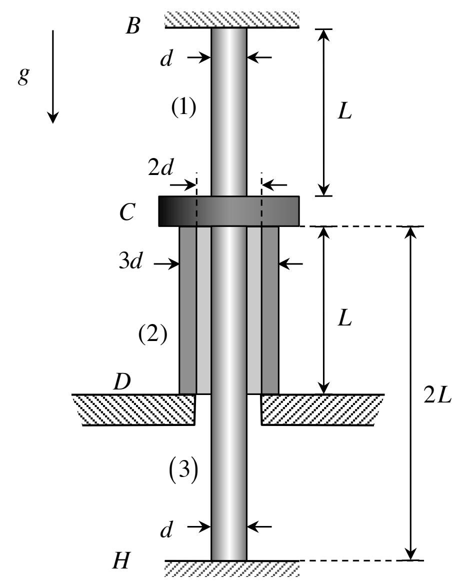
ME 323 Examination # 1 **SOLUTION**

July 3, 2018

PROBLEM NO. 1 – 25 points max.

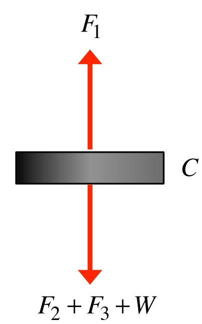
Rigid disk C has a weight of W. This disk is supported from the top by element (1) and from the bottom by elements (2) and (3). Elements (1) and (3) are solid rods with a diameter of d, and lengths of L and $2L$, respectively. Element (2) has a tubular cross section with inner and outer diameters of $2d$ and $3d$, respectively, and has a length of L. All elements are made up of the same material with a Young's modulus of E. The weights of these three elements are negligible compared with the weight of disk C; ignore their weights in your analysis.



For this problem, derive the three equations that are needed in solving for the axial loads (forces) carried by elements (1), (2) and (3). These equations should be written in terms only these three forces and the weight W. You are NOT asked to solve these equations.

Clearly label the steps of equilibrium, force/elongation and compatibility used in your analysis.

1. Equilibrium



$$\sum F = F_1 - F_2 - F_3 - W = 0 \quad \Rightarrow \quad F_1 - F_2 - F_3 = W \tag{1}$$

INDETERMINATE since we have one equation and three unknowns.

2. Force/elongation

$$e_1 = \frac{F_1 L}{EA_1} \ ; \quad A_1 = \pi \left(\frac{d}{2} \right)^2 = \frac{1}{4}\pi d^2 \tag{2}$$

$$e_2 = \frac{F_2 L}{EA_2} \ ; \quad A_2 = \pi \left[\left(\frac{3d}{2} \right)^2 - \left(\frac{2d}{2} \right)^2 \right] = \frac{5}{4}\pi d^2 \tag{3}$$

$$e_3 = \frac{F_3 (2L)}{EA_3} \ ; \quad A_3 = \pi \left(\frac{d}{2} \right)^2 = \frac{1}{4}\pi d^2 \tag{4}$$

3. Compatibility

$$u_C = u_H + e_3 = e_3$$

$$u_B = u_C + e_1 = e_3 + e_1 = 0 \quad \Rightarrow \quad e_1 = -e_3 \tag{5}$$

Also, $e_3 = e_2$ (6)

Combining equations (2)-(6) gives:

$$\frac{4F_1L}{E\pi d^2} = -\frac{8F_3L}{E\pi d^2} \quad \Rightarrow \quad F_1 = -2F_3 \tag{7}$$

$$\frac{4F_2L}{5E\pi d^2} = \frac{8F_3L}{E\pi d^2} \quad \Rightarrow \quad F_2 = 10F_3 \tag{8}$$

4. Solve

Need to solve equations (1), (2) and (3) for the member loads.

<u>*Not required*</u>:

$$-2F_3 - 10F_3 - F_3 = W \quad \Rightarrow \quad F_3 = -\frac{1}{13}W \quad (\textit{compression})$$

$$F_1 = -2\left(-\frac{W}{13}\right) = \frac{2}{13}W \quad (\textit{tension})$$

$$F_2 = 10\left(-\frac{W}{13}\right) = -\frac{10}{13}W \quad (\textit{compression})$$

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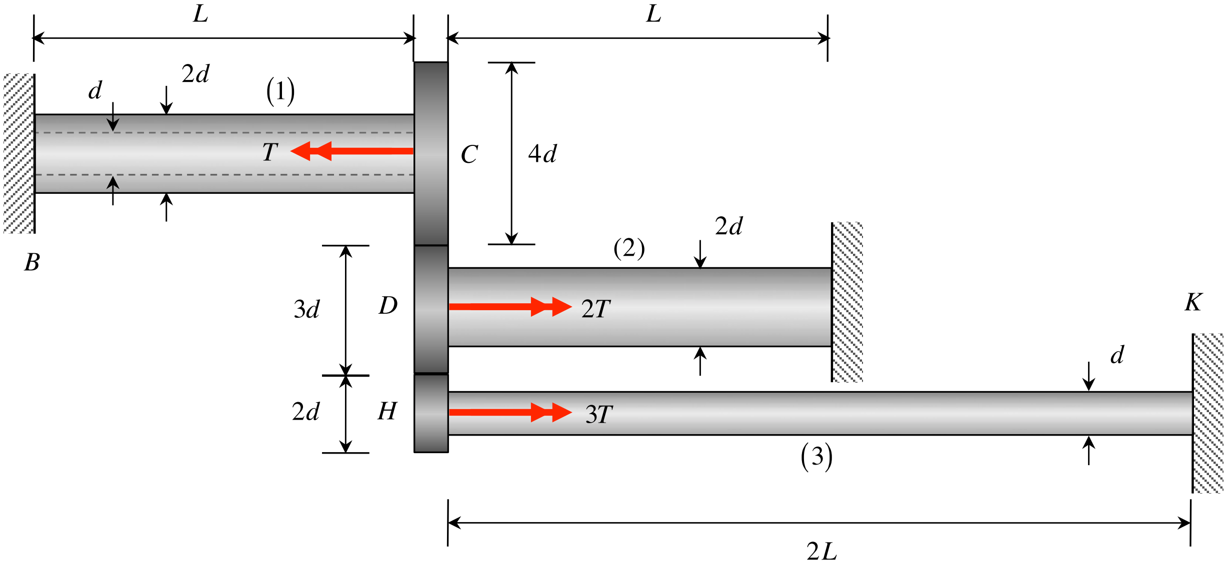
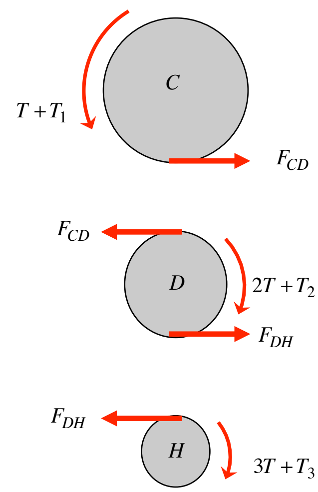
PROBLEM NO. 2 – 25 points max.



The shaft system shown above is made up of shaft components (1), (2) and (3). Components (2) and (3) are solid with outer diameters of $2d$ and d, respectively, and lengths L and $2L$, respectively. Component (1) is tubular with inner and outer diameters of d and $2d$, respectively. Each component is made up of a material with a shear modulus of G. Shaft components (1) and (2) are connected through a meshing gear pair CD, and shaft components (2) and (3) are connected through a meshing gear pair DH. Torques of T, $2T$ and $3T$ act on the rigid gears C, D and H, respectively.

For this problem, derive the three equations that are needed in solving for the shaft torques carried by components (1), (2) and (3). These equations should be written in terms only these three torques and the torque parameter T. You are NOT asked to solve these equations.

Clearly label the steps of equilibrium, torque/rotation and compatibility used in your analysis.



view from left side (showing only forces causing a moment of the gear centers)

1. Equilibrium

Gear C: $\sum M = T + T_1 + F_{CD}(2d) = 0 \quad \Rightarrow \quad F_{CD} = -\frac{1}{2d}(T + T_1)$ (1)

Gear D: $\sum M = F_{CD}\left(\frac{3}{2}d\right) + F_{DH}\left(\frac{3}{2}d\right) - 2T - T_2 = 0$ (2)

Gear H: $\sum M = F_{DH}(d) - 3T - T_3 = 0 \quad \Rightarrow \quad F_{DH} = \frac{1}{d}(3T + T_3)$ (3)

Combining equations (1)-(3):

$$-\frac{1}{2d}\left(T+T_1\right)\left(\frac{3}{2}d\right)+\frac{1}{d}\left(3T+T_3\right)\left(\frac{3}{2}d\right)-2T-T_2=0 \quad \Rightarrow \quad -3T_1-4T_2+6T_3=7T \tag{4}$$

The problem is INDETERMINATE since we have one equation and three unknowns.

2. Torque/rotation

$$\Delta\phi_1=\frac{T_1L}{GI_{P1}} \quad ; \quad I_{P1}=\frac{\pi}{2}\left(\frac{2d}{2}\right)^4-\frac{\pi}{2}\left(\frac{d}{2}\right)^4=\frac{15}{32}\pi d^4 \tag{5}$$

$$\Delta\phi_2=\frac{T_2L}{GI_{P2}} \quad ; \quad I_{P2}=\frac{\pi}{2}\left(\frac{2d}{2}\right)^4=\frac{1}{2}\pi d^4 \tag{6}$$

$$\Delta\phi_3=\frac{T_3\left(2L\right)}{GI_{P3}} \quad ; \quad I_{P3}=\frac{\pi}{2}\left(\frac{d}{2}\right)^4=\frac{1}{32}\pi d^4 \tag{7}$$

3. Compatibility

$$\left(\frac{4d}{2}\right)\Delta\phi_1=\left(\frac{3d}{2}\right)\Delta\phi_2 \quad \Rightarrow \quad \Delta\phi_2=\frac{4}{3}\Delta\phi_1 \tag{8}$$

$$\left(\frac{3d}{2}\right)\Delta\phi_2=-\left(\frac{2d}{2}\right)\Delta\phi_3 \quad \Rightarrow \quad \Delta\phi_2=-\frac{2}{3}\Delta\phi_3 \tag{9}$$

Combining equations (5)-(9):

$$\frac{2T_2L}{G\pi d^4}=\frac{4}{3}\left(\frac{32T_1L}{15G\pi d^4}\right) \quad \Rightarrow \quad T_1=\frac{45}{64}T_2 \tag{10}$$

$$\frac{2T_2L}{G\pi d^4}=-\frac{2}{3}\left(\frac{64T_3L}{G\pi d^4}\right) \quad \Rightarrow \quad T_3=-\frac{3}{64}T_2 \tag{11}$$

4. Solve

Solve equations (4), (10) and (11) for the component torques.

<u>*Not required*</u>:

$$-3\left(\frac{45}{64}T_2\right)-4T_2+6\left(-\frac{3}{64}T_2\right)=7T \quad \Rightarrow \quad T_2=-\frac{448}{409}T$$

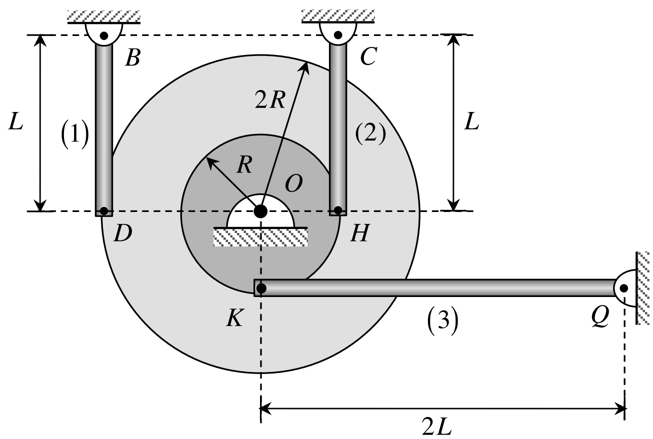
and:

$$T_1=-\frac{315}{409}T$$

$$T_3=\frac{21}{409}T$$

PROBLEM NO. 3 – 25 points max.

A rigid disk is pinned to ground at its center O. Rod elements (1), (2) and (3) are pinned to the disk at radial distances of $2R$, R and R, respectively. Each rod has the same cross-sectional area A and is made up of a material having a Young's modulus of E and a thermal coefficient of expansion of α. For the configuration shown above, elements (1) and (2) are vertically aligned, whereas element (3) is horizontally aligned. With the rod elements being initially unstressed, the temperatures of elements (1) and (3) are increased by ΔT and $2\Delta T$, respectively, with the temperature of element (2) being held constant.



a) Using appropriate FBD(s), write down the equilibrium equation(s) relating the axial loads carried by elements (1), (2) and (3).

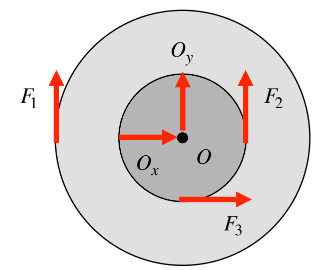
b) Write down the load/elongation equations for the members.

c) Write down the appropriate compatibility equations.

d) Determine the axial load carried by each element. Write your final answers in terms of the parameters of α, ΔT, E and A.

e) Is the *strain* in element (1) compressive, tensile or zero? Provide a written explanation.

f) Is the *stress* in element (1) compressive, tensile or zero? Provide a written explanation.



1. Equilibrium

$$\sum M_O = -F_1(2R) + F_2(R) + F_3(R) = 0 \quad \Rightarrow \quad -2F_1 + F_2 + F_3 = 0 \qquad (1)$$

The problem is INDETERMINATE since we have one equation and three unknowns.

2. Force/elongation

$$e_1 = \frac{F_1 L}{EA} + \alpha \Delta T L \qquad (2)$$

$$e_2 = \frac{F_2 L}{EA} \qquad (3)$$

$$e_3 = \frac{F_3 (2L)}{EA} + \alpha(2\Delta T)(2L) = \frac{2F_3 L}{EA} + 4\alpha \Delta T L \qquad (4)$$

3. Compatibility

For a CCW rotation of $\Delta\theta$ of the disk:

$$\Delta\theta = \frac{e_1}{2R} = -\frac{e_2}{R} \;\;\Rightarrow\;\; e_1 = -2e_2 \tag{5}$$

$$\Delta\theta = -\frac{e_2}{R} = -\frac{e_3}{R} \;\;\Rightarrow\;\; e_3 = e_2 \tag{6}$$

Combining equations (2)-(6) gives:

$$\frac{F_1 L}{EA} + \alpha\Delta TL = -2\frac{F_2 L}{EA} \;\;\Rightarrow\;\; F_1 = -2F_2 - \alpha\Delta TEA \tag{7}$$

$$\frac{2F_3 L}{EA} + 4\alpha\Delta TL = \frac{F_2 L}{EA} \;\;\Rightarrow\;\; F_3 = \frac{1}{2}F_2 - 2\alpha\Delta TEA \tag{8}$$

4. Solve

Solve equations (1), (7) and (8):

$$-2\left(-2F_2 - \alpha\Delta TEA\right) + F_2 + \left(\frac{1}{2}F_2 - 4\alpha\Delta TEA\right) = 0 \;\;\Rightarrow\;\; F_2 = 0$$

$$F_1 = -\alpha\Delta TEA$$

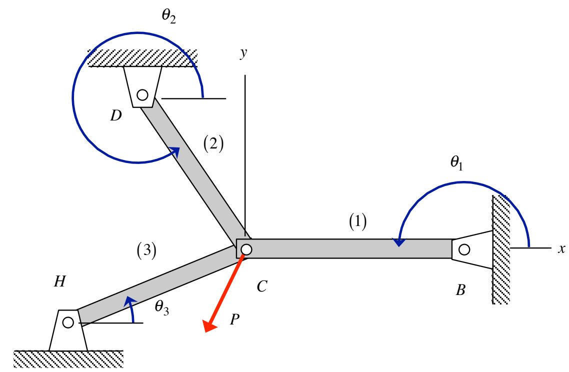
$$F_3 = -2\alpha\Delta TEA$$

From this:

$$\sigma_1 = \frac{F_1}{A} = -\alpha\Delta TE \;\;\left(\mathit{compressive}\right)$$

$$\varepsilon_1 = \frac{F_1}{EA} + \alpha\Delta T = -\alpha\Delta T + \alpha\Delta T = 0 \;\;\left(\mathit{zero\ strain}\right)$$

PROBLEM NO. 4 - PART A –3 points max.



The following compatibility equation is to be used to relate the elemental deformations e_i ; $i = 1,2,3$ to the horizontal and vertical components of displacement of joint C (u_C and v_C) of the truss shown: $e_i = u_C \cos\theta_i + v_C \sin\theta_i$. Circle the most accurate description of the elemental angles θ_i:

Element 1:

a) $0 \le \theta_1 < 90°$

b) $90° \le \theta_1 < 180°$

c) $180° \le \theta_1 < 270°$

d) $270° \le \theta_1 < 360°$

Element 2:

a) $0 \le \theta_2 < 90°$

b) $90° \le \theta_2 < 180°$

c) $180° \le \theta_2 < 270°$

d) $270° \le \theta_2 < 360°$

Element 3:

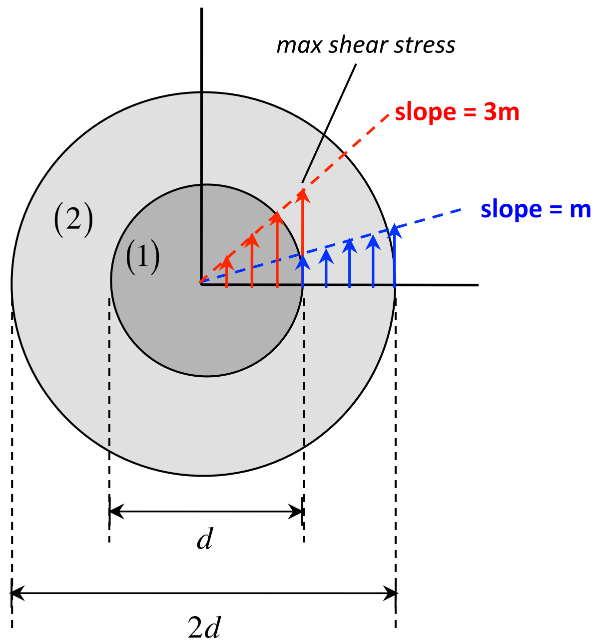
a) $0 \le \theta_3 < 90°$

b) $90° \le \theta_3 < 180°$

c) $180° \le \theta_3 < 270°$

d) $270° \le \theta_3 < 360°$

PROBLEM NO. 4 - PART B – 6 points max.

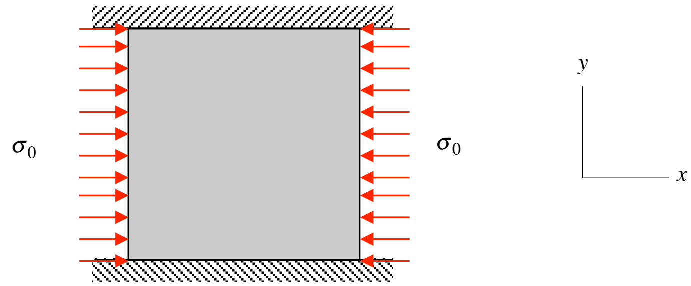


A shaft carrying a torque of T has the cross section shown above, with (1) and (2) being made up of a materials with shear moduli of $3G$ and G, respectively.

a) On the figure above, make a sketch of the shear stress distribution on the cross section.

b) Identify on the sketch the location on the cross section having the largest shear stress in the shaft.

PROBLEM NO. 4 - PART C – 6 points max.

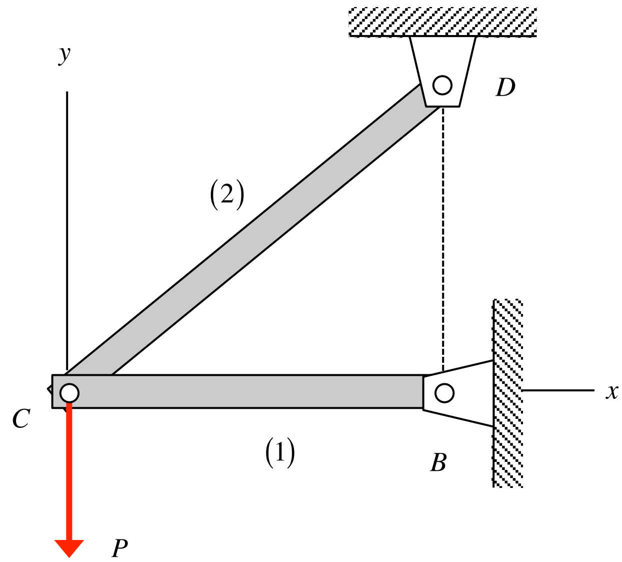


A block of material having a Young's modulus of E and Poisson's ratio of ν is placed between two smooth horizontal surfaces. A uniform, compressive normal stress of σ_0 is applied to the vertical surfaces of the block. The block is unconstrained in the *z*-direction. What are the strains in the *x*- and *y*-directions for the block?

$$\boxed{\varepsilon_y = 0} \;\Rightarrow\; 0 = \frac{1}{E}\left(\sigma_y - \nu\sigma_x\right) \;\Rightarrow\; \sigma_y = \nu\sigma_x = -\nu\sigma_0$$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right) = \frac{1}{E}\left[-\sigma_0 - \nu\left(-\nu\sigma_0\right)\right] = -\frac{\sigma_0}{E}\left(1-\nu^2\right)$$

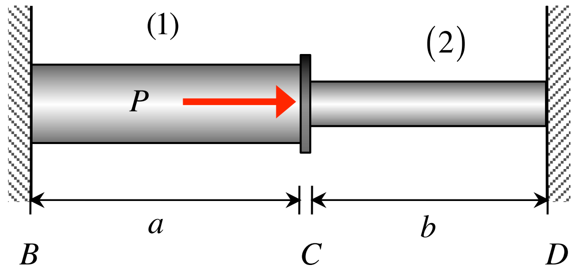
PROBLEM NO. 4 - PART D – 4 points max.



You are reviewing work by a design team at your consulting firm related to the truss bracket shown above. The team's work indicates that if the Young's modulus for element (2) in the truss is doubled, the stress in that element is decreased by a factor of two. You know that this result is not correct. Provide an explanation here to your design team as to why the result is incorrect.

Explanation: The truss is determinate. Member loads are found from equilibrium alone. Therefore, stress does not depend on material properties.

PROBLEM NO. 4 - PART E – 3 points max.



A structure is made up of axial members (1) and (2) shown above with a load of P acting at the rigid connector C. You are asked to re-design the structure by changing the length *b* of member (2) in order to decrease the normal stress in that member. You are *not* able to change any other aspect of the design such as the material or the cross sectional area of the member. Circle the answer below that describes best your design options.

a) A *decrease* in the length *b* of member (2) will decrease the normal stress in the element.
b) An *increase* in the length *b* of member (2) will decrease the normal stress in the element.
c) Changing the length *b* of member (2) cannot change the normal stress in the element.

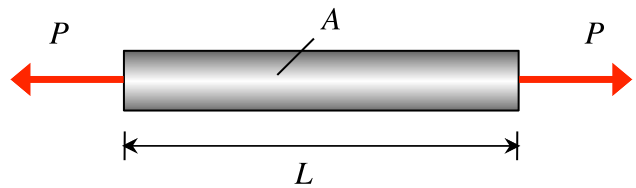
Explanation: The problem is indeterminate. Increasing the length of (2) will decrease its stiffness. This decrease in stiffness will decrease the load carried by member (2), thereby reducing its stress.

Alternately:

$$|e_1| = |e_2| \;\Rightarrow\; \frac{F_1 a}{E_1 A_1} = \frac{|F_2| b}{E_2 A_2} \;\Rightarrow\; \frac{\left(P - |F_2|\right) a}{E_1 A_1} = \frac{|F_2| b}{E_2 A_2} \;\Rightarrow\; |F_2| = \frac{Pa}{a + b\left(E_1 A_1\right)/\left(E_2 A_2\right)}$$

Increasing *b* decreases element load and stress.

PROBLEM NO. 4 - PART F – 3 points max.



An aluminum alloy is known to have a Young modulus, a proportional limit, a yield strength and an ultimate strength of E = 10 ksi, σ_{PL} = 38 ksi, σ_Y = 40 ksi and σ_U = 45 ksi, respectively. A rod having a cross-section area of A = 2 in^2 and length L = 6 in is loaded with an axial force P.

a) What is the largest load P that can be applied to the bar such that the following relationship exists between axial stress and axial strain: $\sigma = E\varepsilon$?
For the linear range, we need $P / A = \sigma < \sigma_{PL} \quad \Rightarrow \quad P < A\sigma_{PL} = 76\ kips$

b) What is the largest load P that can be applied to the bar such that a maximum of 0.1% offset strain remains after the loading is removed?
For this, we need $P / A = \sigma < \sigma_Y \quad \Rightarrow \quad P < A\sigma_Y = 80\ kips$

c) What is the largest load P that can be applied to the bar such that the specimen does not experience necking?
To prevent necking, we need $P / A = \sigma < \sigma_U \quad \Rightarrow \quad P < A\sigma_U = 90\ kips$