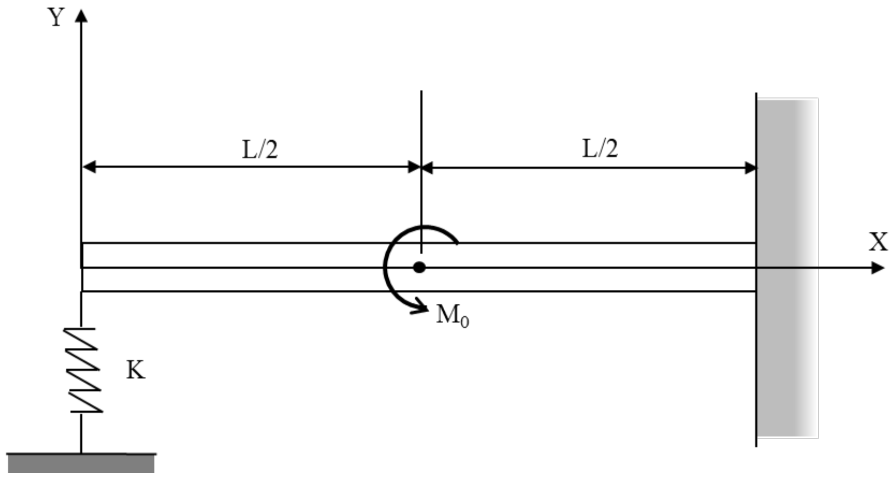
ME 323 Examination #2 | **Name ______________________________**
(Print) (Last) (First)

November 7, 2013 | **Instructor ____________________________**

PROBLEM #1 (15 points)

The cantilever beam shown below is subject to a couple at mid-span and supported by a spring at the left end. Using superposition method (tables shown on page 3), determine the force in the spring; leave your answer in terms of E, I, M_o, L, and K. Spring constant K is known.



Deflection of the left end due to the force applied by the spring

$$\delta_{fs_{left\,end}} = \frac{F_s L^3}{3EI}$$

Deflection of the left end due to couple applied at mid-span

$$\delta_{M_{o\,left\,end}} = -\frac{M_o \frac{L}{2}}{2EI}\left(2L - \frac{L}{2}\right) = -\frac{3M_o L^2}{8EI}$$

Deflection of the left end at the spring

$$\delta_{left\,end} = -\frac{F_s}{K}$$

Therefore,

$$-\frac{F_s}{K} = \frac{F_s L^3}{3EI} - \frac{3M_o L^2}{8EI}$$

Solving for F_s, we obtain,

$$F_s = \frac{\frac{3M_o L^2}{8EI}}{\frac{L^3}{3EI} + \frac{1}{K}}$$

Method II – using singularity function

$$p(x) = F_s < x >^{-1} - M_o < x - \frac{L}{2} >^{-2}$$

$$V(x) = F_s < x >^{0} - M_o < x - \frac{L}{2} >^{-1}$$

$$M(x) = F_s < x >^{1} - M_o < x - \frac{L}{2} >^{0}$$

$$EIv'(x) = \frac{F_s}{2} < x >^{2} - M_o < x - \frac{L}{2} >^{1} + C_1$$

$$EIv(x) = \frac{F_s}{6} < x >^{3} - \frac{M_o}{2} < x - \frac{L}{2} >^{2} + C_1 x + C_2$$

$$v'(x = L) = 0 \rightarrow \frac{F_s}{2}(L)^2 - M_o(\frac{L}{2}) + C_1 = 0 \rightarrow C_1 = -\frac{F_s L^2}{2} + \frac{M_o L}{2}$$

$$v(x = L) = 0 \rightarrow \frac{F_s L^3}{6} - \frac{M_o}{2}\frac{L}{4}^2 + C_1 L + C_2 = 0$$

Substituting for C_1 and solving for C_2, we obtain;

$$C_2 = \frac{F_s L^3}{3} - \frac{3 M_o L^2}{8}$$

$$EIv(x = 0) = \frac{F_s L^3}{3} - \frac{3 M_o L^2}{8} \rightarrow v(x = 0) = \frac{F_s L^3}{3EI} - \frac{3 M_o L^2}{8EI} = -\frac{F_s}{K}$$

Solving for spring force, we have,

$$F_s = \frac{\frac{3 M_o L^2}{8EI}}{\frac{L^3}{3EI} + \frac{1}{K}}$$