ECE 201 – Fall 2012
Exam #2
October 15, 2012
Division 0301: Bermel (11:30 am)

Instructions

1. DO NOT START UNTIL TOLD TO DO SO.
2. Write your name, division, professor, and student ID# on your scantron sheet.
3. This is a CLOSED BOOKS and CLOSED NOTES exam.
4. Calculators are allowed (but not necessary).
5. If extra paper is needed, use back of test pages.
6. Cheating will not be tolerated. Cheating in this exam will result in an F in the course.
7. If you cannot solve a question, be sure to look at the other ones and come back to it if time permits.
8. As described in the course syllabus, we must certify that every student who receives a passing grade in this course has satisfied each of the course outcomes. On this exam, you have the opportunity to satisfy outcomes iii and iv. (See the course syllabus for a complete description of each outcome.) On the chart below, we list the criteria we use for determining whether you have satisfied these course outcomes.

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<tr>
<th>Course Outcome</th>
<th>Exam Questions</th>
<th>Total Questions</th>
<th>Minimum # correct responses required to satisfy course outcome</th>
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<td>1-3</td>
<td>3</td>
<td>2</td>
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<tr>
<td>iv</td>
<td>4-13</td>
<td>10</td>
<td>5</td>
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</table>

If you fail to satisfy any of the course outcomes, don't panic. There will be more opportunities for you to do so.

9. You will find formulas on the final page of this exam. You can tear the page out if you want to.
1. For the circuit below on the left, find the Thevenin equivalent voltage source and resistance values (as shown on the right):

\[ R_{th} = \left( \frac{1}{300} + \frac{1}{120} + \frac{1}{600} \right) \]
\[ = 75 \Omega \]
\[ V_{oc} = R_{th}I_{sc} = 0 \]

Answers:
1. \( V_{oc} = 0 \) V; \( R_{th} = 75 \) Ω
2. \( V_{oc} = 6 \) V; \( R_{th} = 1020 \) Ω
3. \( V_{oc} = 12 \) V; \( R_{th} = 120 \) Ω
4. \( V_{oc} = 15 \) V; \( R_{th} = 180 \) Ω
5. \( V_{oc} = 30 \) V; \( R_{th} = 1020 \) Ω
6. \( V_{oc} = 54 \) V; \( R_{th} = 75 \) Ω
7. \( V_{oc} = 54 \) V; \( R_{th} = 120 \) Ω
8. None of the above

\[ I_{sc} = \frac{18}{300} - \frac{12}{120} + \frac{24}{600} = 0 \]

2. Given the circuit below on the left, find the Norton equivalent current source and resistance values (as shown on the right):

Answers:
1. \( I_{sc} = 0 \) A; \( R_{th} = 10/3 \) Ω
2. \( I_{sc} = 3 \) A; \( R_{th} = 5 \) Ω
3. \( I_{sc} = 7/2 \) A; \( R_{th} = 5 \) Ω
4. \( I_{sc} = 9/2 \) A; \( R_{th} = 10/3 \) Ω
5. \( I_{sc} = 9/2 \) A; \( R_{th} = 10 \) Ω
6. \( I_{sc} = 6 \) A; \( R_{th} = 10/3 \) Ω
7. \( I_{sc} = 6 \) A; \( R_{th} = 10 \) Ω
8. None of the above

At supernode:
\[ I_A + 3 + \frac{V_2}{4} = \frac{V_{AB}}{5} + \frac{V_{AB}}{10} \]

Since \( V_2 = \frac{3}{2} V_{AB} \):
\[ I_A + 3 + \frac{V_{AB}}{4} = \frac{V_{AB}}{5} + \frac{V_{AB}}{10} \]

\[ \therefore I_A = \frac{V_{AB}}{12} - I_{sc} \]

\[ \Rightarrow R_{th} = 5; \quad I_{sc} = 3 \]
3. If the following relationship between input current and output voltage is observed for a linear circuit, what are its Norton equivalent current source and resistance values?

<table>
<thead>
<tr>
<th>$I_A$</th>
<th>$V_{AB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>36</td>
</tr>
<tr>
<td>5</td>
<td>48</td>
</tr>
<tr>
<td>12</td>
<td>76</td>
</tr>
</tbody>
</table>

**Answers:**

1. $I_{sc} = 0$ A; $R_{th} = 18 \, \Omega$
2. $I_{sc} = 2$ A; $R_{th} = 9 \, \Omega$
3. $I_{sc} = 5$ A; $R_{th} = 4.8 \, \Omega$
4. $I_{sc} = 7$ A; $R_{th} = 4 \, \Omega$
5. $I_{sc} = 12$ A; $R_{th} = 3 \, \Omega$
6. $I_{sc} = 14$ A; $R_{th} = 3 \, \Omega$
7. $I_{sc} = 19$ A; $R_{th} = 2 \, \Omega$
8. None of the above

**If** $I_A = \frac{V_{AB} - I_{sc}}{R_{th}}$

Then $Z = \frac{36 - I_{sc}}{R_{th}}$

$\therefore \quad 5 = \frac{48 - I_{sc}}{R_{th}}$

$3 = \frac{12 - I_{sc}}{R_{th}} \rightarrow R_{th} = 4$

$I_{sc} = \frac{36 - 2 \cdot 7}{4}$

4. Find the equivalent inductance (in H) for the network of inductors shown below:

\[ L_{eq} = \frac{1}{L_1 + \frac{1}{L_{234}} + \frac{1}{L_{56}}} \]

\[ L_{234} = \left( \frac{1}{L_2 + \frac{1}{L_3} + \frac{1}{L_4}} \right)^{-1} = \left( \frac{1}{\frac{1}{32} + \frac{1}{80} + \frac{1}{160}} \right)^{-1} = 20 \]

\[ L_{56} = \left( \frac{1}{L_5} + \frac{1}{L_6} \right)^{-1} = \left( \frac{1}{20} + \frac{1}{80} \right)^{-1} = 16 \]

\[ L_{eq} = 70 + 20 + 16 = 106 \]
5. What is the equivalent capacitance of this network of capacitors with equal capacitances C, as measured between terminals A and B?

\[ C_{eq} = \frac{C}{2} \]

6. If \( I = 2\,\text{A} \) (i.e., \( I = 1\,\text{A} \) at \( t = 0 \) s, and \( I = 0.25\,\text{A} \) at \( t = 2 \) s), then how much stored energy (in J) is lost from the inductor between \( t = 0 \) and \( t = 1 \) s?

\[
U = \int_{0}^{1} L\frac{dI}{dt} \, dt
\]

\[
= \frac{1}{2} L \left( I_f^2 - I_i^2 \right)
\]

\[
= \frac{1}{2} \cdot 8 \left( \left(\frac{1}{2}\right)^2 - 1^2 \right)
\]

\[
= -3.5\,\text{J}
\]
7. For the diagram below, what is $V_C(t)$ (in V) for $t>0$? Note: $u(t)=1$ if $t>0$, and 0 otherwise.

$$V_C(t) = V_{oc} + (V_0 - V_{oc}) e^{-\frac{t}{\tau}}$$

Here, $V_C(0) = 0$

$V_{oc} = 40V$

$\tau = RC = 8 \times 10^{-3} \times 25 \times 10^{-6}$

$= 40(1 - e^{-5t})$

$= 200 \times 10^{-3} = 0.2s$

8. What is the value of the current through the inductor $I_L$ (in A) when $t=2$? Note: $u(t)=1$ if $t>0$, and 0 otherwise.
9. Find the voltage $V_c(t)$ (in V), for $t > 0$. Note: $u(t) = 1$ if $t > 0$, and $0$ otherwise.

$$V_{in} = -16u(-t) + 32u(t)$$

Answers:

1. $-16e^{-t/1.024}$
2. $16(1 - e^{-t/1.024})$
3. $32(1 - e^{-t/1.024})$
4. $32 - 48e^{-t/1.024}$
5. $32 - 48e^{-t/2.048}$
6. $32 - 64e^{-t/1.024}$
7. $64(1 - e^{-t/2.048})$
8. None of the above

$$V_c(t < 0) = -16 = V_c(0^+), \quad V_c(t \to \infty) = 32$$

$$\tau = RC = 64 \cdot 1.6 \cdot 10^{-2} = 1.024$$

$$V_c(t > 0) = 32 + (-16 - 32)e^{-t/\tau} = 32 - 48e^{-t/1.024}$$

10. How much energy (in J) will be stored in the capacitor from $t = 0$ to $t \to \infty$?

$$\begin{align*}
20u(t-2) \quad A \quad &\uparrow \\
10 \Omega \quad &\quad 150 \mu F
\end{align*}$$

Answers:

1. 1
2. 2
3. 3
4. 4
5. 5
6. 6
7. 7
8. None of the above

$$P = IV = C \frac{dV}{dt} V$$

$$U = \int P \, dt = \int C \frac{dV}{dt} V \, dt = \frac{1}{2} C (V_f^2 - V_i^2)$$

Since $V_i = 0$ and $V_f = 20 \cdot 10^2$, and $C = 1.5 \cdot 10^{-4} \text{F}$,

$$U = \frac{1}{2} \cdot 1.5 \cdot 10^{-4} (2 \cdot 10^3)^2 = 3$$
11. If the circuit below has an initial voltage across the inductor \( V_L(0) = 20 \text{ V} \), and an initial current \( I_L(0) = 0 \text{ A} \), find the current \( I_L(t) \) for \( t > 0 \):

\[
\omega_0 = \frac{1}{\sqrt{L C}} = \frac{1}{\sqrt{2 \cdot 0.2 \cdot 2 \cdot 10^{-3}}} = \frac{100}{2} = 50
\]

**General solution:**

\[
I_L(t) = A \cos \omega_0 t + B \sin \omega_0 t
\]

\[
V_L(0) = V_c(0) \times 20 = L \frac{dI_L}{dt}
\]

\[
B = \frac{20}{0.2 \cdot 50} = 2 = L \omega_0 \left( B \cos \omega_0 t - A \sin \omega_0 t \right)
\]

Since \( I_L(0) = 0 \), \( A = 0 \)

Answers:
1. 0.2 \( \sin(500t) \)
2. 0.2 \( \cos(500t) \)
3. \( \cos(50t) \)
4. 2 \( \sin(50t) \)
5. 10 \( \cos(50t) \)
6. 20 \( \cos(50t) \)
7. 20 \( \sin(50t) \)
8. None of the above

12. Find the frequency (in Hz) at which this undriven RLC circuit will oscillate:

\[
S_\pm = -\Gamma \pm \sqrt{\Gamma^2 - \omega_0^2}
\]

\[
\Gamma = \frac{R}{2L} = \frac{6}{2 \cdot 1} = 3
\]

\[
\omega_0 = \frac{1}{\sqrt{L C}} = \frac{1}{\sqrt{2 \cdot 10^{-3}}} = 5
\]

\[
S_\pm = -3 \pm \sqrt{3^2 - 5^2}
\]

\[
= -3 \pm 4i
\]

Answers:
1. 1
2. 2
3. 3
4. 4
5. 5
6. 6
7. 7
8. None of the above
13. Find the voltage across the capacitor $V_C$ (in V) as a function of time after the switch moves from left to right at $t=0$:

Answers:
(1) $5(1-e^{-\sqrt{2}})$
(2) $50(1-e^{\sqrt{2}})$
(3) $50e^{-\sqrt{2}}$
(4) $50(1-e^{-2t})$
(5) $50e^{-2t}$
(6) $50(1-e^{-20t})$
(7) $50e^{20t}$
(8) None of the above

For $t<0$: $V_C = 50V$
As $t \to \infty$: $V_C \to 0V$

$$T = \frac{1}{RC} = \frac{1}{(0.5 \Omega)(0.1 \text{F})} = 0.05 \text{s}$$
$$V_C(t > 0) = 50e^{-t/T} = 50e^{-20t}$$
Potentially Useful Formulas

\[ I = \frac{dQ}{dt} \]

\[ V = IR \text{ or } I = GV \]

\[ Q = CV \]

\[ V = L \frac{dl}{dt} \]

In series:

\[ R_{eq} = \sum_k R_k \]

\[ L_{eq} = \sum_k L_k \]

\[ \frac{1}{C_{eq}} = \sum_k \left( \frac{1}{C_k} \right) \]

\[ V_k = \frac{V R_k}{R_{eq}} \]

In parallel:

\[ \frac{1}{R_{eq}} = \sum_k \left( \frac{1}{R_k} \right) \]

\[ \frac{1}{L_{eq}} = \sum_k \left( \frac{1}{L_k} \right) \]

\[ C_{eq} = \sum_k C_k \]

\[ I_k = \frac{V}{R_k} = \frac{I R_{eq}}{R_k} \]

\[ P = I V \]

For RL and RC circuits:

\[ X = X_\infty + (X_0 - X_\infty) e^{-\frac{(t - t_0)}{\tau}} \]

For RL circuits: \( \tau = L/R \)

For RC circuits: \( \tau = RC \)

For series RLC circuits:

\[ s_\pm = -\Gamma \pm \sqrt{\Gamma^2 - \omega_0^2} \]

Where \( \Gamma = R/2L \) and \( \omega_0 = 1/(LC)^{1/2} \)