ME 323 FINAL EXAM

SPRING SEMESTER 2014

QUESTION SHEET (No need to return)

Time allowed: 2 hours

Instructions

1. Begin each problem in the space provided on the solution sheets. If additional space is required, ask for additional paper. Work on one side of each sheet only, with only one problem on a sheet.

2. To obtain maximum credit for a problem, you must present your solution clearly. Accordingly:
   a. Identify coordinate systems
   b. Sketch free body diagrams
   c. State units explicitly
   d. Clarify your approach to the problem including assumptions

3. If your solution cannot be followed, it will be assumed that it is in error.

4. When you finish your exam please hand in the solutions to problems 1, 2, 3 and 4 separately and restaple if necessary.

Formulas

Hooke's law:

$\varepsilon = (L_f - L_i)/L_i$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \quad \gamma_{xz} = \frac{1}{G}\tau_{xz} \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu(\varepsilon_y + \varepsilon_z)\right]$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_y + \nu(\varepsilon_z + \varepsilon_x)\right]$$

$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_z + \nu(\varepsilon_x + \varepsilon_y)\right]$$

Stress transformation and Mohr's circle:

$$\sigma_{x'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta_{xx'} + \tau_{xy}\sin 2\theta_{xx'}$$

$$\sigma_{y'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta_{xx'} - \tau_{xy}\sin 2\theta_{xx'}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta_{xx'} + \tau_{xy}\cos 2\theta_{xx'}$$

$$\sigma_{p1} = \sigma_{avg} + R \qquad \sigma_{p2} = \sigma_{avg} - R$$

$$\tau_{max} = R$$

$$\sigma_{avg} = \left(\frac{\sigma_x + \sigma_y}{2}\right)$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

Axial deformation, thermal expansion:

$$e = \frac{FL}{EA} + L\alpha\Delta T$$

$$F = \frac{EA}{L}(e - L\alpha\Delta T)$$

$$F = K(e - L\alpha\Delta T)$$

$$e = u\cos(\theta) + v\sin(\theta)$$

Torsion:

$$\tau = Gr\frac{\phi}{L}$$

$$\tau = \frac{Tr}{I_p}$$

$$\phi = \frac{TL}{GI_p}$$

$$K = \frac{GI_p}{L}$$

$$T = K\phi$$

$$I_{p\_Circular\_Cross\_Section} = \frac{\pi d^4}{32}$$

$$I_{p\_Hollow\_Circular\_Cross\_Section} = \frac{\pi(d_o^4 - d_i^4)}{32}$$

Stress due to bending moment:

$$\sigma(x,y) = \frac{-E(x)y}{\rho(x)} = \frac{-M(x)y}{I_{zz}}$$

$$I_{zz} = \frac{bh^3}{12} \text{ (rectangle)} \quad I_{zz} = \pi\frac{d^4}{64} \text{ (circle)}$$

Stress due to shear force:

$$\tau(x,y) = \frac{V(x)Q(y)}{I_z b} \quad Q(y) = \int_{A'} \eta\, dA = A'\bar{y}'$$

$$(\tau_{max})_{N.A.} = \frac{3V}{2A} \text{ (rectangle)} \quad (\tau_{max})_{N.A.} = \frac{4V}{3A} \text{ (circle)}$$

Failure criteria, factor of safety:

von Mises Equivalent Stress:

$$\sigma_M = \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_{p_1}-\sigma_{p_2}\right)^2+\left(\sigma_{p_2}-\sigma_{p_3}\right)^2+\left(\sigma_{p_3}-\sigma_{p_1}\right)^2}$$

von Mises Stress (Plane Stress):

$$\sigma_M = \sqrt{\sigma_{p_1}^2 - \sigma_{p_1}\sigma_{p_2} + \sigma_{p_2}^2}$$

$$FS = \frac{Failure\ Stress}{Allowable\ Stress}, \frac{Yield\ Strength}{State\ of\ Stress}$$

Buckling:

Critical buckling load for a pinned-pinned beam

$$P_{cr} = EI\frac{\pi^2}{L^2}$$

Critical buckling for fixed-fixed beam

$$P_{cr} = EI\frac{\pi^2}{(0.5L)^2}$$

Stress in pressure vessels:

$$\sigma_{spherical} = \frac{pr}{2t};\quad \sigma_h = \frac{pr}{t};\quad \sigma_a = \frac{pr}{2t}$$

2nd order equation to solve for deflection curve

$$EI\frac{d^2v(x)}{dx^2} = M(x)$$

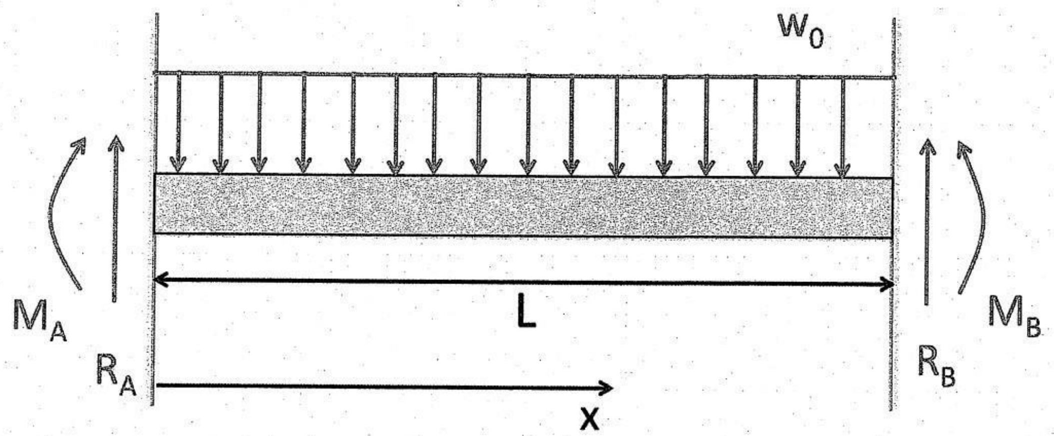
Energy methods:

Strain energy: $U_{tot} = \int \frac{M^2}{2EI}dx + \int \frac{F^2}{2AE}dx + \int \frac{T^2}{2GI_p}dx + \int \frac{f_s V^2}{2AG}dx$ where f_s is the shape factor

- Castigliano's theorem for deflection Δ at a point in the direction of the force $\Delta = \partial U_{tot}/\partial P$ where P is the point force
- Castigliano's theorem for slope θ_C or angle of twist ϕ_C in the direction of an applied moment M or applied torque T is given by $\theta_C = \partial U_{tot}/\partial M$ or $\phi_C = \partial U_{tot}/\partial T$
- Work-energy theorem states that the deflection Δ at the point of application of a force P in the direction of applied force can be calculated by equating $\frac{1}{2}P\Delta = U_{tot}$
- Work-energy theorem states that the slope θ_C or angle of twist ϕ_C in the direction of an applied moment M or applied torque T or can be calculated by equating $\frac{1}{2}M\theta_C = U_{tot}$ or $\frac{1}{2}T\phi_C = U_{tot}$

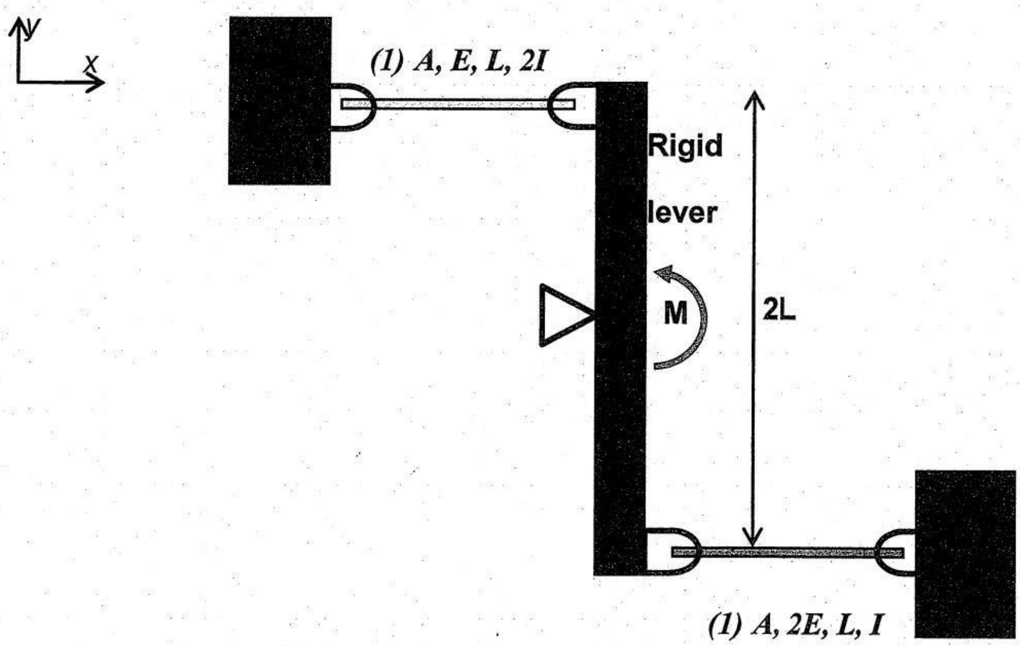
Problem 1 (25 points)

The fixed-fixed beam with flexural rigidity **EI** is subjected to a uniformly distributed load of intensity **w_0**. Use second order integration method to find the reactions at A and B.



Problem 2 (25 points)

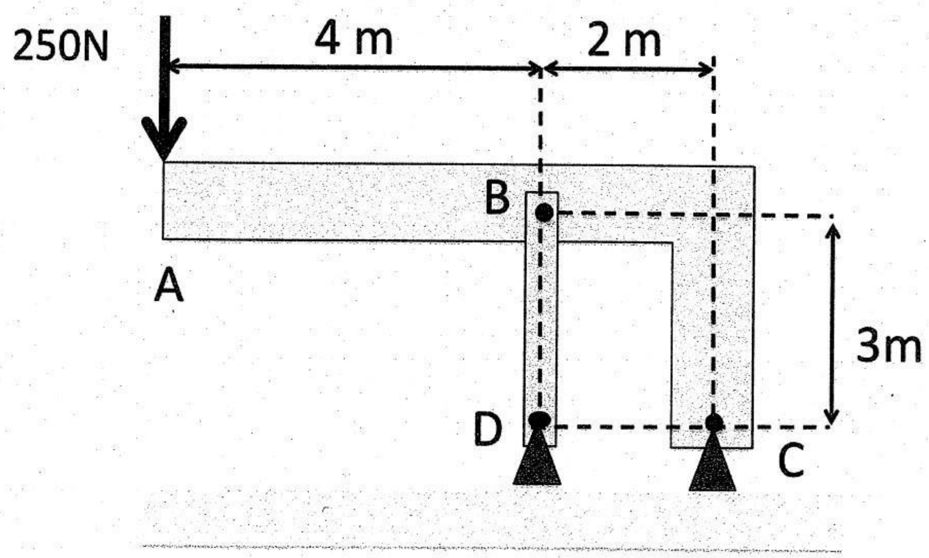
An external moment M is applied to the midpoint of the rigid lever below and is gradually increased from 0. **Assume that the rods (1) and (2) both have pinned-pinned boundary conditions**. Rods (1) and (2) have equal cross sectional areas but different area moments and moduli of elasticity as shown below. Determine the rotation angle θ of the rigid lever in terms of E, I, L, A when the first buckling event occurs. Assume the rotation angle is small so that $\sin(\theta) \sim \theta$ and $\cos(\theta) \sim 1$.



Problem 3

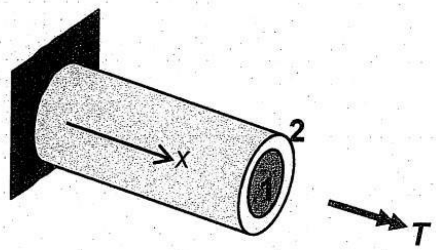
The bar ***ABC*** shown below is attached to a rigid base by a pin joint at ***C***. The bar ***BD*** is a pin-jointed truss member that connects ***ABC*** and the base. The structure is made of aluminum (**E** = ***70GPa***). The cross section of the member **ABC** is a square with side **10cm** and the cross section of **BD** is a square with side **5cm.** Using the Castigliano's theorem determine the **vertical deflection at A**, neglect shear strain energy.

Hint: The member BD is a two force member so it only supports and axial force resultant.



Problem 4 – please circle the correct answer in the solution sheet not here

Problem 4.1 **(15 points)** A composite cylindrical bar consists of a core (1) of shear modulus G_1 and outer diameter d_1 and an outer cladding (2) of shear modulus G_2 and outer diameter d_2. An external torque T is applied to a rigid cap at the end of the bar. State whether the following statements are true or false.



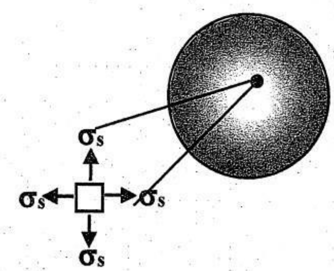
(A) The shear strain is always largest at the outer surface of (2). *True False*

(B) The shear stress maximizes at the interface of (1) and (2) if $G_1 d_1 > G_2 d_2$. *True False*

(C) The ratio of internal torques in (1) to (2) equals the ratio $(GI_p)_1 / (GI_p)_2$. *True False*

Problem 4.2 ***(10 points)***

A thin-walled spherical pressure vessel made of stainless steel of inner radius R and thickness t, contains gas at a gauge pressure of p. If p_{fail} is the critical pressure at which the wall of the vessel fails, then what is the relationship between $(p_{fail})_{tresca}$ the critical pressure predicted by the absolute maximum shear stress theory (Tresca), and $(p_{fail})_{von-Mises}$ by the distortional strain energy theory (von Mises)?



(A) $(p_{fail})_{von-Mises} > (p_{fail})_{tresca}$, (B) $(p_{fail})_{von-Mises} = (p_{fail})_{tresca}$, (C) $(p_{fail})_{von-Mises} < (p_{fail})_{tresca}$