Solution

THIS EXAM SHOULD BE TAKEN ONLY BY STUDENTS REGISTERED FOR SECTION 2

October 24, 2012

ECE201 Linear Circuit Analysis I
Fall 2012, Section 2, MWF 7:30-8:20am
Exam 2

This exam corresponds to learning objective 2.

Solve the following problems. The number of points for each problem is shown in the table below.

Use only the space provided to solve each problem, and copy the answers to the space marked "Answer:..." Do not forget to specify units.

Show all the steps of your solution. Final answers alone will not be considered.

Non-integer answers can be written as \( a/b \) or as \( c.d \) where \( d \) is rounded to 2-3 digits.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>/ 30</td>
</tr>
<tr>
<td>2</td>
<td>/ 20</td>
</tr>
<tr>
<td>3</td>
<td>/ 30</td>
</tr>
<tr>
<td>4</td>
<td>/ 20</td>
</tr>
<tr>
<td>Total</td>
<td>/ 100</td>
</tr>
</tbody>
</table>
Reminder:

The energy absorbed by an inductor in the time interval \([t_0, t_1]\) is equal to
\[w_L(t_0, t_1) = \frac{1}{2}L[i_L^2(t_1) - i_L^2(t_0)].\]

The energy absorbed by a capacitor in the time interval \([t_0, t_1]\) is equal to
\[w_C(t_0, t_1) = \frac{1}{2}Cv_C^2(t_1) - \frac{1}{2}Cv_C^2(t_0).\]

The solution to \(\frac{dx(t)}{dt} + ax(t) = A\) is \(x(t) = \frac{A}{a} + Ke^{-at}\).

For a source-free RL circuit, \(i_L(t) = \frac{R}{L} (t-t_0)\).

For a source-free RC circuit, \(v_C(t) = v_C(t_0)e^{-\frac{t-t_0}{RC}}\).

For a series RL circuit with an independent voltage source \(V_s\), \(i_L(t) = \frac{V_s}{R} + \left[i_L(t_0) - \frac{V_s}{R}\right]e^{-\frac{R}{L}(t-t_0)}\).

For a series RC circuit with an independent voltage source \(V_s\), \(v_C(t) = V_s + [v_C(t_0) - V_s]e^{-\frac{t-t_0}{RC}}\).

For a voltage or current in an RC or RL first-order linear circuit with constant input,
\(x(t) = X_e + \left[x(t_0^+) - X_e\right]e^{-\frac{t-t_0}{\tau}}\).
Problem 1

Using nodal or loop analysis as learned in class, write a first-order differential equation for $i_L(t)$ in the following circuit. Express the equation in the standard form we have seen in class.

\[
\frac{\mathcal{L}_L - 3}{6} + i_L + \frac{\mathcal{L}_L}{3} - 1 = 0
\]

\[
0.5 \mathcal{L}_L + i_L = 1.5
\]

\[
0.5 \cdot 3 \cdot \frac{di_L}{dt} + i_L = 1.5
\]

\[
\frac{di_L}{dt} + \frac{2}{3} i_L = 1
\]

(30 points) Answer (differential equation):

\[
\frac{di_L}{dt} + \frac{2}{3} i_L = 1
\]
Problem 2

Analysis of an RC circuit showed that the voltage across a capacitor satisfies the equation:
\[
\frac{dv_C(t)}{dt} + 3v_C(t) = 6.
\]

It is also known that \(v_C(0) = 4\) V. Find \(v_C(t)\) for \(t \geq 0\).

\[
v_C(t) = \frac{6}{3} + k_2 e^{-3t}
\]

\[u = v_C(0) = 2 + k_2 \quad , \quad k_2 = 2\]

(20 points) Answer: \(v_C(t) = 2 + 2 e^{-3t} \quad \checkmark\)
Problem 3

For the purpose of computing $i_L(t)$, the circuit below is equivalent to a series RL circuit with an independent voltage source.

Draw the equivalent circuit and indicate the values of the inductor, the resistor, and the voltage source.

Assuming that $i_L(0) = 1$ A, use the equivalent circuit and any one of the methods we learned in class to find $i_L(t)$ for $t \geq 0$.

![Circuit Diagram]

\[ i + 1 = \frac{v}{3} + \frac{v-3}{3} = \frac{2v}{3} - 1 \]
\[ 2\frac{v}{3} = i + 2 \]
\[ v = \frac{3}{2} i + 2 \]
\[ u = \frac{3}{2} i + 2 \]

\[ 12 \text{th} \quad u_{oc} \]

Answer:

(20 points) Equivalent RL circuit:

\[ i_L(t) = \frac{V_s}{R} + \left[i_L(t_0) - \frac{V_s}{R}\right] e^{-\frac{V_s}{L}t} = \frac{3}{1.5} + \left(1 - \frac{3}{1.5}\right) e^{-\frac{1.5}{L}t} \]

(10 points) $i_L(t) = 2 - e^{-1.5t} \text{ A}$
Problem 4

In the following circuit, \( v_s(t) = 10u(t)V \), \( i_s(t) = 2u(t)A \), \( i_L(0) = 5A \), \( R_1 = 10\Omega \), \( R_2 = 20/3\Omega \) and \( L = 2H \). Analysis of the circuit yielded the equation

\[
i_L(t) = 3+2e^{-2t} \text{ A for } t \geq 0.
\]

Find \( v_L(t) \), \( v_{R1}(t) \), \( i_{R1}(t) \) and \( i_{R2}(t) \) for \( t > 0 \).

\[
\begin{align*}
\mathcal{U}_L(t) &= 2 \frac{d}{dt} \left( 3+2e^{-2t} \right) = -8e^{-2t} \text{ V} \\
\mathcal{U}_{R1}(t) &= v_s(t) - \mathcal{U}_L(t) = 10 + 8e^{-2t} \text{ V} \\
i_{R1}(t) &= \frac{\mathcal{U}_{R1}(t)}{10} = 1+0.8e^{-2t} \text{ A} \\
i_{R2}(t) &= \frac{\mathcal{U}_L(t)}{20/3} = -8 \cdot \frac{3}{20} e^{-2t} = -1.2e^{-2t} \text{ A}
\end{align*}
\]

Answer:

(5 points) \( v_L(t) = -8e^{-2t} \text{ V} \)

(5 points) \( v_{R1}(t) = 10+8e^{-2t} \text{ V} \)

(5 points) \( i_{R1}(t) = 1+0.8e^{-2t} \text{ A} \)

(5 points) \( i_{R2}(t) = -1.2e^{-2t} \text{ A} \)