ECE 20100 –Fall 2014
Exam #1

September 25, 2014

Section ___

Name Solutions PUID

Instructions

1. DO NOT START UNTIL TOLD TO DO SO.

2. Write your name, section, professor, and student ID# on your Scantron sheet. We may check PUIDs.

3. This is a CLOSED BOOKS and CLOSED NOTES exam.

4. Calculators are NOT allowed (and not necessary).

5. If extra paper is needed, use the back of test pages.

6. Cheating will not be tolerated. Cheating in this exam will result in, at the minimum, an F grade for the course. In particular, continuing to write after the exam time is up is regarded as cheating.

7. Please do not turn in your Scantron sheet before time is up. Please inform the proctors if you need to go to the bathroom.

8. If you cannot solve a question, be sure to look at the other ones, and come back to it if time permits.

9. Exam #1 provides evidence for satisfaction of this ECE 20100 Learning Objective:
   i) An ability to analyze linear resistive circuits.

   The minimum score needed to satisfy this objective will be posted on Blackboard after the exam has been graded. Remediation options will be posted in Blackboard if you fail to satisfy any of the course outcomes.

Honor Pledge: I have neither given nor received unauthorized assistance on this exam

Signature: ________________________________
1. Fig. 1(a) illustrates the charge flow in a conductor. The net charge accumulated on the right side of the cross-section boundary is shown in Fig. 1(b) for time $0 < t < 4$ seconds.

The reference current direction is from left to right, as shown in Fig. 1(a). Which of the following figures shows the correct current flow?
2. The voltage \( V_o \) across a device (in V) has the polarity shown in Fig. 2, and the reference direction of current \( I_o \) (in A) is also shown in Fig. 2. If \( V_o = -3I_o \), then the equivalent resistance of the device (in \( \Omega \)) is:

\[ \text{Equivalent resistance} \rightarrow \text{value of resistor that would give same voltage and current relationship} \]

\[ \text{Ohm's Law: } V_o = I_o R_{\text{eq}} \text{ for passive sign convention} \]

\[ \begin{align*}
+ V_o' & \quad - \\
\rightarrow & R_{\text{eq}}
\end{align*} \]

\[ \Rightarrow R_{\text{eq}} = \frac{V_o'}{I_o} \quad [4] \]

\[ \begin{align*}
\text{So } V_o' &= V_o' \quad [3] \\
\text{and } I_o &= -I_o \\
\therefore V_o &= -3I_o \quad [1]
\end{align*} \]


\[ \frac{V_o'}{I_o} = 3 \quad [5] \]

- Plug [5] into [4]:

\[ R_{\text{eq}} = \frac{V_o'}{I_o} = 3 \quad \boxed{3} \]

(1) 1  (2) 2  (3) 3  (4) 4  (5) 5
(6) -1  (7) -2  (8) -3  (9) -4  (10) -5
3. The equivalent resistance (in $\Omega$) between nodes 1 and 2 is:

\[ R_{eq} = \frac{4 \Omega}{2} = 2 \Omega \]  
\[ R_{eq} \text{ for } N \text{ resistors in parallel is } R_{eq} = \frac{R}{N} \]

- The 4 $\Omega$ resistors are in parallel:

- So circuit can be redrawn:

\[ R_{eq} = \frac{4 \Omega}{2} = 2 \Omega \]

- All these resistors are in series. For resistors in series, \[ R_{eq} = \sum R_k \]

\[ R_{eq} = 1 + 2 + 2 + 3 = 8 \Omega \]
4. Given the following circuit, what is the total power (in Watts) that is absorbed by the unknown circuit elements #1 and #2, according to the passive sign convention?

![Circuit Diagram]

(1) 0  (2) 2  (3) -2  (4) 5  (5) -5  
(6) 7  (7) -7  (8) 10  (9) -10  (10) None of the above

- Power absorbed is calculated as \( P = VI \) when \( V \) and \( I \) are in passive sign convention. Need voltage across and current through each unknown element.

**Element #1** (\( V \) & \( I \) both given)

\[
\begin{align*}
5V & \quad \downarrow \ 12A \\
\ \ \\
V_1 &= 5V \\
I_1 &= 2A
\end{align*}
\]

\[
\begin{align*}
&\text{In passive sign convention} \\
\therefore \quad P_1 &= V_1 I_1 = 5 \times 2 = 10 \text{ W}
\end{align*}
\]

**Element #2**

- \( V_2 \) is known, \( I_2 \) is unknown

- Perform KCL on the surface drawn
  \( I \) entering = \( I \) leaving

\[
2 = 2 + I_2 + 1
\]

\[
\therefore \quad I_2 = -1 \text{ A}
\]

- \( V_2 \) is unknown
- Perform KVL on \( G - A - B - C - G \):

\[
5 + 3 - V_2 - 5 = 0
\]

\[
\therefore \quad V_2 = 3 \text{ V}
\]

Therefore

\[
P_2 = V_2 I_2 = (3)(-1) = -3 \text{ W}
\]

- Total power absorbed by the two elements is \( P_{\text{Total}} = P_1 + P_2 = 10 + (-3) = 7 \text{ W} \)
5. Consider the circuit below. If the independent voltage source $V_s = 4\, \text{V}$, what current $I_2$ (in Amperes) will be generated on the other side?

![Circuit Diagram]

(1) 0  (2) 1  (3) 1.5  (4) 2  (5) 3  
(6) 4  (7) 4.5  (8) 6  (9) 12  (10) None of the above

- Because there is only a single wire connecting Region 1 & Region 2, the current $I_{1-2} = 0$. Thus, the two regions can be analyzed separately. (To see this, perform KCL on a surface enclosing Region 1 completely, but does not enclose any portion of Region 2).

- To find $I_2$, we can use Ohm's Law on the 1Ω resistors in series:

\[
\begin{align*}
\text{\text{\text{\text{V_2 = 3V_1 \hspace{1cm} (resistor \& voltage source are in parallel)}}}} \\
I_2 = \frac{V_2}{1} \hspace{1cm} (\text{[2]})
\end{align*}
\]

- To solve [1] for $I_2$, we need $V_1$. $V_1$ can be found by voltage division across the 2Ω resistors in Region 1 (because they are in series):

\[
\begin{align*}
4V &= V_s \hspace{1cm} (2Ω) \\
V_1 &= \frac{2V}{2+2} = 2V \hspace{1cm} (\text{[2]})
\end{align*}
\]

- Plugging [2] into [1], we find $I_2$:

\[
I_2 = \frac{3V_1}{2} = \frac{3(2)}{2} = \boxed{3\, \text{A}}
\]
6. The value of voltage $V_1$, in Volts, for the circuit below is:

\[
\begin{array}{c}
\text{2 A} \\
\text{+} \\
\text{I}_1 \\
\text{V}_1 \parallel 1 \Omega \\
\text{-} \\
\text{3 A}
\end{array}
\]

(1) 1  (2) 2  (3) 3  (4) 4  (5) 5
(6) 6  (7) -1  (8) -2  (9) -3  (10) -4

*With passive sign convention, ohm's law can be used to find $V_1$:

\[V_1 = I_1 (1)\]

* $I_1$ is unknown and can be found by KCL @ A:

I entering = I leaving

\[2 + 3 = I_1\]

$\therefore I_1 = 5$A

\[\therefore V_1 = I_1 (1) = 5 \text{ V}\]
7. Choose the node equation that can be used to correctly calculate the voltage $V_a$ in this diagram:

$$\begin{align*}
(1) \quad & 0.2(V_a - 12) + 0.2(V_a - V_o) + V_a = 0 \\
(2) \quad & 5(V_a - 12) + 5(V_o - V_a) + V_a = 0 \\
(3) \quad & 5(V_a + 12) + 5(V_o + V_o) + 2V_a = 0 \\
(4) \quad & 11(V_a - 12 - V_o) = 0 \\
(5) \quad & (V_a - 12)/5 + (V_o - V_o)/5 + V_a = 0 \\
(6) \quad & V_a = (V_o + 12)/2 \\
(7) \quad & V_a = 12 + V_o + 0.2 + 0.2 + 1 \\
(8) \quad & \text{None of the above}
\end{align*}$$

- **Performing KCL @ a:**
  $$\sum I_{\text{leaving}} = 0$$
  $$I_1 + I_2 + I_3 = 0 \quad \square$$

- **Need $I_1$, $I_2$, & $I_3$:**
  From Ohm's law with passive sign convention,
  $$I_1 = \frac{V_a}{0.2} = \frac{V_a - 12}{0.2} = 5(V_a - 12)$$
  $$I_2 = \frac{V_o}{0.2} = \frac{V_a - V_o}{0.2} = 5(V_a - V_o)$$
  $$I_3 = \frac{V_3}{1} = \frac{V_a - 0}{1} = V_a$$

- **Plug these into $\square$**
  $$5(V_a - 12) + 5(V_a - V_o) + V_a = 0$$
8. Which of the following equations correctly describes its corresponding node in the diagram below?

\[ \frac{V_1}{10} + \frac{V_2 - V_1}{15} + 10 = 0 \text{ (node 1)} \]
\[ 15(V_2 - V_1) + 5V_2 + 0.4V_x = 0 \text{ (node 2)} \]
\[ 0.4V_x + 10 + \frac{V_3 - V_4}{10} + \frac{V_3}{5} = 0 \text{ (node 3)} \]
\[ \frac{V_3 - V_4}{10} + 5 = 0 \text{ (node 4)} \]
\[ \frac{V_1}{10} + \frac{V_2}{5} + \frac{V_x}{5} = 0 \text{ (Ground node)} \]
\[ \frac{V_2 - V_1}{15} + \frac{V_2}{5} + 5 + \frac{V_3}{5} = 0 \text{ (node 2)} \]
\[ -10 - 0.4V_x + \frac{V_3}{5} + \frac{V_3 - 5}{10} = 0 \text{ (node 3)} \]
\[ \frac{V_2 - V_1}{15} + \frac{V_2}{5} + \frac{V_3}{5} + \frac{V_3 - V_4}{10} + 10 = 0 \text{ (node 2)} \]
9. What is the current $I$ (in Amperes) flowing through the 1 Ω resistor depicted in the figure below?

![Circuit Diagram]

(1) 1 (2) 2 (3) 3 (4) 4 (5) 5
(6) -4 (7) -3 (8) -2 (9) -1 (10) 0

- To find $I$, perform KCL @ A:
  \[ I \text{-entering} = I \text{-leaving} \]
  \[ I = I_1 + I_3 \] [1]

- By Ohm's Law with passive sign convention:
  \[ I = \frac{V_A - V_B}{1} = V_A - V_B \] [2]
  \[ I_3 = \frac{V_3}{3} = \frac{V_A - 0}{3} = \frac{V_A}{3} \] [3]

  \[ 2 = (V_A - V_B) + \left(\frac{V_A}{3}\right) \]
  \[ 6 = 3V_A - 3V_B + V_A \]
  \[ 4V_A = 6 + 3V_B \]
  \[ V_A = \frac{6 + 3V_B}{4} \] [4]

- $V_B$ is given by the voltage source:
  \[ V_B = 0 - 6 = -6V \] [5]

- Plug [5] into [4]:
  \[ V_A = \frac{6 + 12}{4} = \frac{18}{4} = -3V \] [6]

  \[ I = V_A - V_B = -3 - (-6) = \boxed{3 A} \]
10. Find the value of the voltage $V_1$ (in Volts) across the independent current source in this diagram:

![Diagram of electrical circuit]

- $V_1$ can be found by performing KVL along $A - B - C - A$:
  \[ V_1 - V_2 - V_3 = 0 \quad \rightarrow \quad V_1 = V_2 + V_3 \quad [1] \]

- $V_2$ and $V_3$ can be calculated by Ohm's law with passive sign convention:
  \[ V_2 = I_2 \cdot (1) = I \cdot (i) = (1)(1) = 1 \text{ V} \quad [2] \]

  Because resistor and $I_1$ source are in series

  \[ V_3 = I_3 \cdot (2) \quad [4] \]

- $I_3$ is needed. It can be found by KCL at Node C:
  \[ I_{\text{entering}} = I_{\text{leaving}} \]
  \[ I_2 + I_4 = I_3 \quad \left\{ \begin{array}{c} I_3 = 3V_2 + 1 \quad [3] \\ = I_1 = 1 \quad \Rightarrow \quad V_2 \end{array} \right. \]

- Plugging [2] into [3]:
  \[ I_3 = 3(1) + 1 = 4 \text{ A} \quad [5] \]

  \[ V_3 = 4(2) = 8 \text{ V} \quad [6] \]

  \[ V_1 = 1 + 8 = \boxed{9 \text{ V}} \]
11. Find the voltage across \( R_3 \), \( V_3 \), as a function of the independent sources \( V_s \) and \( I_s \) for the circuit below, if \( R_1 = R_2 = R_3 = 2 \Omega \):

\[
\begin{align*}
\text{(1) } & V_s/2 + I_s \\
\text{(2) } & V_s/2 + 2I_s \\
\text{(3) } & 2V_s + I_s \\
\text{(4) } & V_s/3 + 2I_s \\
\text{(5) } & 2V_s/3 + 6I_s \\
\text{(6) } & V_s/2 + 2I_s \\
\text{(7) } & V_s + 6I_s \\
\text{(8) } & V_s/2 + 4I_s \\
\text{(9) } & 2V_s/3 + 2I_s \\
\text{(10) } & \text{None of the above}
\end{align*}
\]

To find the coefficients of \( V_s \) and \( I_s \), we need to calculate how each source impacts \( V_3 \) by itself. Thus, then we can apply superposition since voltage \( V_3 \) is linear across a resistor in a linear circuit.

- **Zero out \( I_s \):** Let \( I_s = 0 \) \( \to \) \( I_s \) branch is open

\[
\begin{align*}
V_s + V_3 &= 0 \\
\text{There is no current} & \text{ through } R_2, \text{ so it} \\
\text{acts like an open} & \text{ with the same voltage} \\
\text{at each node} & \text{ it is} \\
\text{connected to} & \text{(V=0)}
\end{align*}
\]

- \( R_1 \) \& \( R_3 \) are in series now, so we can use voltage division:

\[
V_3 = V_s \frac{R_3}{R_1 + R_3} = V_s \frac{2}{3} = \frac{V_s}{2} \quad [1]
\]

- **Zero out \( V_s \):** Let \( V_s = 0 \) \( \to \) \( V_s \) branch is shorted

Combine:

\[
V_3 = V_3 + V_3 = [1] + [2] = \frac{V_s}{2} + I_s
\]

By Ohm's Law, \( V_{32} = I_s (1) \quad [2] \)
12. Find the voltage $V_{out}$ (in Volts) across the resistor on the right, given a voltage source of 64 V.

You could approach this problem with source transformation, but it seems I may have to deal with some ugly fractions. Let's see if the linearity of the circuit makes this easy to solve. I will guess that $V_{out} = \frac{1}{2} V_{out,g} = 2 \text{ V}$ (making $I_{g} = 1 \text{ A}$).

So...

\[ I_{1,g} = 1 \text{ A} \quad \text{Ohm's law} \]

\[ I_{2,g} = I_{1,g} (2+2) = 4 \text{ V} \quad \text{Ohm's law} \]

\[ I_{3,g} = \frac{V_{g}}{4} = 1 \text{ A} \quad \text{Ohm's law} \]

\[ I_{3,1,g} = I_{3,g} + I_{2,g} = 2 \text{ A} \quad \text{KCL @ A} \]

\[ V_{B,g} = V_{A,g} + I_{3,1,g} (2) = 4 + 2(2) = 8 \text{ V} \quad \text{Ohm's law} \]

\[ I_{4,g} = 2 \text{ A} \quad \text{Ohm's law} \]

\[ I_{5,g} = 4 \text{ A} \quad \text{KCL @ B} \]

\[ V_{C,g} = 8 + 4(2) = 16 \text{ V} \quad \text{Ohm's law} \]

\[ I_{6,g} = \frac{16}{4} = 4 \text{ A} \quad \text{Ohm's law} \]

\[ I_{7,g} = 4 + 4 = 8 \text{ A} \quad \text{KCL @ C} \]

\[ V_{D,g} = 16 + 8(2) = 32 \text{ V} \quad \text{Ohm's law} \]

\[ I_{8,g} = \frac{32}{4} = 8 \text{ A} \quad \text{Ohm's law} \]

\[ I_{9,g} = 8 + 8 = 16 \text{ A} \quad \text{KCL @ D} \]

\[ V_{E,g} = 32 + 16(2) = 64 \text{ V} \quad \text{Ohm's law} \]

In actuality, $V_E = 64 \text{ V}$. Due to linearity, the actual $V_{out}$ is related to $V_{out,g}$ by:

\[ V_{out} = \frac{V_{E}}{V_{out,g}} \rightarrow V_{out} = \frac{64}{64} \rightarrow V_{out} = 2 \text{ V} \]