Name (Print)____Solution__________________NB____
(Last) (First)

ME 323 - Mechanics of Materials
Exam # 1
Date: July 3, 2014 Time: 11:00 – 12:00 PM (in class)
Location: ME 1130
Instructor: Nasir Bilal

Instructions:

Begin each problem in the space provided on the examination sheets. If additional space is required, use the extra paper provided.

Work on one side of each sheet only, with only one problem on a sheet.

Please remember that for you to obtain maximum credit for a problem, you must present your solution clearly.

Accordingly,

- coordinate systems must be clearly identified,
- free body diagrams must be shown,
- units must be stated,
- write down clarifying remarks,
- state your assumptions, etc.

If your solution cannot be followed, it will be assumed that it is in error.

When handing in the test, make sure that ALL SHEETS are in the correct sequential order.

Remove the staple and restaple, if necessary.

Prob. 1 ______________________

Prob. 2 ______________________

Prob. 3 ______________________

Total ________________________

Useful Equations

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \quad \gamma_{xz} = \frac{1}{G}\tau_{xz} \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$G = \frac{E}{2(1+\nu)}$$

$$\tau = \frac{Tr}{I_p}$$

$$\phi = \frac{TL}{GI_p}$$

$$I_p = \frac{\pi d^4}{32}$$ Polar moment of inertia for solid circular cross section

$$I_p = \frac{\pi d^4}{32}(d_o^{\,4} - d_i^{\,4})$$ Polar moment of inertia for a circular hollow cross section

$$e = \frac{FL}{EA} + L\alpha\Delta T$$

$$e = u\cos\theta + v\sin\theta$$

$$Factor\ of\ Safety = \frac{Yield\ Strength}{Allowable\ Stress}$$

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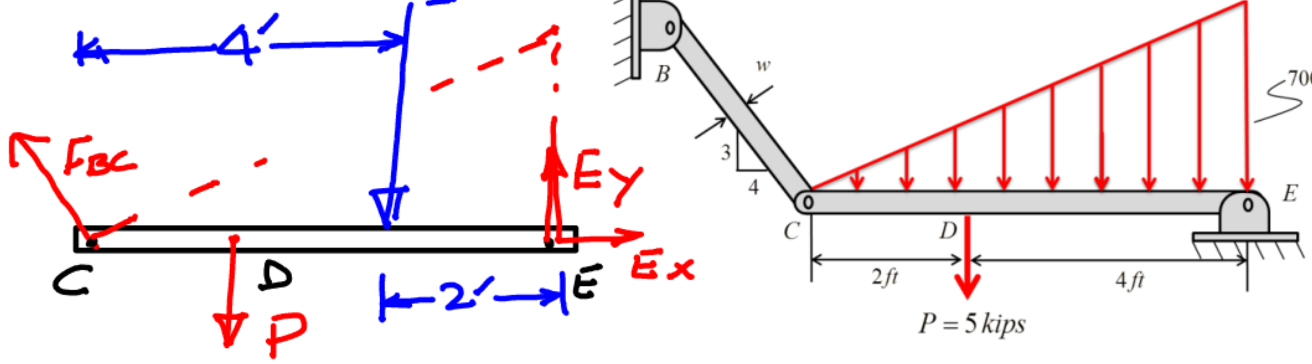
PROBLEM #1 (30 points)

The beam CDE is supported by a pin E and a rectangular tension bar BC also connected to CDE by a pin at C as shown in figure. The loads acting on the beam are shown in the figure. The tension bar BC is to be made of structural steel with a yield strength of $\sigma_Y = 36\ ksi$, while pins at C and E are to be of high-strength steel with a shear yield strength $\tau_Y = 48\ ksi$.

a) If the factor of safety with respect to yielding is $FS = 3.0$, and the width of member BC is $w = 1.5\ in.$, determine the required thickness, t, of the tension bar BC.
b) The pin at point C is in SINGLE shear and the pin at point E is in DOUBLE shear. Determine the required pin diameters at C and E for the factor of safety $FS = 3.3$ for shear yielding.

$$F = \frac{1}{2}(6)(700) = 2.1\ \text{kips}$$



$$\curvearrowleft M_E = 0 \Rightarrow -F_{BC}\left(\frac{3}{5}\right)(6) + 5(4) + (2.1)(2) = 0$$

$$\Rightarrow \boxed{F_{BC} = 6.72\ \text{kips}}$$

$$+\uparrow \sum F_y = 0$$

$$\Rightarrow \frac{3}{5}F_{BC} - 5 - 2.1 + E_y = 0$$

$$\Rightarrow \boxed{E_y = 3.1\ \text{kips}}$$

$$\xrightarrow{+} \sum F_x = 0 \quad -\frac{4}{5}F_{BC} + C_x = 0$$

$$\Rightarrow \boxed{E_x = +5.38\ \text{kips}}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(3.1)^2 + (+5.38)^2}$$

$$\boxed{E = 6.209\ \text{kips}}$$

$$\sigma = \frac{F_{BC}}{A_{BC}} = \frac{F_{BC}}{w.t}$$

$$\Rightarrow t = \frac{F_{BC}}{w.\sigma} = \frac{F_{BC}}{w.\frac{\sigma_Y}{FS}}$$

$$t = \frac{F_{BC}.FS}{w.\sigma_Y}$$

$$= \frac{6.72\times10^3 \times 3}{(1.5).36\times10^3}$$

$$\boxed{t = 0.3733\,\text{in}}$$

For pin C:-

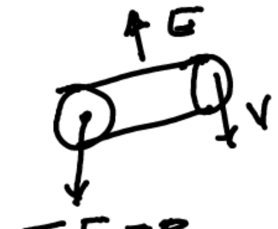
$$\tau = \frac{F_{BC}}{A_{BC}} = \frac{F_{BC}}{\frac{\pi}{4}d^2}$$

$$d = \sqrt{\frac{4F_{BC}}{\pi.\tau}} = \sqrt{\frac{4F_{BC}.FS}{\pi.\tau.}}$$

$$= \sqrt{\frac{4\times 6.72\times 3.3}{\pi.48}}$$

$$\boxed{d = .767\ \text{in}}$$

For pin E:



$$\Sigma F = 0$$

$$\Rightarrow E - 2V = 0$$

$$\Rightarrow V = \frac{E}{2}$$

$$F_E - 2Y = 0$$

$$Y = \frac{F_E}{2}$$

$$\tau = \frac{F_E}{2A_E}$$

$$\tau = \frac{F_E}{2\frac{\pi}{4}d^2}$$

$$\tau = \frac{2F_E}{\pi d^2}$$

$$d = \sqrt{\frac{2F_E}{\pi\tau}}$$

$$d = \sqrt{\frac{2F_E}{\pi\tau_{allow}}\times FS}$$

$$= \sqrt{\frac{2\times 6.21\times10^3\times 3.3}{\pi\times 48\times10^3}}$$

$$\boxed{d = .521\ \text{in}}$$

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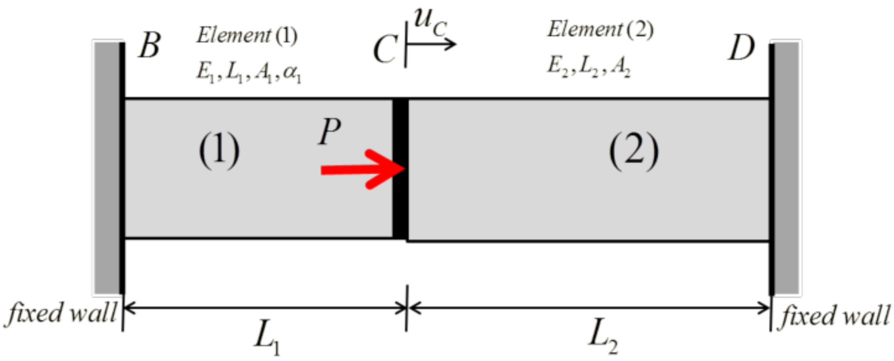
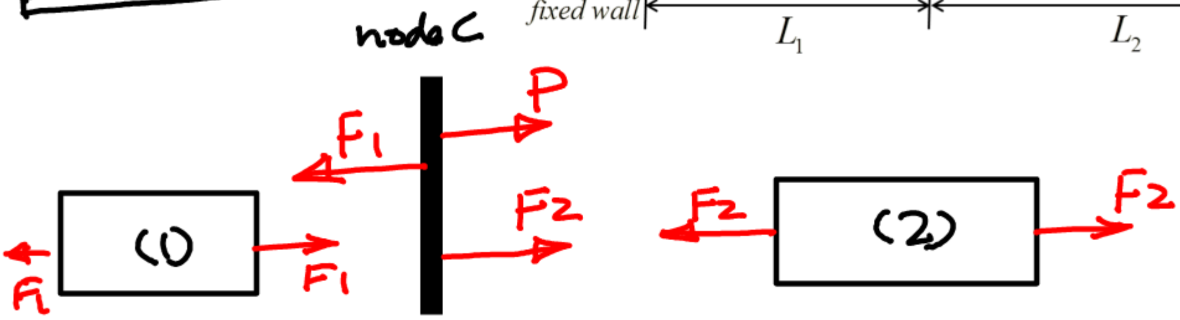
PROBLEM #2 (30 points)

A cylindrical rod element (1) is connected to another cylindrical rod element (2) at point C, and both elements are attached at the other end to fixed wall as shown in the figure. A load P is applied at node C. The temperature of element (1) is INCREASED by ΔT, while the temperature of element (2) is NOT changed. Data for the problem are provided in the figure. Determine:

1. the internal forces in elements (1) and (2).
2. the stresses in elements (1) and (2)
3. the horizontal displacement u_C of point C.



Solution:



Step #01: **Equilibrium:**

$$\xrightarrow{+} \sum F_x = 0 \Rightarrow P + F_2 - F_1 = 0$$

$$\Rightarrow \boxed{F_1 = F_2 + P} \longrightarrow (1)$$

Step #02 **Force-Deformation Relationship.**

$$\boxed{\begin{aligned} e_1 &= f_1 F_1 + \alpha_1 \Delta T L_1 \\ e_2 &= f_2 F_2 \end{aligned}} \longrightarrow (2)$$

Step # 03 Compatibility:

$$\boxed{\begin{aligned} e_1 + e_2 &= 0 \\ e_1 &= -e_2 \end{aligned}} \longrightarrow (3)$$

$\therefore$

$$f_1 F_1 + \alpha_1 \Delta T L_1 = -f_2 F_2$$

$$F_2 = \frac{-f_1 F_1 + \alpha_1 \Delta T L_1}{f_2}$$

sub in (1)

$$F_1 = -\frac{f_1 F_1 + \alpha_1 \Delta T L_1}{f_2} + P$$

$$f_2 F_1 + f_1 F_1 = P f_2 - \alpha_1 \Delta T L_1$$

$$\boxed{F_1 = \frac{P f_2 - \alpha_1 \Delta T L_1}{f_1 + f_2}}$$

where $f_1 = \dfrac{L_1}{A_1 E_1}$; $f_2 = \dfrac{L_2}{A_2 E_2}$

$$F_2 = \frac{P f_2 - \alpha_1 \Delta T L_1}{f_1 + f_2} - P$$

$$= \frac{\cancel{P f_2} - \alpha_1 \Delta T L_1 - P f_1 - \cancel{P f_2}}{f_1 + f_2}$$

$$\boxed{F_2 = -\frac{(P f_1 + \alpha_1 \Delta T L_1)}{f_1 + f_2}}$$

Now sub for f_1 and f_2 i.t.o. A, E, L.

$$F_1 = \frac{\frac{PL_2}{A_2E_2} - \alpha_1 \Delta T L_1}{\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2}}$$

$$\boxed{F_1 = \frac{(PL_2 - \alpha_1 \Delta T . L_1 A_2E_2) A_1E_1}{L_1 A_2E_2 + L_2 A_1 E_1}}$$

$$\Rightarrow \sigma_1 = \frac{F_1}{A_1}$$

$$\boxed{\sigma_1 = \frac{(PL_2 - \alpha_1 \Delta T L_1 A_2E_2) E_1}{L_1 A_2E_2 + L_2 A_1E_1}}$$

$$F_2 = \frac{-\left(\frac{PL_1}{A_1E_1} + \alpha_1 \Delta T L_1\right)}{\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2}}$$

$$\boxed{F_2 = -\frac{(PL_1 + \alpha_1 \Delta T L_1 A_1E_1) A_2E_2}{L_1 A_2E_2 + L_2 A_1E_1}}$$

$$\boxed{\sigma_2 = \frac{F_2}{A_2} = -\frac{(PL_1 + \alpha_1 \Delta T L_1 A_1E_1) E_2}{L_1 A_2E_2 + L_2 A_1E_1}}$$

(iii) Displacement of pt. C.

$$e_1 = U_c - \cancelto{0}{U_B}$$

$$U_c = f_1 F_1 - \alpha_1 \Delta T L_1$$

$$= \frac{L_1}{A_1\cancel{E_1}}\left[\frac{PL_2 - \alpha_1 \Delta T L_1 . A_2E_2}{L_1 A_2E_2 + L_2 A_1E_1}\right] A_1\cancel{E_1} + \alpha_1 \Delta T L_1$$

$$\boxed{U_c = \frac{PL_1L_2 - \alpha_1 \Delta T L_1^2 . A_2E_2}{L_1 A_2E_2 + L_2 A_1E_1} + \alpha_1 \Delta T . L_1}$$

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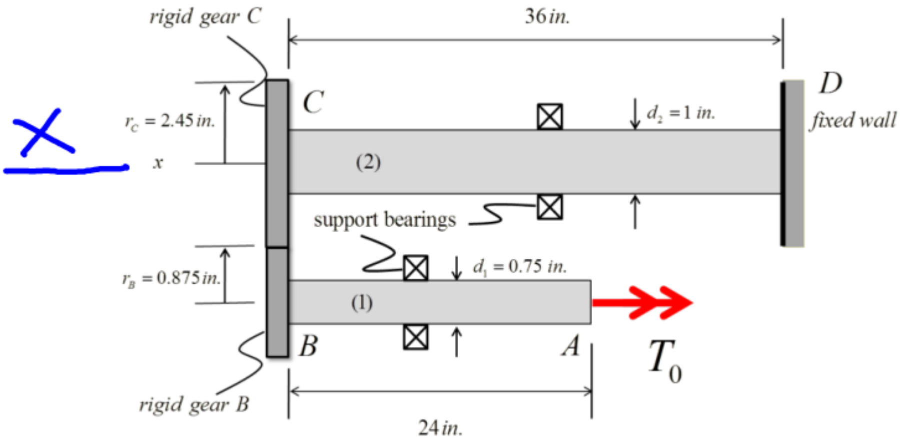
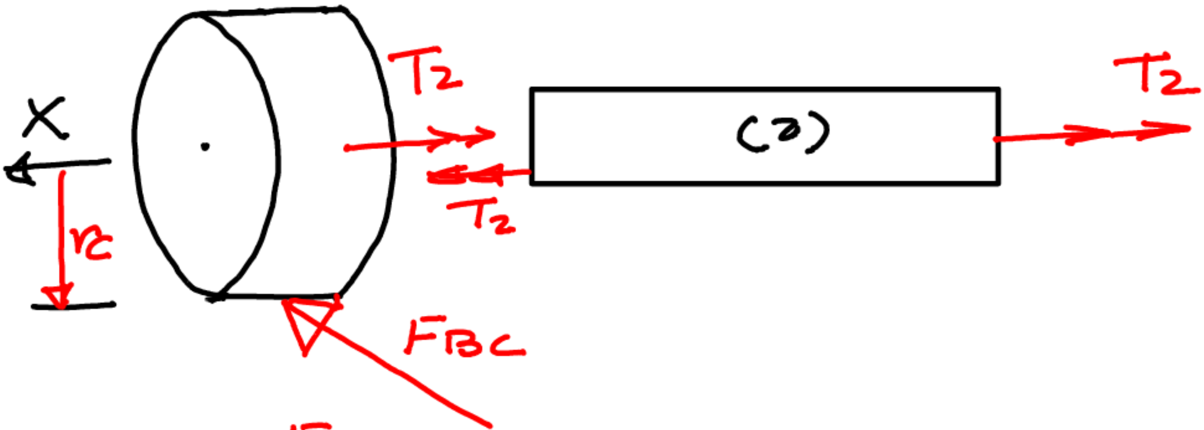
PROBLEM #3 (40 points)

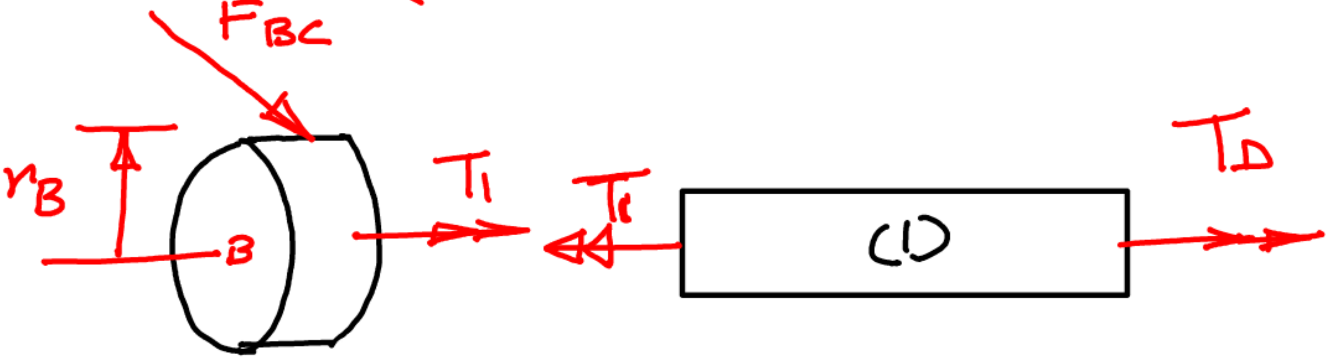
Two solid steel shafts are connected by the gears shown in figure. The shear modulus for each shaft, $G = 11.2 \times 10^6\ psi$ and the allowable shearing stress is $8\ ksi$. Determine:

1. the largest torque T_0 that may be applied to end A of shaft AB and,
2. the corresponding angle through which end A of shaft AB rotates.



Solution:





$$\sum M_x = 0$$

shaft #1:

$$T_1 - T_0 = 0 \Rightarrow \boxed{T_1 = T_0} \rightarrow (1)$$

$$-T_1 - F_{BC} r_B = 0$$

$$\boxed{F_{BC} = \frac{-T_1}{r_B}} \longrightarrow (2)$$

$$-F_{BC} \cdot r_C - T_2 = 0$$

$$\boxed{T_2 = -F_{BC} \cdot r_C} \longrightarrow (3)$$

$$② \Rightarrow \quad F_{BC} = \frac{-T_0}{r_{BC}}$$

$$T_2 = T_0 \frac{r_C}{r_B} = \frac{2.45}{0.875} T_0$$

$$\boxed{T_2 = 2.80\ T_0}$$

<u>Compatibility:</u>

$$\phi_B r_B = -\phi_C r_C$$

$$\boxed{\phi_B = -\phi_C \frac{r_C}{r_B}}$$

$$\boxed{\phi_B = -2.8\ \phi_C}$$

part (1): Max torque.

$$T_1 = T_0$$

$$\tau = \frac{T \cdot r}{J} = \frac{16 T}{\pi d^3}$$

$$8000 = \frac{16\ T_0}{\pi (0.75)^3}$$

$$\Rightarrow \boxed{T_0 = 663\ \text{lb-in}}$$

For shaft 2 :-

$$(2.8\, T_o) = \frac{8000\, \pi (1)^3}{16}$$

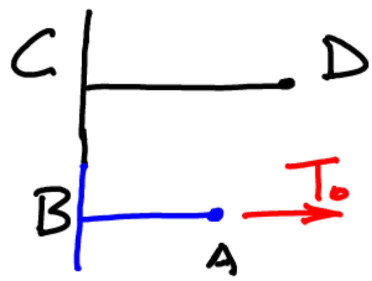
$$\boxed{T_o = 561 \text{ lb-in}}$$

The max torque that could be applied is the smaller value of T_o.

$$\boxed{T_o = 561 \text{ lb-in}}$$

Part (b) :-

angle of rotation ϕ_A ?



$$\phi_1 = \phi_B - \phi_A \longrightarrow (i)$$

$$\phi_B = -\phi_c \frac{r_c}{r_B} \longrightarrow (ii)$$

$$\phi_2 = \phi_c - \cancel{\phi_D}^{\,0} \longrightarrow (iii)$$

$$\phi_B = -\phi_2 \frac{r_c}{r_B}$$

$$\phi_A = -\phi_1 + \phi_B$$

$$\phi_A = -\phi_1 - \phi_2 \left(\frac{r_c}{r_B}\right)$$

$$\phi_A = -\frac{T_1 L_1}{G J_1} - \frac{T_2 L_2}{G J_2}\left(\frac{2.45}{0.875}\right)$$

$$= -\frac{(561)(24)}{11.2\times10^6 \times \frac{\pi (0.75)^4}{32}} - \frac{(2.8)(561)(36)}{11.2\times10^6 \times \frac{\pi}{32}}\left(\frac{2.45}{0.875}\right)$$

$$\boxed{\begin{aligned}\phi_A &= -0.1827 \text{ rad} \\ &= -10.468 \text{ deg}\end{aligned}}$$