Name: KWON JAE-SUNG **Instructor:** Sadeghi Koslowski
Print (Last) (First) (Circle one)

ME 323 EXAM 2
SPRING SEMESTER 2010
28 MARCH, 2012 8:00 PM – 9:30 PM

Instructions

1. Write your name **clearly** on the top of **each sheet**.
2. Begin each problem in the space provided on the examination sheets. If additional space is required, use the paper provided at the exam venue. Work on one side of each sheet only, with one problem on a sheet.
3. The points for each problem are indicated next to the problem number.
4. To obtain maximum credit for a problem, you must present your solution clearly. Accordingly:
   a. Identify coordinate systems
   b. Sketch neat, well labeled free body diagrams
   c. State units explicitly
   d. Clarify your approach
   e. State any assumptions
5. Partial credit will be given for correct approaches, if clearly presented.
6. If your solution cannot be followed, it will be assumed to be incorrect.

Problem 1 (20 pts)	
Problem 2 (25 pts)	
Problem 3 (25 pts)	
Problem 4 (30 pts)	
Total (100 pts)	

6:50 PM ~ 8:15 PM

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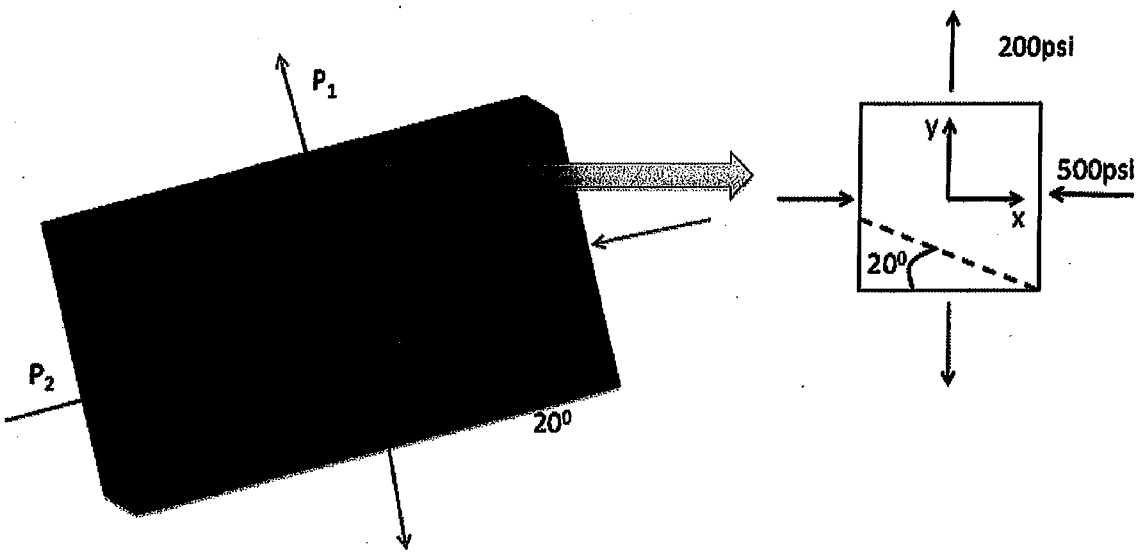
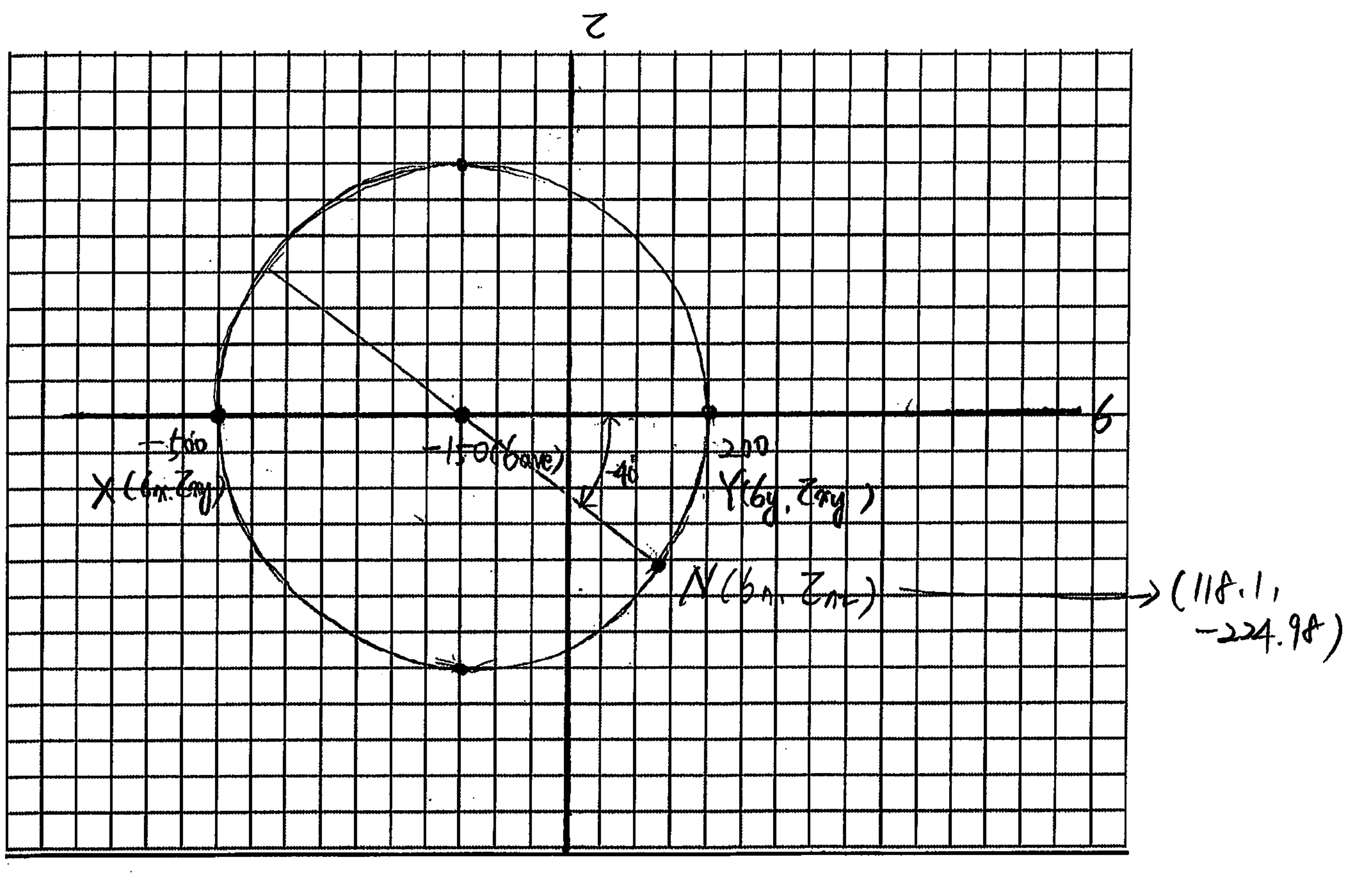
Problem 1 (20 points) A rectangular wooden plate is subjected to forces P_1 and P_2 as shown in the Figure.

1. Draw a Mohr's circle of the state of stress shown in the square element.
2. Apply the Mohr's circle to find the shear and normal stresses acting along the direction of the fiber (-20^0 with the x axis).



Sol)

1.



$\sigma_x = -500$ psi , $\sigma_y = 200$ psi , $\tau_{xy} = 0$

$X(\sigma_x\ \tau_{xy}) = (-500, 0)$

$Y(\sigma_y, -\tau_{xy}) = (200, 0)$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{(-500-200)}{2} = -350 \text{ psi}$$

$$R = \sqrt{(-350)^2 + 0^2} = 350$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-500+200}{2} = -150 \text{psi}$$

2. $\sigma_n = 350 \times \cos 40^\circ - 150 = 350 \times 0.766 - 150 = 118.1$ [psi]

$\tau_{nt} = -350 \times \sin 40^\circ = -224.98$ [psi]

$\therefore (\sigma_n, \tau_{nt}) = (118.1, -224.98)$ [psi]

//

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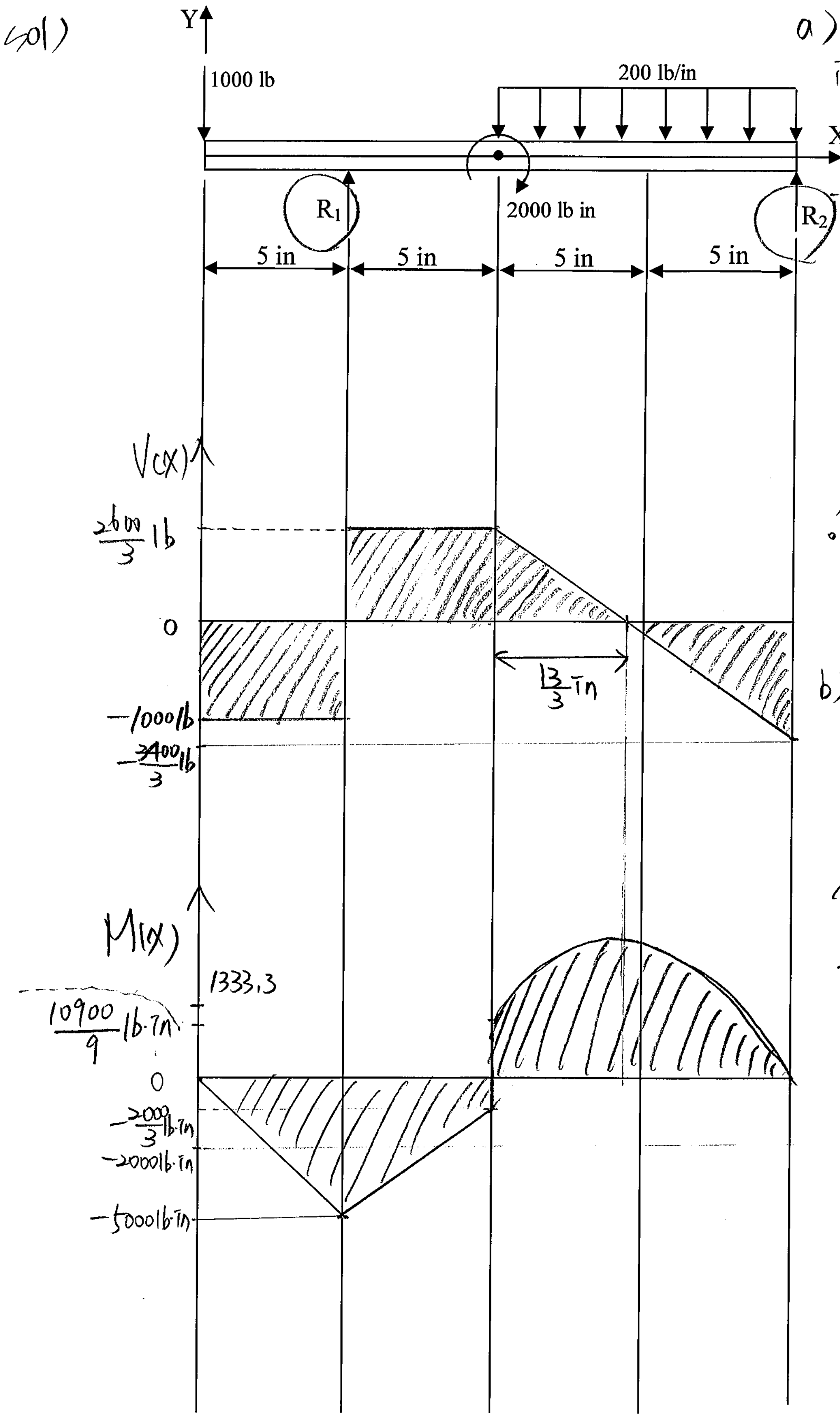
Problem 2 (25 points)

The simply supported beam is subject to the loading shown. Determine the reaction forces R_1 and R_2. Draw appropriate shear and moment diagrams; please indicate the scale and slopes on vertical and horizontal axes carefully.

sol)



a)

i) $\Sigma F = R_1 + R_2 - 1000 - 200 \times 10 = 0$

$\therefore R_1 + R_2 = 3000.$

ii) $\Sigma M = -2000 - 5 \times 2000 + R_2 \times 10 - R_1 \times 5 + 10000 = 0$

$\therefore R_1 - 2R_2 = -400$

By coupling i) and ii)

$\therefore R_2 = \frac{3400}{3}$ lb

$R_1 = \frac{5600}{3}$ lb

b) The shear diagram is drawn like the left side. In order to draw the momentum diagram, $\frac{dM}{dx} = V(x)$ is used.

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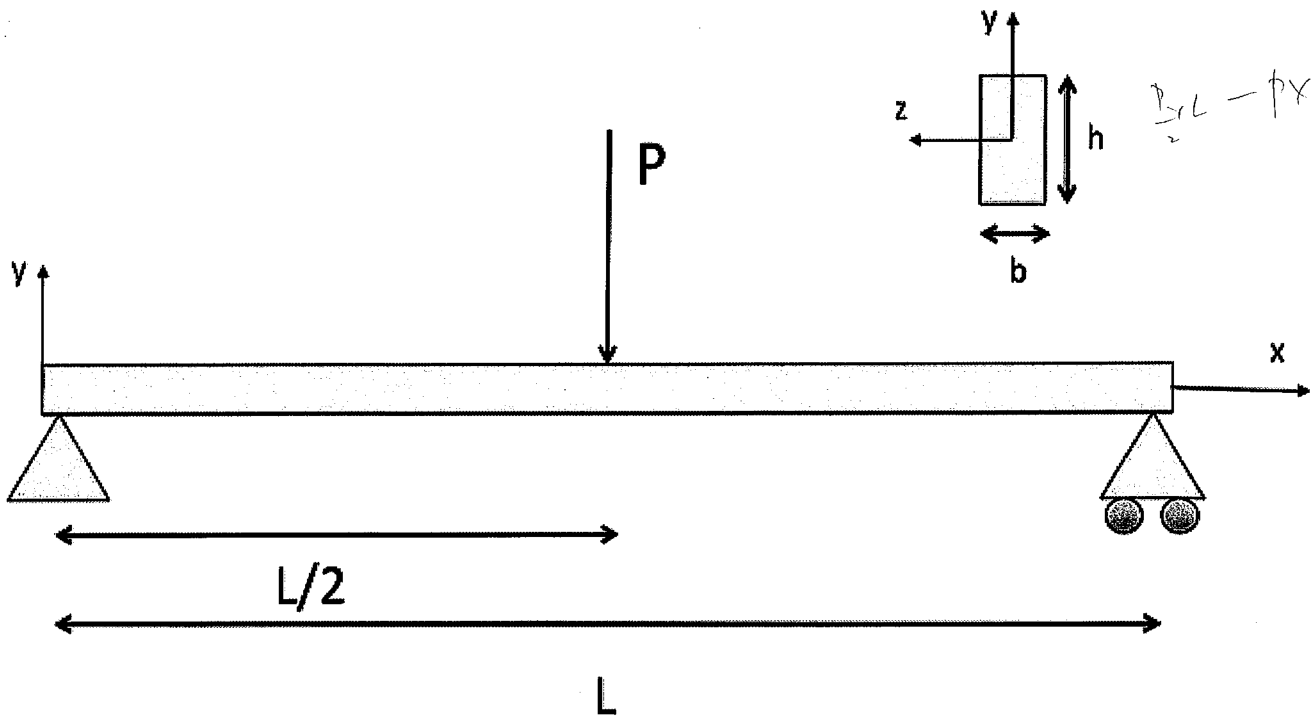
Problem 3 (25 points)

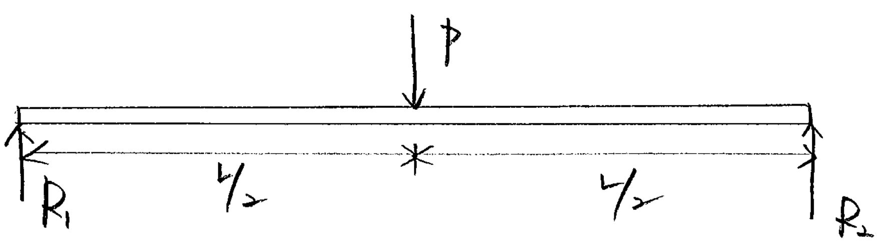
tensile

For the simple supported beam shown in the figure:

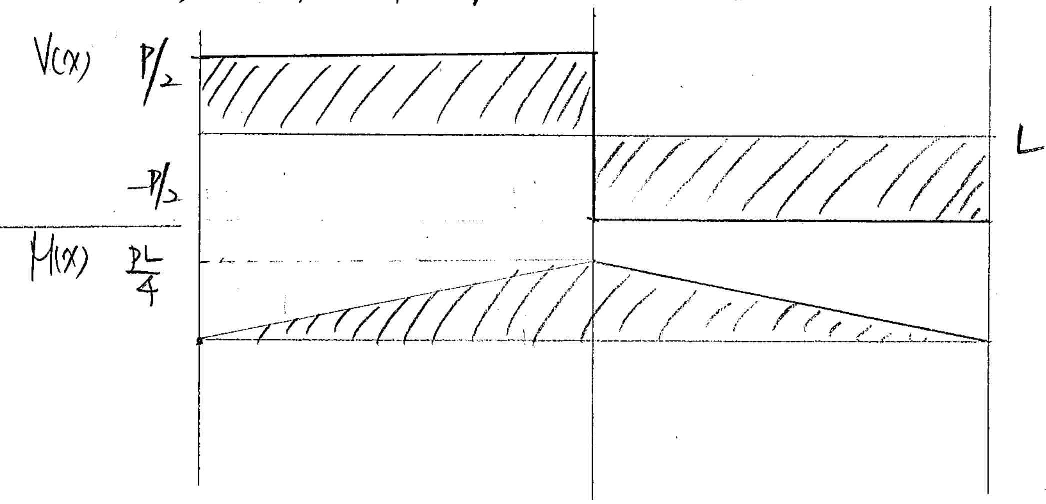
1. Calculate the maximum bending (flexural) stress and state the location (x and y coordinates)
2. Calculate the maximum shear stress and state the location (x and y coordinates)
3. State all the assumptions you made in items 1. and 2. Based on these assumptions calculate the ratio between the maximum shear stress and the maximum bending stress and show which one is larger.



sol) FBD)



$\Sigma F = 0$; $R_1 + R_2 = P$, $\Sigma M = 0$; $\frac{L}{2} \cdot P - R_1 L = 0$ $\therefore R_1 = R_2 = P/2$



① $\sigma_{max} = -\frac{My}{I}\Big|_{max} = -\frac{\frac{PL}{4} \times -\frac{h}{2}}{\frac{bh^3}{12}} = +\frac{\frac{PhL}{\cancel{8}\,2}}{\frac{bh^{\cancel{3}2}}{\cancel{12}\,3}} = +\frac{3PL}{2bh^2}$

$+\frac{h}{2}$ → <compressive>

$\therefore\ \sigma_{max} = \frac{3PL}{2bh^2}$ at $x = L/2,\ y = -\frac{h}{2}$ → <tensile> //

② $\tau_{max} = \frac{VQ}{It}\Big|_{max} = \frac{\frac{P}{2} \times b \times \frac{h}{2} \times \frac{h}{4}}{\frac{bh^3}{12} \times b} = \frac{\frac{P\cancel{b}h^{\cancel{2}}}{\cancel{16}\,4}}{\frac{b^{\cancel{2}}h^{\cancel{3}}}{\cancel{12}\,3}} = \frac{3P}{4bh}$

$\therefore\ \tau_{max} = \frac{3P}{4bh}$ at $y = 0$, along with the beam //

③ $\frac{\sigma_{max}}{\tau_{max}} = \frac{\frac{\cancel{3P}L}{\cancel{2}bh^{\cancel{2}}}}{\frac{\cancel{3P}}{2\cancel{4}b\cancel{h}}} = \frac{\frac{L}{h}}{\frac{1}{2}} = \frac{2L}{h}$

Since the given structure has the dimension of $L \gg h$,

$$\frac{\sigma_{max}}{\tau_{max}} = \frac{2L}{h} \gg 1$$

$\therefore\ \sigma_{max} \gg \tau_{max}$

The above thing is based on these assumptions;

i) the distribution of flexural stress on a given cross section is not affected by the deformation due to shear.

ii) The cross-section of the beam has the dimension of $h \geq 2b$.

iii) Error in the flexural stress calculated by the flexure formula is small if the "L" of the beam is large, compared with the "h".

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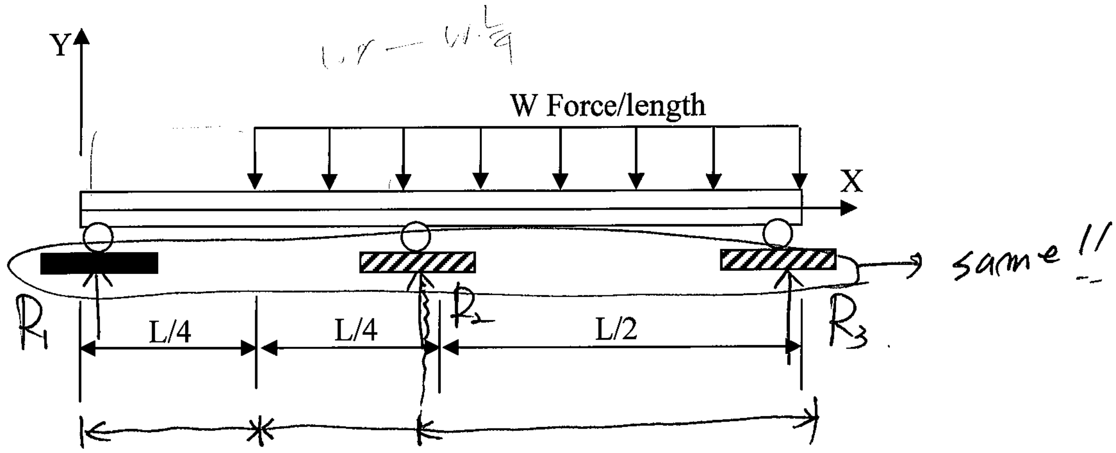
Problem 4 (30 points)

The beam shown below is subject to the loading shown. Using the singularity (Macauley) function; determine the reaction forces R_1, R_2 and R_3. Show all work.



Sol)

$$P(x) = R_1\langle x-0\rangle^{-1} - W\langle x-\tfrac{L}{4}\rangle^0 + R_2\langle x-\tfrac{L}{2}\rangle^{-1} + R_3\langle x-L\rangle^{-1}$$

$$V(x) = \int P(\xi)\cdot d\xi$$

$$= R_1\langle x\rangle^0 - W\langle x-\tfrac{L}{4}\rangle^1 + R_2\langle x-\tfrac{L}{2}\rangle^0 + R_3\langle x-L\rangle^0$$

$\because M(0)=0$

$$M(x) = R_1\langle x\rangle^1 - \frac{W}{2}\langle x-\tfrac{L}{4}\rangle^2 + R_2\langle x-\tfrac{L}{2}\rangle^1 + R_3\langle x-L\rangle^1 + \cancel{M(0)\langle x\rangle^0}$$

$$EI\theta(x) = \frac{R_1}{2}\langle x\rangle^2 - \frac{W}{6}\langle x-\tfrac{L}{4}\rangle^3 + \frac{R_2}{2}\langle x-\tfrac{L}{2}\rangle^2 + \frac{R_3}{2}\langle x-L\rangle^2 + EIv'(0)\langle x\rangle^0$$

$= EIv'$

$$EIv(x) = \frac{R_1}{6}\langle x\rangle^3 - \frac{W}{24}\langle x-\tfrac{L}{4}\rangle^4 + \frac{R_2}{6}\langle x-\tfrac{L}{2}\rangle^3 + \frac{R_3}{6}\langle x-L\rangle^3 + EIv'(0)\langle x\rangle^1 + \cancel{EIv(0)\langle x\rangle^0}$$

$\because v(0)=0$

i) $V(L) = R_1 - \frac{3}{4}WL + R_2 + R_3 = 0$ $\quad\therefore R_1 + R_2 + R_3 = \frac{3}{4}WL$ — ⓐ

ii) $M(L) = R_1L - \frac{9}{32}WL^2 + \frac{R_2L}{2} = 0$ $\quad\therefore R_1 + \frac{R_2}{2} = \frac{9}{32}WL$ — ⓑ

iii) $v(L) = \frac{R_1}{6}L^3 - \frac{W}{24}\cdot\frac{81}{256}L^4 + \frac{R_2}{6}\cdot\frac{L^3}{8} + EIv'(0)\cdot L = 0$. —①

iv) $v\left(\frac{L}{2}\right) = \frac{R_1}{6}\cdot\frac{L^3}{8} - \frac{W}{24}\cdot\frac{L^4}{256} + EIv'(0)\cdot\frac{L}{2} = 0$. —②

From ②,

$$EIv'(0)\cdot L = \frac{W}{12}\cdot\frac{L^4}{256} - \frac{R_1L^3}{24}$$

Plugging it into ①,

$$\frac{R_1}{6}\cdot L^3 - \frac{W}{24}\cdot\frac{81}{256}L^4 + \frac{R_2L^3}{48} + \frac{W}{12}\cdot\frac{L^4}{256} - \frac{R_1L^3}{24} = 0.$$

$\times 24$; $\cancel{4}R_1L^3 - \frac{81}{256}WL^4 + \frac{R_2L^3}{2} + \frac{2WL^4}{256} - \cancel{R_1}L^3 = 0.$

$$= 3R_1\cancel{L^3} + \frac{R_2}{2}\cancel{L^3} - \frac{79}{256}WL^{\cancel{4}} = 0.$$

$$\therefore\ 3R_1 + \frac{R_2}{2} = \frac{79}{256}WL \quad \text{—ⓒ}$$

Coupling ⓐ, ⓑ and ⓒ,

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & \frac{1}{2} & 0 \\ 3 & \frac{1}{2} & 0 \end{pmatrix}\begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} = \begin{pmatrix} \frac{3}{4}WL \\ \frac{9}{32}WL \\ \frac{79}{256}WL \end{pmatrix}$$

$\therefore\ R_1 = 0.0137\,WL$, $R_2 = 0.5352\,WL$, $R_3 = 0.2012\,WL$.