ECE 20100 –Fall 2014
Exam #2
October 22, 2014

Section (circle below)

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Name Solutions PUID

Instructions

1. DO NOT START UNTIL TOLD TO DO SO.
2. Write your name, section, professor, and student ID# on your Scantron sheet. We may check PUIDs.
3. This is a CLOSED BOOKS and CLOSED NOTES exam.
4. Calculators are NOT allowed (and not necessary).
5. If extra paper is needed, use the back of test pages.
6. Cheating will not be tolerated. Cheating in this exam will result in, at the minimum, an F grade for the course. In particular, continuing to write after the exam time is up is regarded as cheating.
7. If you cannot solve a question, be sure to look at the other ones, and come back to it if time permits.
8. Exam #2 provides evidence for satisfaction of this ECE 20100 Learning Objective:
   ii) An ability to analyze 1st order linear circuits with sources and/or passive elements.
   The minimum score needed to satisfy this objective will be posted on Blackboard after the exam has been graded. Remediation options will be posted in Blackboard if you fail to satisfy any of the course outcomes.
9. The last page of the exam contains potentially useful formulas.

Honor Pledge: I have neither given nor received unauthorized assistance on this exam

Signature: ________________________________
1. For the Thevenin equivalent circuit seen by the load resistor $R_L$, what is the Thevenin equivalent voltage $V_{TH}$?

\[ V_{TH} = \begin{align*} 
(1) & \quad 0 \text{ V} \\
(2) & \quad -3 \text{ V} \\
(3) & \quad -6 \text{ V} \\
(4) & \quad -9 \text{ V} \\
(5) & \quad -12 \text{ V} \\
(6) & \quad -15 \text{ V} \\
(7) & \quad -18 \text{ V} \\
(8) & \quad -\infty \text{ V} \\
(9) & \quad \text{None of the above}
\end{align*} \]

$V_{TH}$ is $V_{oc}$, the voltage across $A \& B$ when $R_L$ is replaced by an open:

\[ \begin{array}{c}
\text{2 A} \\
3 \text{ A} \\
\text{I}_y \text{ (4 A)} \\
\text{V}_{oc}
\end{array} \]

* $V_{TH} = V_{oc} = V_{AB}$ is the voltage across the $4 \text{ A}$ resistor:
  1. $V_{AB} = I_y \times (4 \text{ A})$ → Ohm's law (with $I_y$ & $V_{AB}$ in passive notation)
  2. $I_y = -3 \text{ A}$ → because $4 \text{ A}$ resistor is in series with $3 \text{ A}$ source

\[ \begin{array}{c}
\text{1} \rightarrow \text{2} \\
V_{AB} = -3 \times 4 = -12 \text{ V}
\end{array} \]

\[ \therefore V_{TH} = -12 \text{ V} \]
2. For the Norton equivalent circuit seen by the inductor $I_i$, what is the Norton equivalent resistance $R_N$?

$$3 \text{ V}$$

$$2 \Omega$$

$$+$$

$$-$$

$$1 \Omega$$

$$+$$

$$-$$

$$2v_x$$

$$2 \Omega$$

$$L_1 \rightarrow I_N$$

$$R_N$$

(1) $R_N = 1 \Omega$
(2) $R_N = 2 \Omega$
(3) $R_N = 3 \Omega$
(4) $R_N = 4 \Omega$
(5) $R_N = 5 \Omega$
(6) $R_N = 6 \Omega$
(7) $R_N = 7 \Omega$
(8) $R_N = 0 \Omega$
(9) $R_N = \infty \Omega$
(10) None of the above

* To find $R_N$, zero all independent source and replace $L_1$ by a test source. $R_N = \frac{V_T}{I_T}$

* I choose a 1A source (arbitrarily) for my test source:

$$2 \Omega$$

$$+$$

$$-$$

$$2v_x$$

$$2 \Omega$$

$$V_T$$

$$1A$$

$V_T$ is voltage across 2Ω resistor @ right (parallel w/ 1A source)

$\therefore V_T = i_2 (2) \quad \rightarrow \quad $i's law

Due to KCL @ A:

$0 = i_2 = 2v_x + 1$

Because there is no source on the left side of the circuit,

$V_X = i_1 (1) \quad \rightarrow \quad $i's law

$= 0 (1) = 0$

Plug ③ into ②:

$0 = 0 + 1 = \boxed{1}$

Plug ④ into ①:

$V_T = (1)(2) = 2$

$\therefore R_N = \frac{V_T}{I_T} = \frac{2}{1} = 2 \Omega$
3. What value of resistor $R_L$ will result in the maximum power being absorbed by $R_L$?

- Max power absorbed by $R_L$ when $R_L = R_{TH}$, where $R_{TH}$ is the Thevenin equivalent resistance of the circuit to which $R_L$ is connected.
- Thevenin equivalent resistance is found by zeroing out independent resistance and using resistor combination (since there are no dependent sources).

$I_x$ is $OA$ since it is in series with an open.

\[ V_A = V_C \] due to $\text{E}^\text{L}'s$ law:

The only thing between $A$ & $B$ is a $2 \Omega$ resistor.

\[ R_L = R_{TH} = R_{eq} = 2 \Omega \]

For maximum power absorbed by $R_L$. 

\[ 2 \Omega \]
4. Find the equivalent capacitance seen between nodes A and B, in terms of capacitance \(C\).

\[
\begin{align*}
(1) & \quad 0.5C \\
(2) & \quad 2C \\
(3) & \quad 2.5C \\
(4) & \quad 4C \\
(5) & \quad 4.5C \\
(6) & \quad 6C \\
(7) & \quad 6.5C \\
(8) & \quad 8C \\
(9) & \quad 8.5C \\
(10) & \quad \text{None of the above}
\end{align*}
\]

- \(\frac{1}{C}\) combines like \(R\)
- \(C_4\) and \(C_5\) are connected to node D and nothing else is. Therefore, \(C_4\) and \(C_5\) are in series:
  \[R_{\text{in series}} \rightarrow R_{eq} = R_4 + R_5\]
  \[C_{\text{in series}} \rightarrow \frac{1}{C_{eq}} = \frac{1}{C_4} + \frac{1}{C_5}\]
  \[\therefore C_6 = (\frac{1}{C_4} + \frac{1}{C_5})^{-1} = 0.5\,C\]

- So now,

- \(C_1, C_2, C_3,\) and \(C_6\) are all connected to both node A and node B. Therefore \(C_1, C_2, C_3,\) and \(C_6\) are in parallel.
  \[R_{\text{in parallel}} \rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_6}\]
  \[C_{\text{in parallel}} \rightarrow C_{eq} = C_1 + C_2 + C_3 + C_6\]
  \[\therefore C_{AB} = 2C + 2C + 2C + 0.5C = 6.5\,C\]
5. What is the voltage $v_4(t)$ across the 4 H inductor?

- $i_s(t) = 4t$ A
- $v_5(t)$ 4 H
- $v_4(t)$
- $6$ H
- $6$ H

(1) 2 V  (2) 4 V  (3) 8 V  (4) 16 V  (5) 2t V  
(6) 4t V  (7) 8t V  (8) 16t V  (9) 0 V  (10) None of the above

- $v_4(t) = v_5(t)$ because they are parallel
- If I find $\text{Leq} = L_{AB}$ then
  
  $v_5(t) = \text{Leq} \left( \frac{d i_{\text{eq}}}{dt} \right) = \text{Leq} \frac{d i_s}{dt}$

  \[ \because \text{Leq and } i_s \text{ source are in series} \]

- Finding $\text{Leq}$: $L$ combines like $R$

  \[ \therefore \text{Leq} : \frac{4}{(1 + 6/6)} = \frac{4}{(1 + 3)} = \frac{4}{4} = 2 \ H \]

- Plug 2 into 0:

  \[ v_4 = v_5 = 2 \left( \frac{d}{dt} (4t) \right) = 2(4) = 8 \ V \]

\[ \therefore v_4(t) = 8 \ V \]
6. A 2 μF capacitor is charged to a voltage of 20 V in 50 ms. Assuming the energy stored in the capacitor was initially 0 J, what is the energy stored in the capacitor at time \( t = 50 \text{ ms} \) ?

\[
\begin{array}{c}
\begin{array}{c}
\text{2 \( \mu \)F} \\
\uparrow \\

v_C(50 \text{ ms}) = 20 \text{ V}
\end{array}
\end{array}
\]

\[
(1) \ W_C = 0 \text{ J} \\
(2) \ W_C = 20 \text{ mJ} \\
(3) \ W_C = 40 \text{ mJ} \\
(4) \ W_C = 80 \text{ mJ} \\
(5) \ W_C = 100 \text{ µJ} \\
(6) \ W_C = 200 \text{ µJ} \\
(7) \ W_C = 300 \text{ µJ} \\
(8) \ W_C = 400 \text{ µJ}
\]

(9) None of the above

- Total energy stored in a capacitor at time \( t \) is

\[
\mathcal{W}_C(t) = \frac{1}{2} C v^2(t) \]

- In this problem,

\[
t = 50 \text{ ms} = 50 \times 10^{-3} \text{s} \\
v_C(t) = v_C(50 \text{ ms}) = 20 \text{ V} \\
C = 2 \mu \text{F} = 2 \times 10^{-6} \text{ F}
\]

- Plugging those into \( \mathcal{W}_C(t) \):

\[
\mathcal{W}_C(50 \text{ ms}) = \frac{1}{2} (2 \times 10^{-6})(20)^2
\]

\[
\therefore \mathcal{W}_C(50 \text{ ms}) = 400 \times 10^{-6} = 400 \mu\text{J}
\]
7. A 9 mF capacitor has an initial voltage of $v_c(0) = 5$ V. If the current through the capacitor is given by $i_c(t) = 9 - t^2$ mA for $t \geq 0$ s, what is $v_c(3)$?

![Circuit Diagram]

(1) 1 V  (2) 2 V  (3) 3 V  (4) 4 V  (5) 5 V  
(6) 6 V  (7) 7 V  (8) 8 V  (9) 9 V  (10) None of these

- If the current through a capacitor is known, the voltage drop across it is:
  \[ v_c(t) = v_c(t_0) + \frac{1}{C} \int_{t_0}^{t} i_c(t') \, dt' \]
  for $v_c$ and $i_c$ in passive notation

  for $t \leq t_0$, if $v_c(t_0)$ is known (and, obviously, if the circuit doesn't change between $t_0$ and $t$)

- For our circuit,
  
  $t_0 = 0$

  $v_c(0) = v_c(t_0) = 5$ V

  $C = 9 \text{ mF} = 9 \times 10^{-3} \text{ F}$

  $i_c(t') = 9 - t'^2$ mA = $(9 - t'^2) \times 10^{-3}$

- Plugging these into 0, for $t \geq 0$:
  \[ v_c(t) = 5 + \frac{1}{9 \times 10^{-3}} \int_{0}^{t} 10^{-2} \times (9 - t'^2) \, dt' \]

  \[ = 5 + \frac{1}{9} (9t' - \frac{1}{3} t'^3) \bigg|_{t'=0}^{t} \]

  \[ = 5 + \frac{1}{9} (9t - \frac{1}{3} t^3) = 5 + \frac{3}{27} t^3 \]

  \[ \therefore v_c(3) = 5 + 3 - \frac{(3)^3}{27} = 7 \text{ V} \]
Use this circuit for questions 8, 9, and 10. The switch is closed for a long time and then opened at \( t = 1 \) s. At \( t = 0^- \), the inductor current is known to be \( i_L(0^-) = 0 \) A.

8. For \( 0 \leq t < 1 \) s, what is the time constant, \( \tau \)?

\[
\begin{align*}
(1) \ \tau &= 0 \ s \\
(2) \ \tau &= \frac{1}{4} \ s \\
(3) \ \tau &= \frac{1}{2} \ s \\
(4) \ \tau &= 1 \ s \\
(5) \ \tau &= 2 \ s \\
(6) \ \tau &= 4 \ s \\
(7) \ \tau &= 8 \ s \\
(8) \ \tau &= 16 \ s \\
(9) \ \tau &= \infty \ s \\
(10) \ \text{None of the above}
\end{align*}
\]

- \( \tau \) for an inductor is \( \frac{L}{R_{\text{Th}}} \) where \( R_{\text{Th}} \) is the Thevenin equivalent resistance of the circuit to which \( L \) is connected.
- For \( 0 \leq t < 1 \) s, the circuit connected to \( A \) and \( B \) looks like (with the independent sources zeroed):

\[
\begin{align*}
&\text{(4) } R_{\text{Th}} = 4 \\
\therefore \quad R_{\text{AB}} &= R_{\text{Th}} = 4 \ \Omega \\
\therefore \quad \tau &= \frac{L}{R_{\text{Th}}} = \frac{2 \Omega}{4 \ \Omega} = 0.5 \ s
\end{align*}
\]
Use this circuit for questions 8, 9, and 10. The switch is closed for a long time and then opened at \( t = 1 \) s. At \( t = 0^- \), the inductor current is known to be \( i_L(0^-) = 0 \) A.

\[ 12u(t) \text{ V} \]

\[ 4 \Omega \]

\[ t = 1 \text{ s} \]

\[ 2 \text{ H} \]

\[ 4 \Omega \]

\[ i_L \]

9. For \( t \geq 1 \) s, what is the time constant, \( \tau \)?

(1) \( \tau = 0 \) s  
(2) \( \tau = \frac{1}{4} \) s  
(3) \( \tau = \frac{1}{2} \) s  
(4) \( \tau = 1 \) s  
(5) \( \tau = 2 \) s  
(6) \( \tau = 4 \) s  
(7) \( \tau = 8 \) s  
(8) \( \tau = 16 \) s  
(9) \( \tau = \infty \) s  
(10) None of the above

- Trying to find \( \tau = \frac{L}{R_{TH}} \)

- At \( t \geq 1 \) s, with independent sources zeroed, circuit to left of \( A-B \) looks like:

\[ R_{TH} = R_{AB} = 4 \Omega \]

\[ \therefore \tau = \frac{L}{R_{TH}} = \frac{2 \text{ H}}{4 \Omega} = \frac{1}{2} \text{ s} \]
Use this circuit for questions 8, 9, and 10. The switch is closed for a long time and then opened at \( t = 1 \text{ s} \). At \( t = 0^- \), the inductor current is known to be \( i_L(0^-) = 0 \text{ A} \).

10. As \( t \to \infty \), what is \( i_L(\infty) \)?

(1) \( i_L(\infty) = 1 \text{ A} \)  (2) \( i_L(\infty) = 2 \text{ A} \)  (3) \( i_L(\infty) = 3 \text{ A} \)  (4) \( i_L(\infty) = 4 \text{ A} \)

(5) \( i_L(\infty) = 5 \text{ A} \)  (6) \( i_L(\infty) = 6 \text{ A} \)  (7) \( i_L(\infty) = 8 \text{ A} \)  (8) \( i_L(\infty) = 12 \text{ A} \)

(9) \( i_L(\infty) = 0 \text{ A} \)  (10) None of the above

* In the time interval of \( t \to \infty \), the circuit looks like:

* \( i_L(\infty) \) is \( i_L \) if the inductor is replaced by a short (in the circuit of this time interval):

* \( i_L \) is same as current through the 4 \( \Omega \) resistor:

\[
\frac{12 \text{ A}}{4 \Omega} = 3 \text{ A}
\]

\( \therefore i_L(\infty) = 3 \text{ A} \) for \( t \to \infty \).
11. The current through an inductor was kept at \( i_L(t) = 8 \) A for all time until \( t = 0^- \). The inductor was then connected to the circuit as shown below at \( t = 0 \). Find the current through the inductor, \( i_L(t) \), for \( t \geq 2 \) s.

\[
\text{\begin{tikzpicture}
    \node at (0,0) [pair] {$\text{4} u(t-2) \text{ A}$};
    \node at (2,0) [current] {$i_L$};
    \node at (4,0) [resistor] {$4 \ \Omega$};
    \node at (6,0) [inductor] {$2 \ \text{H}$};
\end{tikzpicture}}
\]

(1) \( i_L(t) = 1 - e^{-2t} - 8e^{-2t-4} \) A

(2) \( i_L(t) = 1 - e^{-2t} + 8e^{-2t-4} \) A

(3) \( i_L(t) = 1 - 8e^{-2t} - e^{-2t+4} \) A

(4) \( i_L(t) = 1 + 8e^{-2t} - e^{-2t+4} \) A

(5) \( i_L(t) = 4 - 4e^{-2t} - 8e^{-2t-4} \) A

(6) \( i_L(t) = 4 - 4e^{-2t} + 8e^{-2t-4} \) A

(7) \( i_L(t) = 4 - 8e^{-2t} - 4e^{-2t+4} \) A

(8) \( i_L(t) = 4 + 8e^{-2t} - 4e^{-2t+4} \) A

(9) None of the above

- During a time interval when the circuit does not change (beginning at \( t = 0 \)),
  \[ i_L(t) = i_L(0) = i_L(0^+) \]
  \[ i_L(t) = \left( i_L(0^-) - i_L(0^+) \right) e^{-\frac{t}{\tau}} \]

- For \( 0 \leq t \leq 2.5 \): Circuit same for \( t = 0 \) to \( t = 2.5 \)
  \[ 4A \begin{array}{c}
      \text{\begin{tikzpicture}
          \node at (0,0) [pair] {$\text{4} $};
        \end{tikzpicture}}
    \end{array} \]
  \[ \begin{array}{c}
      \text{\begin{tikzpicture}
        \node at (0,0) [inductor] {$2 \ \text{H}$};
        \node at (2,0) [current] {$i_L$};
      \end{tikzpicture}}
    \end{array} \]
  \[ \text{current source zero} \]
  \[ t_o = 0 \]
  \[ i_L(0^-) = i_L(0) \text{ short} = 0 \]

- For \( t \geq 2.5 \): Circuit same for all time after \( t = 2.5 \)
  \[ 4A \begin{array}{c}
      \text{\begin{tikzpicture}
          \node at (0,0) [pair] {$\text{4} $};
        \end{tikzpicture}}
    \end{array} \]
  \[ \begin{array}{c}
      \text{\begin{tikzpicture}
        \node at (0,0) [inductor] {$2 \ \text{H}$};
        \node at (2,0) [current] {$i_L$};
      \end{tikzpicture}}
    \end{array} \]
  \[ \text{current source on} \]
  \[ t_o = 2 \]
  \[ i_L(0^-) = i_L(0) \text{ short} = 4 \text{A} \]

- Replace \( i \) by \( 4 \text{A} \)
  \[ i_L(2^+) = i_L(2^-) \]
  \[ i_L(t) = 8e^{-2t} \]

- \( \tau = \frac{L}{R} = \frac{2}{4} = \frac{1}{2} \text{ s} \)

- \[ i_L(t) = 4 + (8e^{-4t-4}) e^{-\frac{t-2}{\frac{1}{2}}} \]

\[ i(t) = 4 + 8e^{-2t-4} e^{-2t+4} \text{ A, } t \geq 2.5 \]
12. A capacitor is charged to \( v_c(0^-) = 4 \text{ V} \). At \( t = 0 \), the switch in the circuit below is closed.

At what time \( t_i \) in seconds will the capacitor voltage be \( v_c(t_i) = 3 \text{ V} \)?

\[ \begin{align*}
(1) & \quad \frac{1}{2} \ln \frac{5}{2} \\
(2) & \quad \frac{1}{2} \ln 10 \\
(3) & \quad 2 \ln \frac{5}{2} \\
(4) & \quad 2 \ln 10 \\
(5) & \quad \frac{5}{2} \ln \frac{1}{2} \\
(6) & \quad \frac{5}{2} \ln 2 \\
(7) & \quad 10 \ln \frac{1}{2} \\
(8) & \quad 10 \ln 2 \\
(9) & \quad \text{None of the above}
\end{align*} \]

• During a time interval when the circuit does not change (beginning @ \( t = t_0 \)):

\[ v_c(t) = v_c(\infty) + (v_c(t_0) - v_c(\infty)) e^{-\frac{t-t_0}{\tau}} \]

• For \( t > 0 \), the circuit stays the same:

\[ \begin{align*}
& v_o(0^-) = 2 \text{ V} \\
& v_o(0^+) = 4 \text{ V} \\
& \tau = R \cdot C = 2 \cdot \left( \frac{5}{2} \right) = 10 \text{ s} \\
& \text{Zero-out 1A source}
\end{align*} \n
\[ v_c(t) = 2 + (4 - 2) e^{-\frac{t}{\tau}} = 2 + 2 e^{-\frac{t}{10}} \text{ V} \]

• Want to find \( t_i \) such that \( v_c(t_i) = 3 \text{ V} \). Use (9):

\[ v_c(t_i) = 2 + 2 e^{-\frac{t_i}{10}} = 3 \quad e^{-\frac{t_i}{10}} = \frac{1}{2} \quad \rightarrow \quad \frac{t_i}{10} = \ln 2 \quad \text{or} \quad 0 \quad \text{or} \quad t_i = -10 \ln 2 \]

\( t_i = 10 \ln 2 \)
Potentially Useful Formulas

\[ x(t) = x(\infty) + \left[ x(t_0^+) - x(\infty) \right] e^{-(t-t_0)/\tau}, \quad \text{where} \quad \tau = R_{\text{th}} C \quad \text{or} \quad \tau = \frac{L}{R_{\text{th}}} \]

\[ v_L(t) = L \frac{di_L(t)}{dt} \]

\[ i_L(t) = i_L(t_0) + \frac{1}{L} \int_{t_0}^{t} v_L(t') dt' \]

\[ W_L(t_0, t_1) = \frac{L}{2} \left[ (i_L(t_1))^2 - (i_L(t_0))^2 \right] \]

\[ i_C(t) = C \frac{dv_C(t)}{dt} \]

\[ v_C(t) = v_C(t_0) + \frac{1}{C} \int_{t_0}^{t} i_C(t') dt' \]

\[ W_C(t_0, t_1) = \frac{C}{2} \left[ (v_C(t_1))^2 - (v_C(t_0))^2 \right] \]

\[ -\ln x = \ln \frac{1}{x} \]