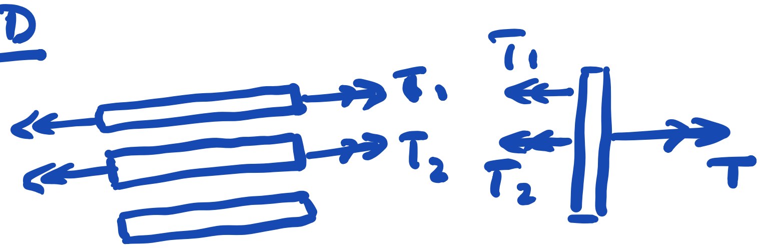
PROBLEM # 2 CONT.

FBD



Equilibrium : $T - T_1 - T_2 = 0$ $\boxed{T_1 + T_2 = T}$ (1)

(a) One equation, 2 unknowns : statically indeterminate

(b) $\Delta\phi_1 = \dfrac{T_1 L}{G I_{p_1}}$ $\Delta\phi_2 = \dfrac{T_2 L}{2G I_{p_2}}$

$$I_{p_1} = \frac{\pi}{2}\left[\left(\frac{3d}{2}\right)^4 - \left(\frac{\cancel{2}d}{\cancel{2}}\right)^4\right] = \frac{\pi}{2}\left[\frac{81}{16}d^4 - d^4\right]$$

$$\boxed{I_{p_1} = \frac{65\pi}{32}d^4}$$

$$I_{p_2} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \boxed{\frac{\pi d^4}{32} = I_{p_2}}$$

Compatibility : $\boxed{\Delta\phi_1 = \Delta\phi_2}$

$$\frac{T_1 \cancel{L}}{\cancel{G}\left(\frac{65\,\cancel{\pi}\,\cancel{d^4}}{\cancel{32}}\right)} = \frac{T_2 \cancel{L}}{2\cancel{G}\left(\frac{\cancel{\pi d^4}}{\cancel{32}}\right)} \qquad \boxed{T_2 = \frac{2T_1}{65}}$$

Replace in (1)

$$T_1 + \frac{2T_1}{65} = T \qquad \boxed{T_1 = \frac{65}{67}T = 0.97\,T}$$

$$T_2 = \frac{2T_1}{65} = \frac{2}{\cancel{65}}\left(\frac{\cancel{65}}{67}T\right)$$

$$\boxed{T_2 = \frac{2}{67}T = 0.03\,T}$$

(c) $$(\tau_1)_{max} = \frac{T_1\left(\frac{3d}{2}\right)}{I_{p_1}} = \frac{\left(\frac{65}{67}T\right)\left(\frac{3d}{2}\right)}{\frac{65\pi d^4}{32}}$$

$$\boxed{(\tau_1)_{max} = \frac{48}{67}\frac{T}{\pi d^3} = 0.228\frac{T}{d^3}}$$

$$(\tau_2)_{max} = \frac{T_2\left(\frac{d}{2}\right)}{I_{p_2}} = \frac{\left(\frac{2}{67}T\right)\left(\frac{d}{2}\right)}{\frac{\pi d^4}{32}}$$

$$\boxed{(\tau_2)_{max} = \frac{32}{67}\frac{T}{\pi d^3} = 0.152\frac{T}{d^3}}$$

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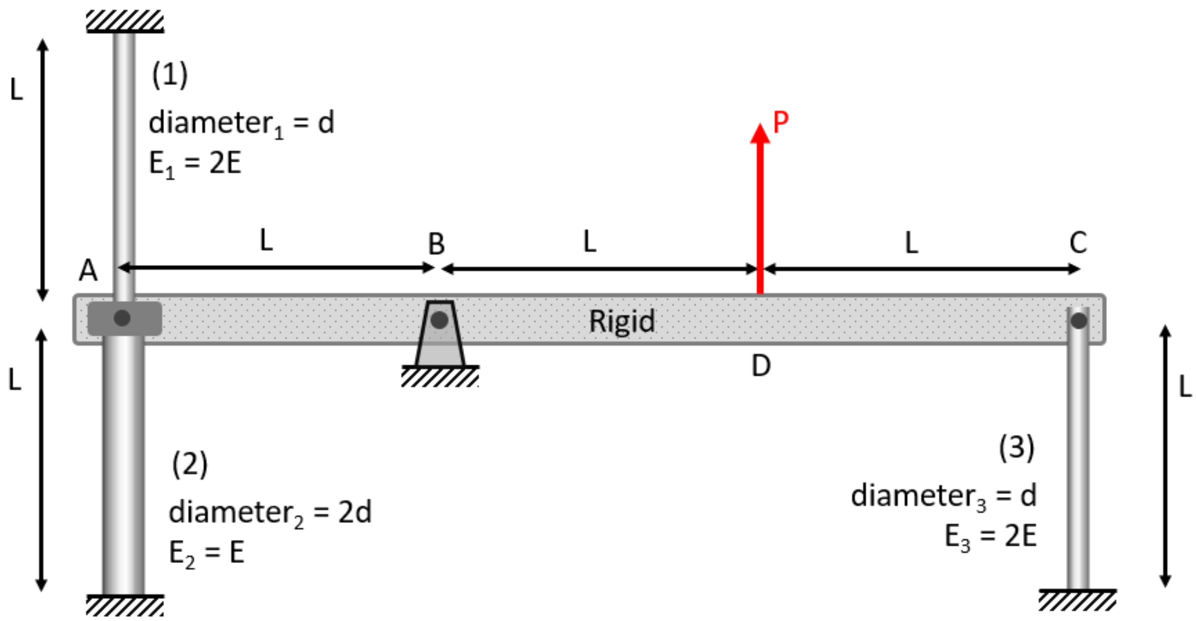
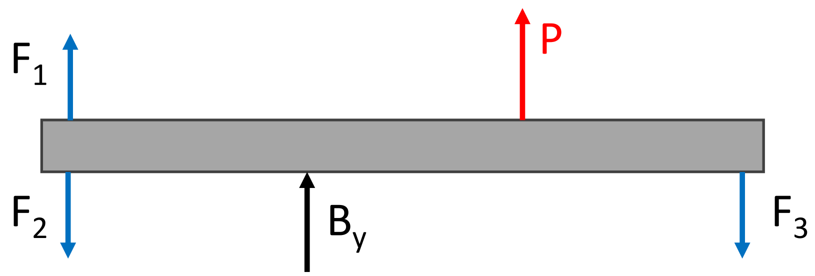
PROBLEM # 3 (25 points)

Rigid bar ABC is held by a pin joint at B and connected to three deformable rods. Two deformable rods are connected at A and one deformable rod is connected at C. A force P is applied at point D. The diameters and elastic moduli of the deformable rods are listed in the figure.

a) Determine whether the assembly is determinate or indeterminate.
b) What are the axial stresses on each of the deformable members? Leave your answers in terms of P, L, E, and/or d.
c) If the yield stress (σ_Y) is 200 MPa, what is the maximum value of P that can be applied to maintain a factor of safety (FoS) of 2? Use values of **L = 1 m**, **E = 70 GPa**, and **d = 0.02 m**.
Assume that none of the members buckle under compression.



1. Equilibrium



(3)

$$\left(\sum M\right)_B = 0 = -F_1L + F_2L + PL - 2F_3L$$ (3)

(a) 1 equation, 3 unknowns
→ indeterminate (1)

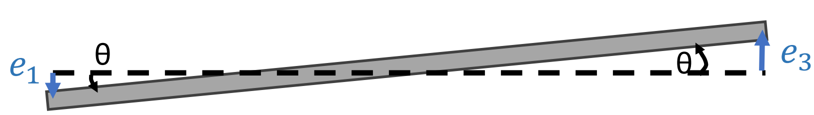
2. Force-elongation

$$e_1 = \frac{F_1L_1}{E_1A_1} = \frac{F_1L}{2E\pi\left(\frac{d}{2}\right)^2} = \frac{2F_1L}{E\pi d^2}$$ (1)

$$e_2 = \frac{F_2L_2}{E_2A_2} = \frac{F_2L}{E\pi\left(\frac{2d}{2}\right)^2} = \frac{F_2L}{E\pi d^2}$$ (1)

$$e_3 = \frac{F_3L_3}{E_3A_3} = \frac{F_3L}{2E\pi\left(\frac{d}{2}\right)^2} = \frac{2F_3L}{E\pi d^2}$$ (1)

3. Compatibility



$$\tan\theta \sim \theta = \frac{e_1}{L} = -\frac{e_2}{L} = \frac{e_3}{2L}$$

→ $e_1 = -e_2$ (3) $2e_1 = e_3$ (3)

4. Solve

$e_1 = -e_2 \rightarrow -2F_1 = F_2$

$2e_1 = e_3 \rightarrow 2F_1 = F_3$

$0 = -F_1L + F_2L + PL - 2F_3 \rightarrow 0 = -F_1 - 2F_1 + P - 4F_1$

$\rightarrow 7F_1 = P$

$F_1 = \frac{P}{7} \qquad F_1 = \frac{-2P}{7} \qquad F_1 = \frac{2P}{7}$ (2)

(b)

$\sigma_1 = \frac{F_1}{A_1} = \frac{4P}{7\pi d^2}$ (1)

$\sigma_2 = \frac{F_2}{A_2} = \frac{-2P}{7\pi d^2}$ (1)

$\sigma_3 = \frac{F_3}{A_3} = \frac{8P}{7\pi d^2}$ (1)

(c) σ_3 has the largest stress and will be the limiting member. (1)

$FS = \frac{\sigma_y}{\sigma_{allow}} \qquad 2 = \frac{200\ MPa}{\sigma_{allow}} \rightarrow \sigma_{allow} = 100\ MPa$ (2)

$\frac{8P}{7\pi d^2} = \sigma_{allow} = 100\ MPa \rightarrow P = \frac{(7\pi(0.02)^2)}{8}(100 * 10^6\ Pa)$

$\rightarrow P = 35000\pi$ N = 110 000 N = 110 kN (1)

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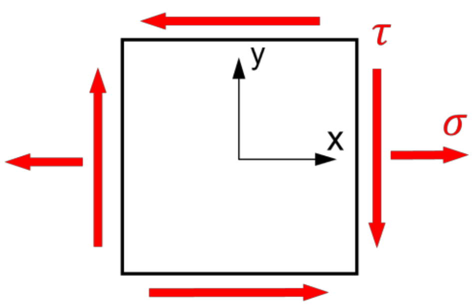
PROBLEM # 4 (25 points)

Part 4.1 (6 points)

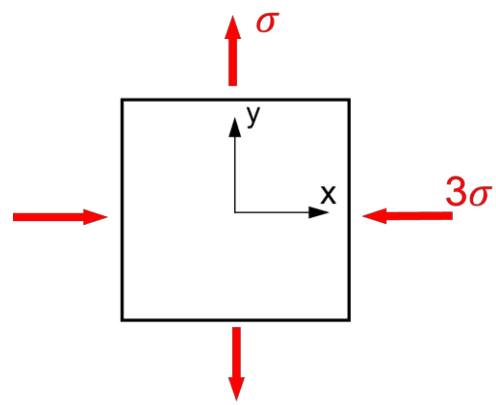
For each state of plane stress shown below, i.e., for configurations (a) and (b), indicate whether each component of the state of strain is:

- = 0 (equal to zero)
- > 0 (greater than zero)
- < 0 (less than zero)

The material is linear elastic with Poisson's ratio ν ($0 < \nu < 0.5$), and the deformations are small.



(a)



(b)

	(a)	(b)
ϵ_x	>0	<0
ϵ_y	<0	>0
ϵ_z	<0	>0
γ_{xy}	<0	0
γ_{yz}	0	0
γ_{zx}	0	0

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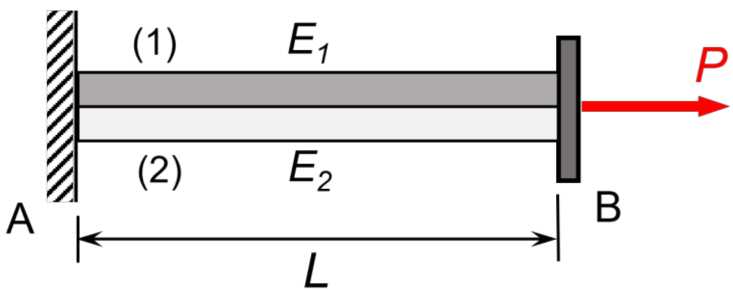
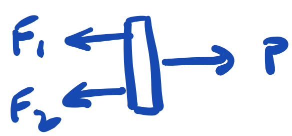
Part 4.2 (4 points)

A rod is made up of elements 1 and 2, each having a length of L and cross-sectional area A. Element 1 has an elastic modulus of E_1, and element 2 has a modulus of E_2, with $E_2 > E_1$. Let F_1 and F_2 represent the axial load carried by elements 1 and 2, respectively. Circle the correct answer below:

- $|F_1| > |F_2|$
- $|F_1| = |F_2|$
- $|F_1| < |F_2|$





$e_1 = e_2$

$$\frac{F_1 \cancel{L}}{\cancel{A} E_1} = \frac{F_2 \cancel{L}}{\cancel{A} E_2}$$

$$\frac{F_1}{F_2} = \frac{E_1}{E_2}$$

But $E_2 > E_1 \Rightarrow \frac{F_1}{F_2} < 1$

$|F_2| > |F_1|$

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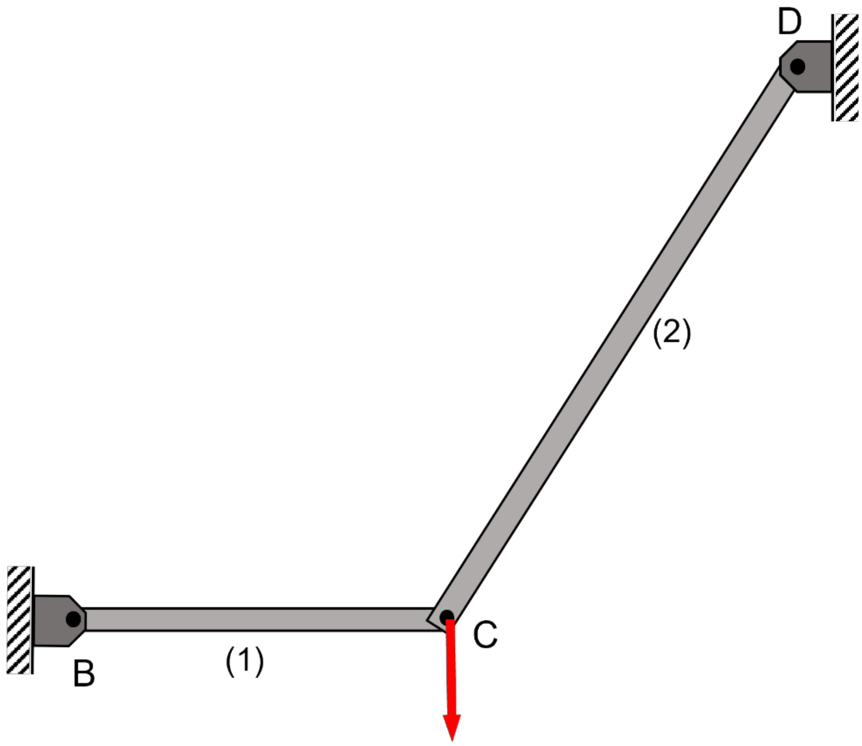
Part 4.3 (2 points)



Consider the truss above that is made up of elements (1) and (2).

TRUE or *FALSE*: The stress in element (1) depends on the material makeup of element (2).

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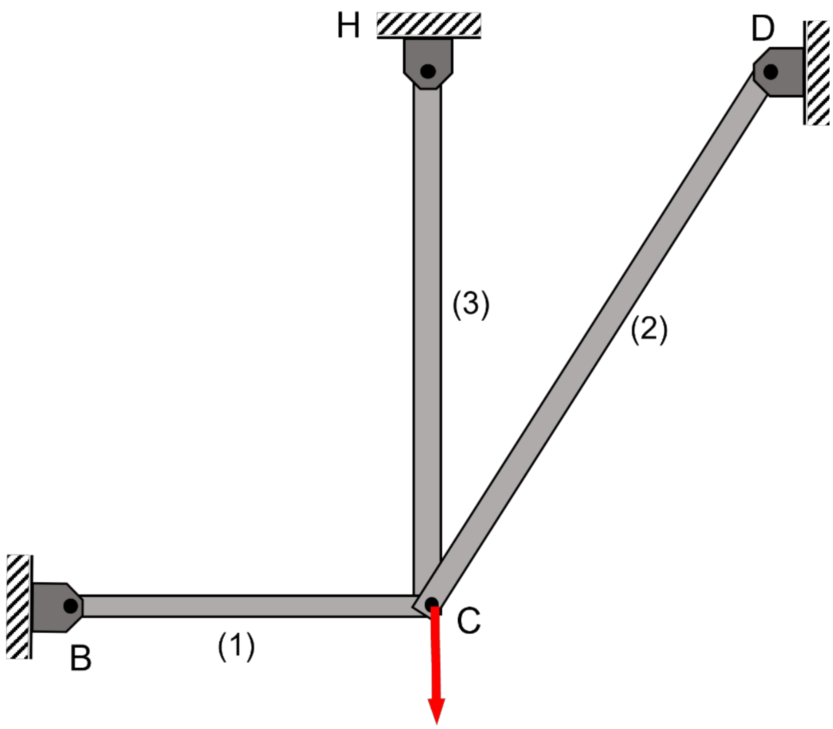
Part 4.4 (3 points)



Consider the truss above that is made up of elements (1), (2) and (3).

TRUE or *FALSE*: The stress in element (1) depends on the material makeup of elements (2) and (3).

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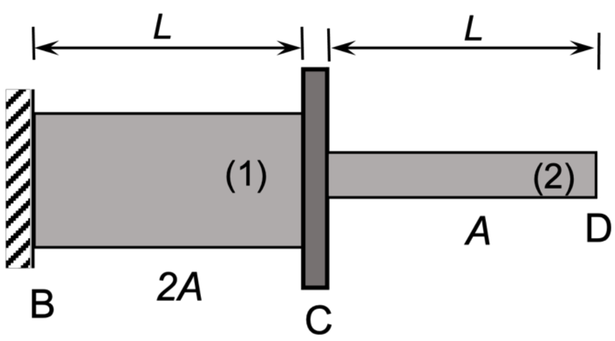
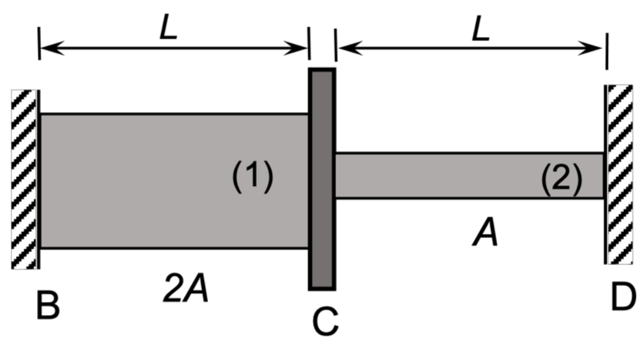
Part 4.5 (6 points)



Rod I



Rod II

a) Rod I shown above is made up of a material with a Young's modulus of E and thermal expansion coefficient α. The cross-sectional areas of elements (1) and (2) are given by $2A$ and A, respectively. Both elements are heated in such a way that each has a temperature increase of ΔT. Let σ_1 and σ_2 represent the stress in elements (1) and (2), respectively. Circle the correct description below of these two stresses:

i. $|\sigma_1| > |\sigma_2|$
ii. $|\sigma_1| = |\sigma_2|$
iii. $|\sigma_1| < |\sigma_2|$

F_1 F_2 F_2

$F_2 = 0 = F_1$

b) Rod II is exactly the same as Rod I, except its right end is attached to a rigid wall. Again, both elements are heated to the same temperature increase ΔT. Circle the correct description below of the stresses in the two elements:

i. $|\sigma_1| > |\sigma_2|$
ii. $|\sigma_1| = |\sigma_2|$
iii. $|\sigma_1| < |\sigma_2|$

F_1 F_2

$F_2 = F_1$

$\sigma_1 = \dfrac{F_1}{2A}$ $\sigma_2 = \dfrac{F_2}{A}$

$|\sigma_2| > |\sigma_1|$

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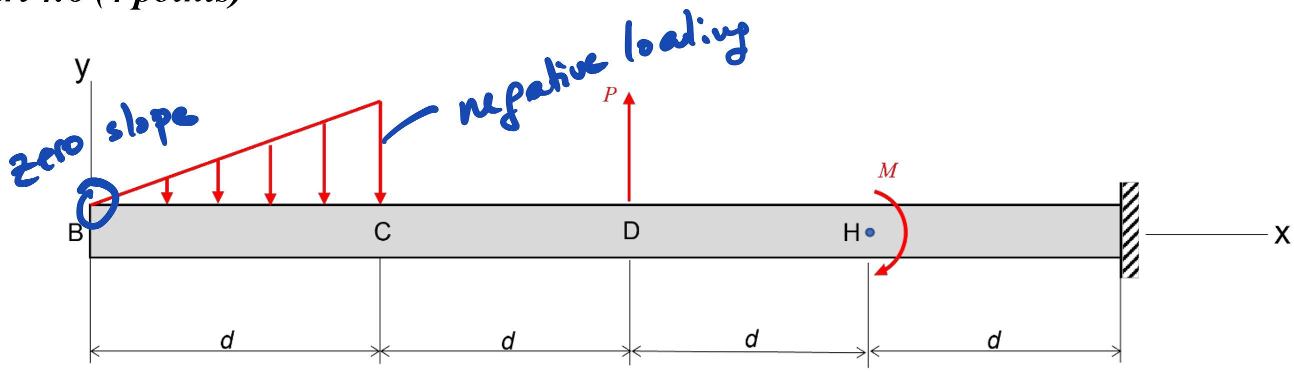
Part 4.6 (4 points)



The above loading is applied to a cantilevered beam. Circle the figure below which most accurately describes the internal shear force resultant in the beam between locations B and C.