

**ME 323 FINAL EXAM  
FALL SEMESTER 2012  
1:00 PM – 3:00 PM  
Dec. 11, 2012**

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**Instructions**

1. Begin each problem in the space provided on the examination sheets. If additional space is required, use the paper provided. Work on one side of each sheet only, with only one problem on a sheet.
2. Each problem is of value as indicated below.
3. To obtain maximum credit for a problem, you must present your solution clearly. Accordingly:
  - a. Identify coordinate systems
  - b. Sketch free body diagrams
  - c. State units explicitly
  - d. Clarify your approach to the problem including assumptions
4. **If your solution cannot be followed, it will be assumed that it is in error.**

Problem #1: (27)	_____
Problem #2: (27)	_____
Problem #3: (27)	_____
Problem #4: (19)	_____
_____	
<b>TOTAL</b>	<b>(100)</b> _____

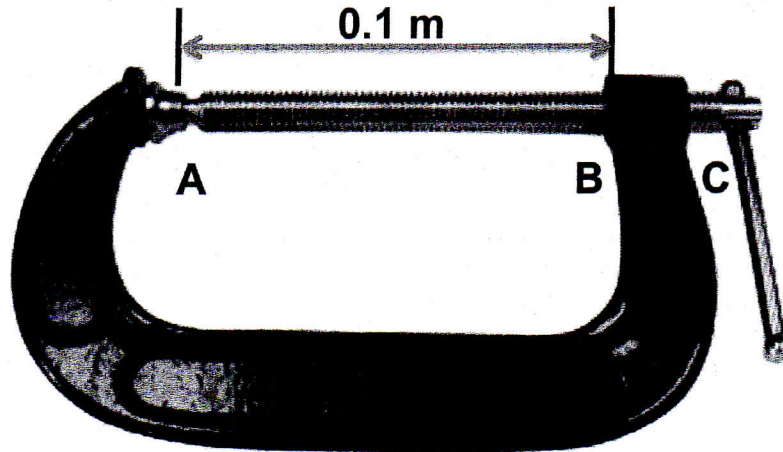
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**PROBLEM #1 (27 points)**

The C-clamp shown is made of steel with elastic modulus of  $E=200 \text{ GPa}$ , and yield strength  $\sigma_{\text{Yield}}=750 \text{ MPa}$ . The clamp is to allow for a clamping force of  $P=5000 \text{ N}$ .

The task is to analyze the clamp bolt **ABC**, which is assumed as a bar with circular cross section of diameter  $d$ . At point **A**, a ball joint is used which allows for rotation of the clamp bolt. At point **B**, the clamp bolt is inside the threaded hole of the clamp body, and thereby is constrained.

Given the distance  $AB=0.1 \text{ m}$ , determine the required diameter  $d$  of the clamp bolt. Consider that the factor of safety for buckling is  $FS_B=3.0$  and that for yielding  $FS_Y=2.0$ .



$$P_c = \frac{\pi^2 EI}{L_{\text{eff}}^2}$$

$$L_{\text{eff}} \text{ (pin-fixed)} = 0.7L$$

$$I = \frac{\pi d^4}{64} \quad FS = \frac{P_c}{P} \Rightarrow P = \frac{P_c}{FS}$$

$$P = \frac{P_c}{FS} = \frac{\pi^2 EI}{FS \cdot L_{\text{eff}}^2} = \frac{\pi^2 E \pi d^4}{FS \cdot L_{\text{eff}}^2 \cdot 64} = \frac{\pi^3 E}{64 FS \cdot L_{\text{eff}}^2} d^4$$

$$d = \sqrt[4]{\frac{64 FS \cdot P \cdot L_{\text{eff}}^2}{\pi^3 E}} = \sqrt[4]{\frac{64 \cdot 3.0 \cdot (0.7 \cdot 0.1)^2 \cdot 5000}{\pi^3 \cdot 200 \times 10^9}} = 5.2 \text{ mm}$$

$$G = \frac{P}{A} \quad A = \frac{\pi d^2}{4} \quad FS = \frac{\sigma_Y}{G} \Rightarrow G = \frac{\sigma_Y}{FS}$$

$$\frac{P}{A} = \frac{\sigma_Y}{FS} \Rightarrow P = \frac{\sigma_Y A}{FS} = \frac{\sigma_Y \pi d^2}{FS \cdot 4} = \frac{\pi \sigma_Y}{4 FS} d^2$$

$$d = \sqrt{\frac{4 FS \cdot P}{\pi \sigma_Y}} = \sqrt{\frac{4 \cdot 2 \cdot 5000}{\pi \cdot 750}} = 4.1 \text{ mm}$$

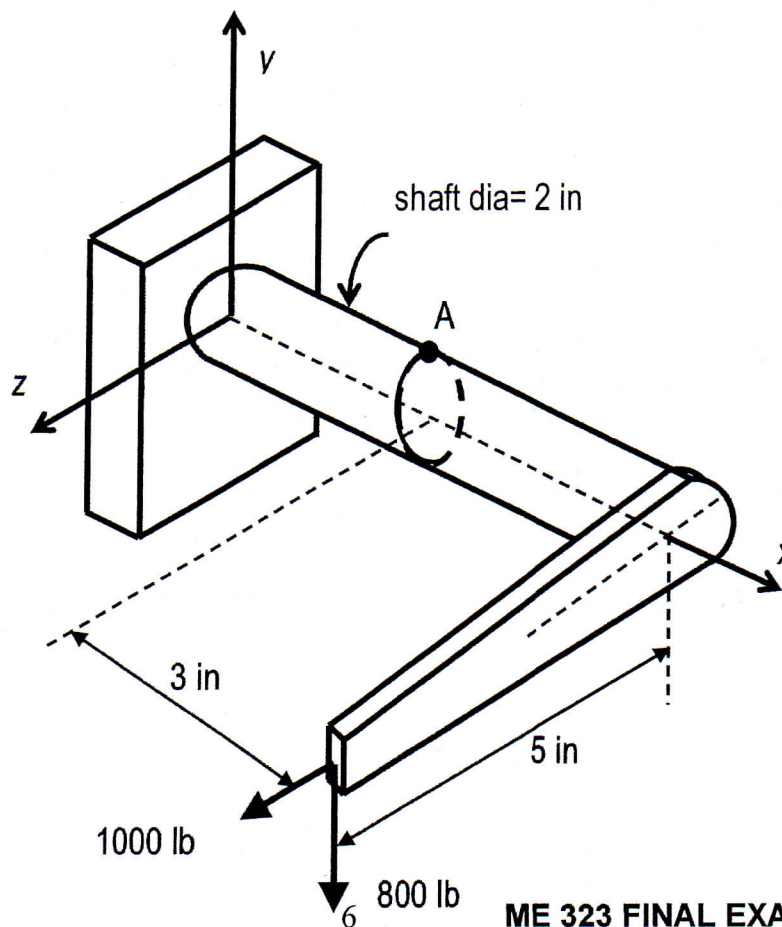
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required  $d = 5.2 \text{ mm}$  (bigger)

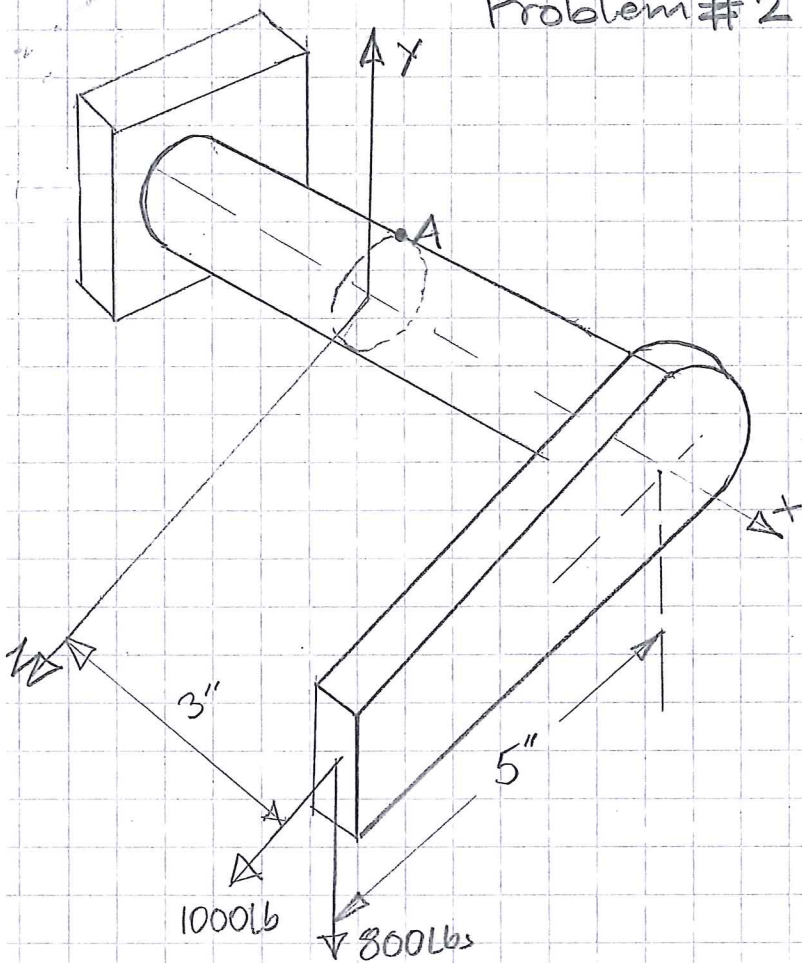
**PROBLEM #2: (27 points)**

A 2-in diameter bracket is securely fastened to the wall as shown in the Figure below. It carries the given horizontal and vertical loads.

- (a) Determine the stresses acting on point **A** on the upper surface of the shaft. Clearly indicate the direction of all stresses on a stress element.
- (b) Determine the maximum resultant stresses (principal stresses) at point **A** of the shaft. Show your stresses on a properly oriented stress element.
- (c) If the direction of **800-lb** load is reversed, specify for each of the following with reasoning whether the stresses increase, decrease or remain unchanged:
  - a. Normal stresses on the element at A.
  - b. Shear stresses on the element at A.
- (d) Determine factor of safety using the maximum-shear-stress theory. The yield strength of the material  $\sigma_{Yield} = 5000$  psi.
- (e) Determine factor of safety using maximum-distortion-energy (Von Mises) theory. The yield strength of the material  $\sigma_{Yield} = 5000$  psi.



# Problem #2



$$\textcircled{1} \sum F_x = 0$$

$$\Rightarrow V_x = 0$$

$$\textcircled{2} \sum F_y = 0$$

$$\Rightarrow -V_y - 800 = 0$$

$$\Rightarrow V_y = -800 \text{ lb} \quad (2)$$

$$\textcircled{3} \sum F_z = 0$$

$$-V_z + 1000 \text{ lb} = 0$$

$$V_z = 1000 \text{ lb} \quad (2)$$

$$\textcircled{4} \sum M_x = 0$$

$$-M_x + 800(5) = 0$$

$$M_x = 4000 \text{ lbs} \quad (2)$$

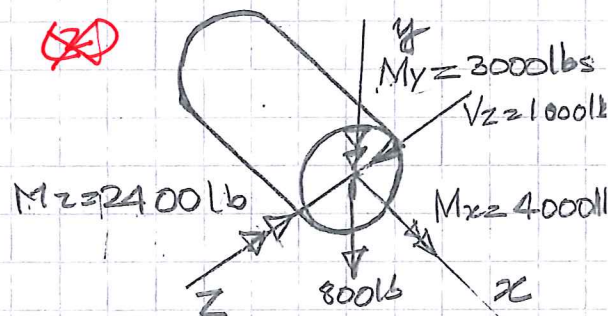
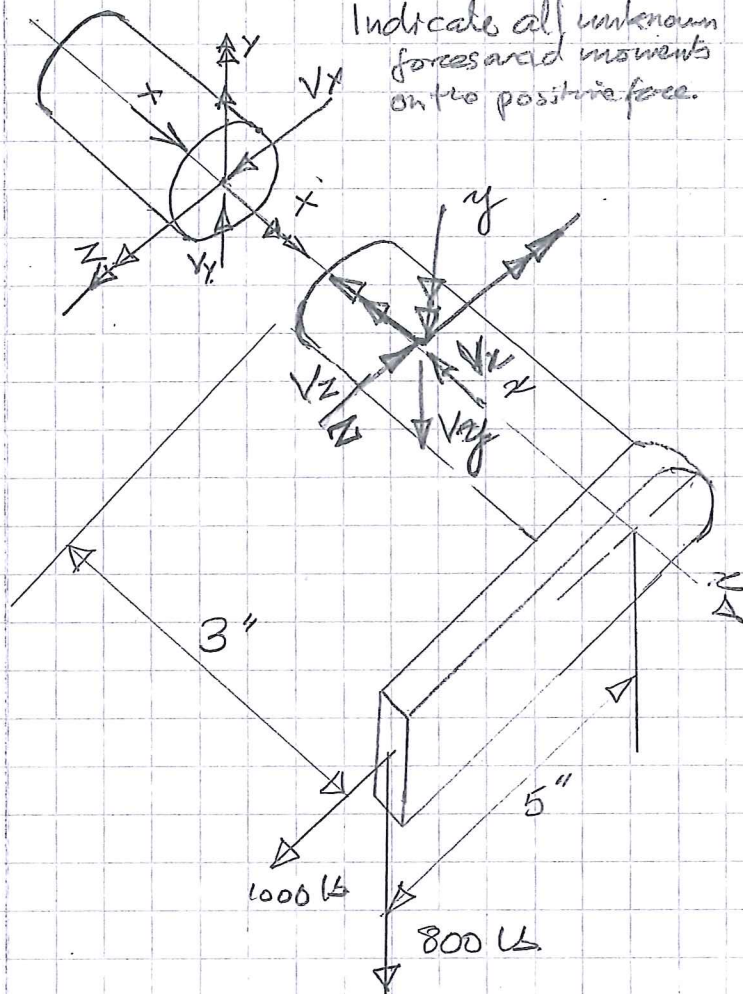
$$\textcircled{5} \sum M_y = 0$$

$$-M_y - 1000(3) = 0$$

$$M_y = -3000 \text{ lb-in} \quad (2)$$

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Indicate all unknown forces and moments on the positive face.



$$\textcircled{6} \sum M_z = 0$$

$$-M_z - 800(3) = 0$$

$$M_z = -2400 \text{ lb-in} \quad (2)$$

Section Properties

$$A = \frac{\pi d^2}{4} = \frac{\pi (2)^2}{4} = \pi \text{ in}^2$$

$$I_{yy} = I_{zz} = \frac{\pi (d^4)}{64} = \frac{\pi (2)^4}{64}$$

$$= 0.785398 \text{ in}^4$$

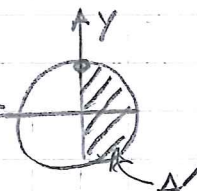
$$I_p = \frac{\pi (d^4)}{32} = \frac{\pi (2)^4}{32} = 1.5708 \text{ in}^4$$

① No stress at pt. A due  $V_y$

②  $\tau_3 = \frac{VQ}{It}$   $[A_y = \frac{2d}{3\pi}]$

$Q = \frac{1}{2} \left[ \frac{\pi}{4} d^2 \right] \cdot \left( \frac{2}{3} \frac{d}{\pi} \right)$  (2)

$Q = \frac{d^3}{12} = \frac{(2)^3}{12} = 0.667 \text{ in}^3$

$\tau_3 = \frac{(1000)(0.667)}{(0.785)(2)}$  

$\tau_3 = 424.841 \text{ psi}$

③ Shear stress due to torque  $M_x$ .

$\tau_4 = \frac{T r_o}{J}$  ;  $T = M_x$  (2)

$= \frac{4000(1)}{1.5708} = 2547 \text{ psi}$

$\tau_4 = 2547 \text{ psi}$

④ Bending moment 'My' doesn't cause any stress at any pt. A.

⑤ Bending 'Mz' causes a normal stress.

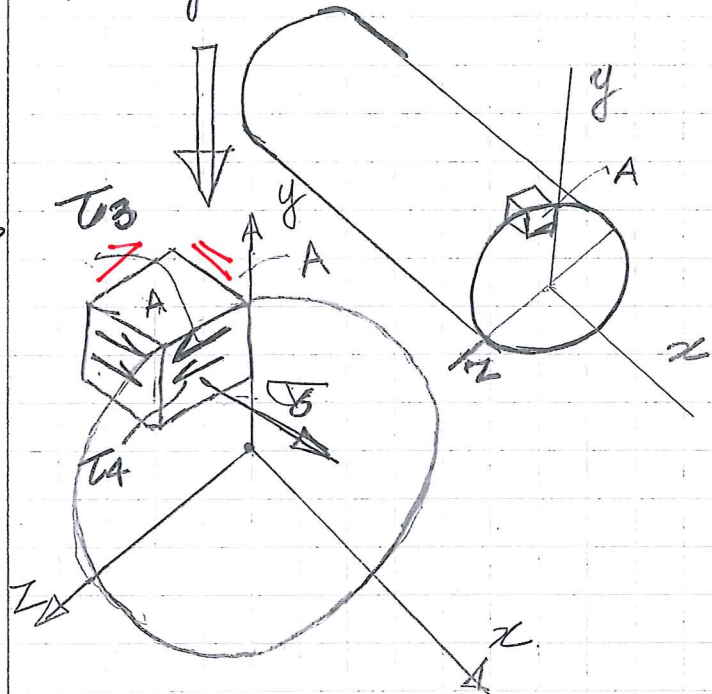
$\sigma = \frac{Mz (r_o)}{I_{zz}}$  (2)

$= \frac{2400(1)}{0.7854}$

$\sigma = 3055.77$

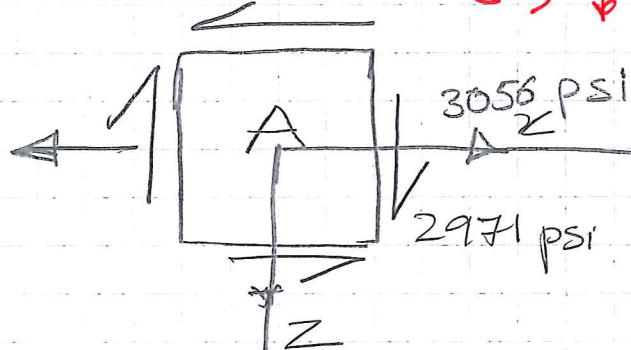
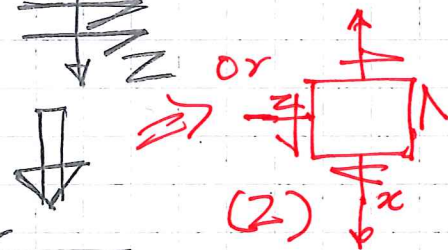
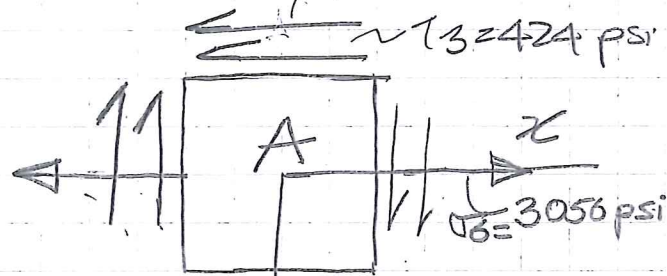
$\sigma = 3056 \text{ psi}$

Looking down: (2)



Extract the stress elements:

look down on the surface on which there are no stresses acting i.e. look down the Z-axis.  $\tau_4 = 2547$



## Principal stresses

$$\sigma_{avg} = \frac{\sigma_x + \sigma_z}{2} = \frac{3056}{2}$$

$$\sigma_{avg} = 1528 \text{ psi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xy}^2}$$

$$R = \sqrt{\left(\frac{3056 - 0}{2}\right)^2 + (2971)^2}$$

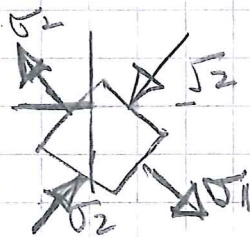
$$R = 3340.9 \text{ psi}$$

$$\sigma_1 = 1528 + 3341 = 4869 \text{ psi}$$

$$\begin{aligned} \sigma_2 &= \sigma_{avg} - R \\ &= 1528 - 3341 \\ \sigma_2 &= -1813 \text{ psi} \end{aligned} \quad (2)$$

## Mohr's circle

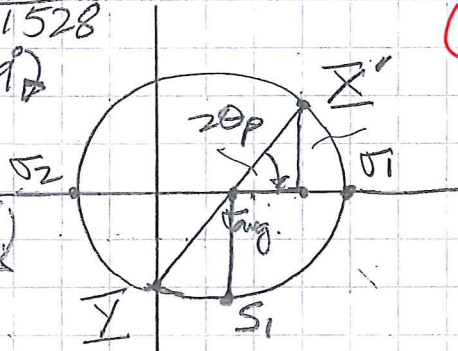
(i)  $\sigma_x = +3056$   
 $\sigma_z = 0$   
 $\tau_{xz} = -$



$$\tan 2\theta_p = \frac{2971}{1528}$$

$$\theta_p = 31.39^\circ$$

$$\theta_s = 76.39^\circ$$



## F.S. using Max. shear stress

$$\sigma_1 = 4869 \text{ psi}$$

$$\sigma_2 = 1813 \text{ psi} \quad (2)$$

$$\sigma_x = 5000 \text{ psi}$$

$$F.S. = \frac{\sigma_y}{|\sigma_1 - \sigma_2|}$$

$$= \frac{5000}{|4869 - (-1813)|}$$

$$F.S. = 0.75 \quad \text{material fails} \\ \text{Not good.}$$

## F.S. using Von Mises

$$\begin{aligned} \sigma_M &= \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} \\ &= \sqrt{(4869)^2 - (4869)(-1813) + (-1813)^2} \end{aligned}$$

$$\sigma_M = 5985.12 \quad (2)$$

$$F.S. = \frac{\sigma_y}{\sigma_M}$$

$$= \frac{5000}{5985.12}$$

$$F.S. = 0.8354 \quad \text{N.G.}$$

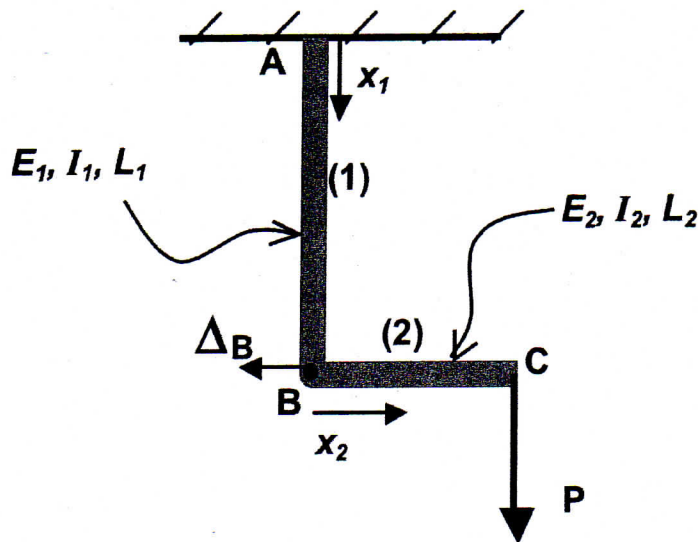
The material still fails.

If the direction is reversed,  
 Normal stress will change di  
 (i) ~~No change in normal stress~~ (2)  
 (ii) The shear stress will reduce  
 b/c the direction of  
 shear due to torque reverse  
 and so stresses decreases.

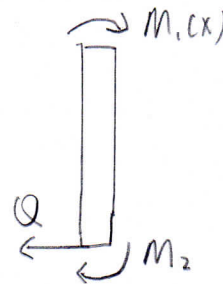
**PROBLEM #3: (27 points)**

The slender L-shaped beam consisting of two members (1) and (2) is welded to the wall (cantilevered) and is subject to a vertical force  $P$  as shown. Assuming that bending dominates the deformation (i.e. strain energy in axial deformation and shear deformation of the structure are negligible) derive an expression for the horizontal deflection  $\Delta_B$  at the point  $B$ . Neglect gravity.

The final answer for  $\Delta_B$  needs to have all the integrals worked out and the answer should be given in terms of the applied load  $P$  as well as  $E_1, I_1, L_1$  and  $E_2, I_2, L_2$  which are respectively the Young's modulus, Area moment, and length of the two members.

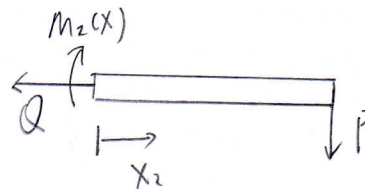


Add a dummy load  $Q$  to  $BC$  as shown in the diagram



$$M_1(x) = (-Qx_1 - PL_2)$$

$$M_2 = -PL_2$$



$$M_2(x) = (-Px_2)$$

$$U = U_1 + U_2$$

$$U = \int_0^{L_1} \frac{(M_1(x_1))^2}{2E_1 I_1} dx_1 + \int_0^{L_2} \frac{Q^2 dx_2}{2E_2 A_2} + \int_0^{L_2} \frac{M_2(x_2)^2}{2E_2 I_2} dx_2$$

$$U = \int_0^{L_1} \frac{(-Qx_1 - PL_2)^2}{2E_1 I_1} dx_1 + \int_0^{L_2} \frac{Q^2 dx_2}{2E_2 A_2} + \int_0^{L_2} \frac{P^2 x_2^2}{2E_2 I_2} dx_2$$

$$U = \frac{Q^2 L_1^3}{6E_1 I_1} + \frac{2QL_1^2 PL_2}{4E_1 I_1} + \frac{P^2 L_2^2 L_1}{2E_1 I_1} + \frac{P^2 L_2^3}{6E_2 I_2} + \frac{Q^2 L_2^3}{2E_2 A_2}$$

$$\left. \frac{\partial U}{\partial Q} \right|_{Q=0} = \Delta_B = \frac{PL_1^2 L_2}{2E_1 I_1}$$

since  $Q=0$

**PROBLEM #4: (19 points) No partial credit is given in individual answers.**

**4.1: (5 points)** From the table below rank the three materials 1-3 for their stiffness and strength.

Material	Young's modulus, E	Poisson's Ratio	Yield Stress $\sigma_Y$	Percent elongation over 2 in. gage length	Stiffness	Strength
Steel	200 GPa	0.3	050 MPa	20	1	2
Nylon	2.8 GPa	0.45	55 MPa	50	3	3
Titanium	115 GPa	0.28	830 MPa	10	2	1

**4.2: (4 points)** If the stress  $\sigma_x$  is applied to a bar the strain  $\epsilon_x$  is measured. Thus, if a stress  $\hat{\sigma}_x = (3/2)\sigma_x$  is applied, the strain  $\hat{\epsilon}_x$  is equal to

$$\hat{\epsilon}_x = \frac{3}{2}\epsilon_x$$

This type of behavior is described by

- (1) Hooke's law
- (2) Poisson's law
- (3) Euler's law.

**4.3: (5 points)** The bar consists of two sections with material A and B, respectively, and the two sections possess equal length L. The bar is fixed between two rigid walls. Material A possesses modulus E and coefficient of thermal expansion  $\alpha$ , and material B possesses modulus 2E and  $2\alpha$ . If the temperature is increased the point joining the two sections moves:

- (1) To the left
- (2) To the right
- (3) Does not move

$$\delta_{free} = L\alpha\Delta T + L2\alpha\Delta T = L3\alpha\Delta T$$

$$\delta_{elas} = \frac{\sigma}{E}L + \frac{\sigma}{2E}L = \frac{3\sigma}{2E}L$$

$$-\frac{3\sigma}{2E}L = 3\alpha\Delta T L$$

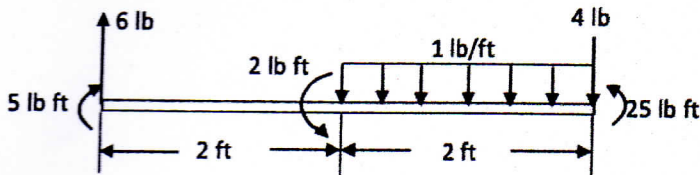
$$\sigma = 2E\alpha\Delta T$$

$$\Delta: \epsilon = -\frac{2E\alpha\Delta T}{E} + \alpha\Delta T = -2\alpha\Delta T + \alpha\Delta T = -\alpha\Delta T < 0$$

move left

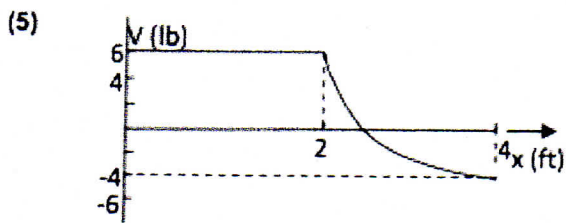
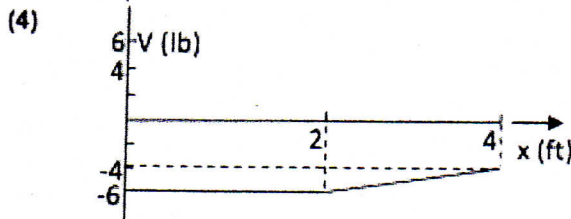
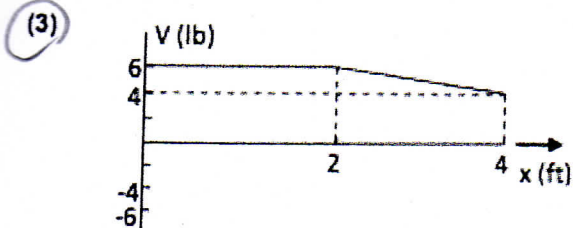
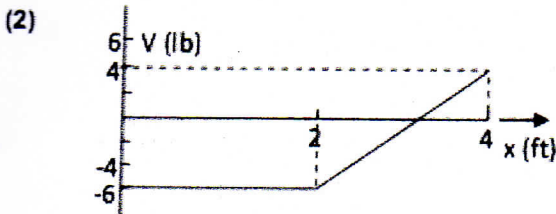
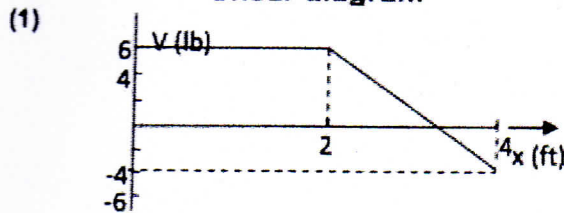


4.4: (5 points) For the beam shown below



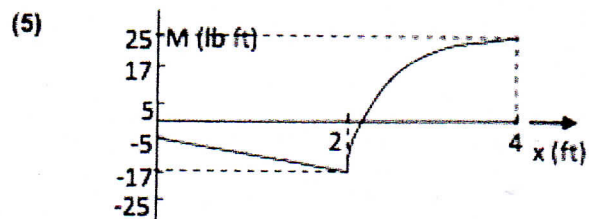
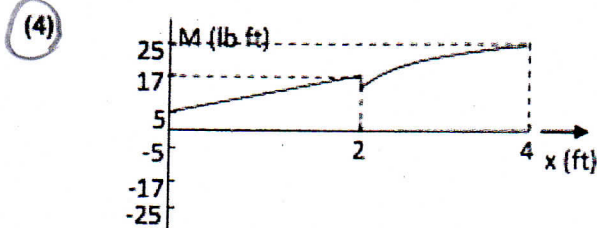
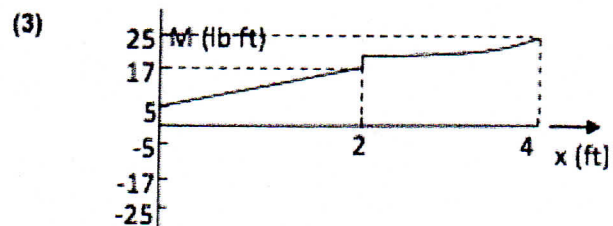
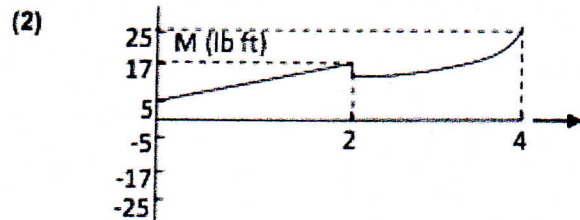
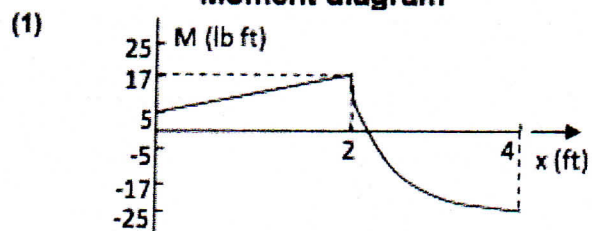
- Choose the correct shear diagram from the column on the left.
  - Choose the correct moment diagram from the column on the right
- Note: The moment diagrams on the right do not necessarily correspond to the diagrams on the left. Circle answers from following choices.

**Shear diagram**



(6) None of above

**Moment diagram**



(6) None of above