Name: _______________, _______________ **Instructor: Susilo Koslowski Raman**
(Print) (Last) (First) (Circle one)

ME 323 MIDTERM # 2

FALL SEMESTER 2011

Time allowed: 1hour

Instructions

1. Begin each problem in the space provided on the examination sheets. If additional space is required, use the yellow paper provided. Work on one side of each sheet only, with only one problem on a sheet.
2. Each problem is of equal value.
3. To obtain maximum credit for a problem, you must present your solution clearly. Accordingly:
    a. Identify coordinate systems
    b. Sketch free body diagrams
    c. State units explicitly
    d. Clarify your approach to the problem including assumptions
4. If your solution cannot be followed, it will be assumed that it is in error.
5. When handing in the test, make sure that ALL SHEETS are in the correct sequential order. Remove the staple and restaple, if necessary.

Prob. 1 _______________

Prob. 2 _______________

Prob. 3 _______________

Total _______________

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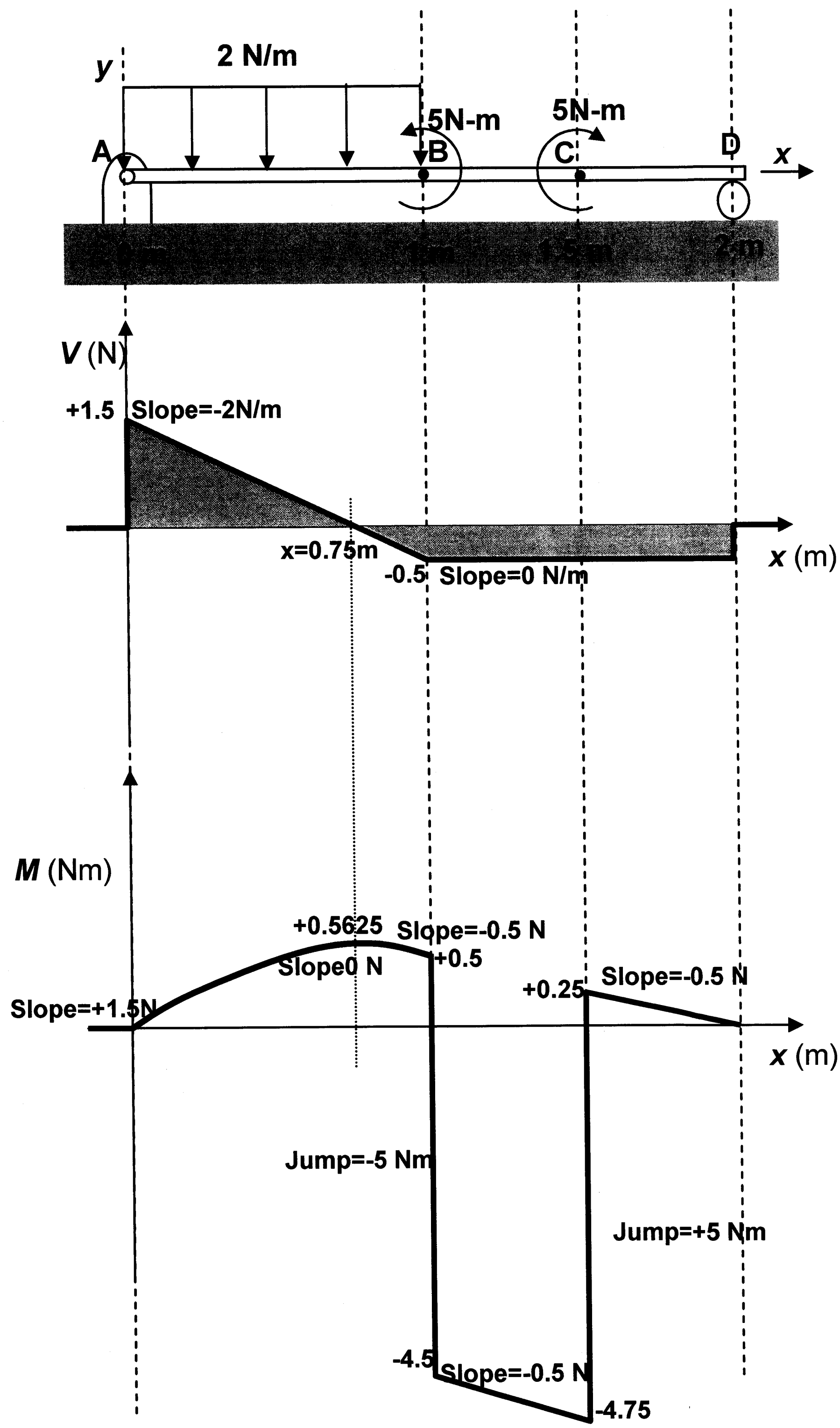
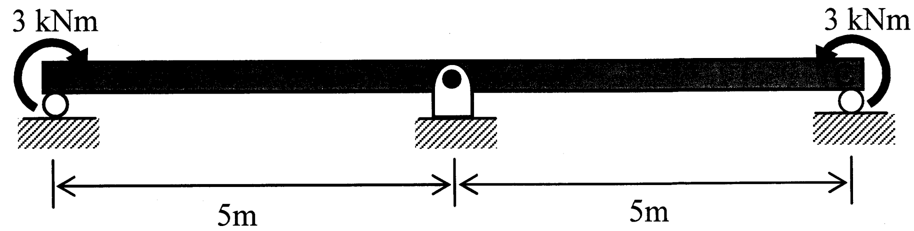
Problem 1

Sketch the shear force-bending moment diagrams for the beam shown below, stating clearly the peak values of shear force (V) and bending moment (M) and the slopes of the M and V with respect to x at A, B, C, D.

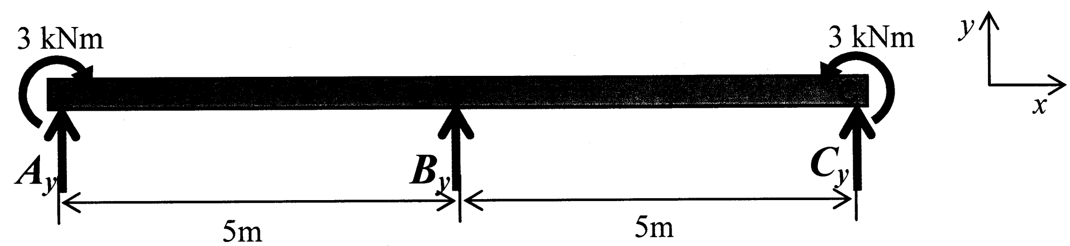


Consider the beam shown below. E = 200GPa, I = 10^{-8} m^4



a. Draw the free body diagram for the beam and write down the equilibrium equations.
b. Find the reaction force at roller A.
c. Find the deflection slope at A.

SOLUTION



$$\sum M_{z,@A} = 0 \quad \rightarrow \quad (5m)B_y + (10m)C_y + 3000Nm - 3000Nm = 0$$

$$B_y = -2C_y$$

$$\sum F_y = 0 \quad \rightarrow \quad A_y + B_y + C_y = 0$$

Statically Indeterminate!

#1. Finding Bending Moment Equation

$$p(x) = 3000\langle x-0\rangle^{-2} + A_y\langle x-0\rangle^{-1} + B_y\langle x-5\rangle^{-1} + C_y\langle x-10\rangle^{-1} - 3000\langle x-10\rangle^{-2}$$

$$V(x) = 3000\langle x-0\rangle^{-1} + A_y\langle x-0\rangle^{0} + B_y\langle x-5\rangle^{0} + C_y\langle x-10\rangle^{0} - 3000\langle x-10\rangle^{-1}$$

$$M(x) = 3000\langle x-0\rangle^{0} + A_y\langle x-0\rangle^{1} + B_y\langle x-5\rangle^{1} + C_y\langle x-10\rangle^{1} - 3000\langle x-10\rangle^{0}$$

#2. Moment-Deflection Equation

$$EIv'' = M(x)$$

$$EIv'' = 3000\langle x-0\rangle^{0} + A_y\langle x-0\rangle^{1} + B_y\langle x-5\rangle^{1} + C_y\langle x-10\rangle^{1} - 3000\langle x-10\rangle^{0}$$

$$EIv' = 3000\langle x-0\rangle^{1} + \frac{A_y}{2}\langle x-0\rangle^{2} + \frac{B_y}{2}\langle x-5\rangle^{2} + \frac{C_y}{2}\langle x-10\rangle^{2} - 3000\langle x-10\rangle^{1} + c_1$$

$$EIv = 1500\langle x-0\rangle^{2} + \frac{A_y}{6}\langle x-0\rangle^{3} + \frac{B_y}{6}\langle x-5\rangle^{3} + \frac{C_y}{6}\langle x-10\rangle^{3} - 3000\langle x-10\rangle^{2} + c_1x + c_2$$

#3. Boundary Conditions

Roller at **A**

$$v(x=0)=0 \quad \rightarrow \quad c_2=0$$

Pin at **B**

$$v(x=5)=0 \quad \rightarrow \quad 1500\left(5^2\right)+\frac{A_y}{6}\left(5^3\right)+5c_1=0$$

Roller at **C**

$$v(x=10)=0 \quad \rightarrow \quad 1500\left(10^2\right)+\frac{A_y}{6}\left(10^3\right)+\frac{B_y}{6}\left(5^3\right)+10c_1=0$$

***Solve* 4 unknowns (A_y, B_y, C_y, c_1)**

$$\begin{cases} \boxed{Eq.1} B_y=-2C_y \quad \rightarrow \quad C_y=-\frac{1}{2}B_y \\ \boxed{Eq.2} A_y+B_y+C_y=0 \\ \boxed{Eq.3} \frac{5^3}{6}A_y+5c_1=-1500\left(5^2\right) \\ \boxed{Eq.4} \frac{10^3}{6}A_y+\frac{5^3}{6}B_y+10c_1=-1500\left(10^2\right) \end{cases}$$

Substitute $C_y=-\frac{1}{2}B_y$ into Eq. 2, 3, 4,

$$\begin{cases} \boxed{Eq.2B} A_y+\frac{1}{2}B_y=0 \quad \rightarrow \quad B_y=-2A_y \\ \boxed{Eq.3B} \frac{5^3}{6}A_y+5c_1=-1500\left(5^2\right) \\ \boxed{Eq.4B} \frac{10^3}{6}A_y+\frac{5^3}{6}B_y+10c_1=-1500\left(10^2\right) \end{cases}$$

Substitute $B_y=-2A_y$ into Eq. 3B and 4B,

$$\begin{cases} \boxed{Eq.3C} \frac{125}{6}A_y+5c_1=-1500\left(5^2\right) \\ \boxed{Eq.4C} 125A_y+10c_1=-1500\left(10^2\right) \end{cases}$$

$\boxed{Eq.4C}-2\times\boxed{Eq.3C}$:

$$\left(125-\frac{125}{3}\right)A_y=-1500\left(10^2\right)+3000\left(5^2\right)$$

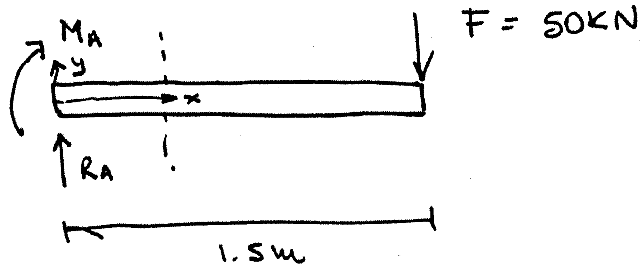
$$A_y=-900N$$

Slope at **A**: Substitute $x=0$ into the v' equation

$$v'=\frac{c_1}{EI}$$

PROBLEM 2 SOLUTION

a)



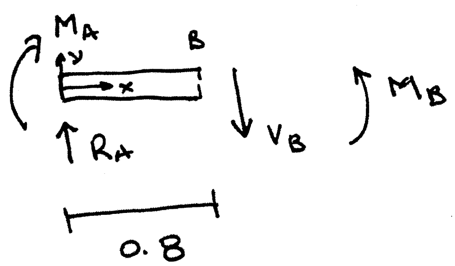
$\Sigma F_y = 0$

$\Sigma M = 0$

$R_A = 50 kN$

$M_A = - F_A \cdot 1.5m$

$= -75 \, kN.m$



$V_B = 50 kN$

$M_B = M_A + R_A \cdot 0.8m$

$M_B = -35 \, kN.m$

b) at $y = 2.5$ cm

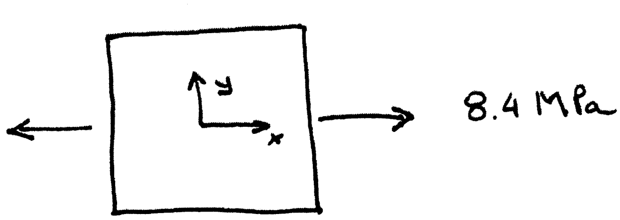
$$\sigma_x = \frac{-M_B \cdot y}{I}$$

$I = 10.4 \; 10^{-7} \; m^4$

$y = h/2 = 0.025 m$

$\sigma_x = 8.4 \, MPa$

$\tau = 0$



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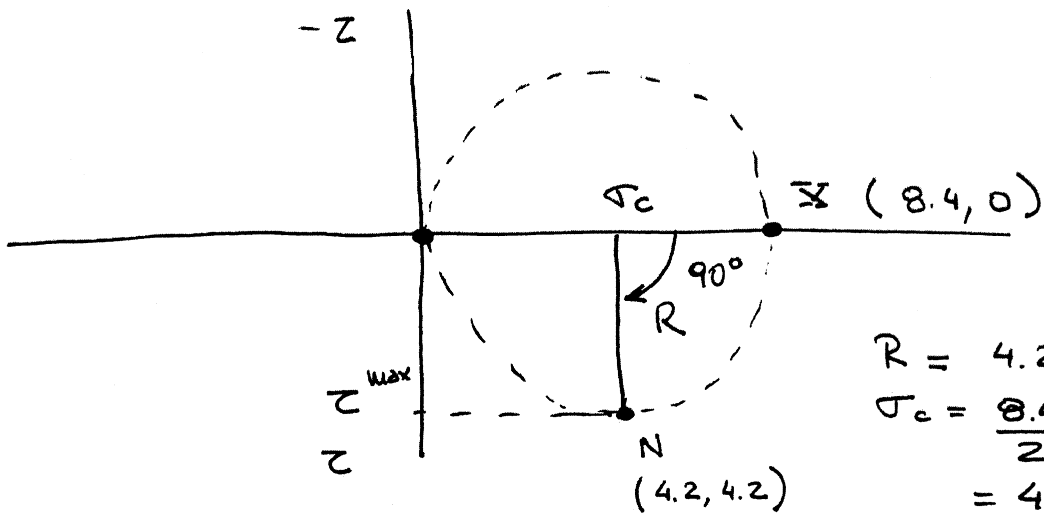
c)



$R = 4.2$ MPa

$\sigma_c = \frac{8.4}{2}$ MPa $= 4.1$ MPa